

**Engineering Thermodynamics**  
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**Lecture 15**  
**Quasi-equilibrium, moving boundary work**

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**Learning objective**

- Examine the moving boundary work or  $P dV$  work commonly encountered in reciprocating devices such as automotive engines and compressors.
- Identify the first law of thermodynamics as simply a statement of the conservation of energy principle for closed (fixed mass) systems.
- Develop the general energy balance applied to closed systems.
- Define the specific heat at constant volume and the specific heat at constant pressure.
- Relate the specific heats to the calculation of the changes in internal energy and enthalpy of ideal gases.
- Describe incompressible substances and determine the changes in their internal energy and enthalpy.
- Solve energy balance problems for closed (fixed mass) systems that involve heat and work interactions for general pure substances, ideal gases, and incompressible substances.

Welcome back. Now we are going to start a new topic. It is energy analysis of closed system. So this is the objective of this particular topic. The first we are going to discuss  $P dV$  of boundary work and some aspect of the work we have already covered. So let us go and do a formal introduction to work, particularly the boundary work.

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**Work**

Work is usually defined as a force  $F$  acting through a displacement  $x$  where the displacement is in the direction of the force.

$W = \int_1^2 F dx$

Work is done by a system if the sole effect on the surroundings (everything external to the system) could be the raising of a weight.

Work is a form of energy in transit, recognized at the boundary.

So that work is usually defined in terms of force acting through a distance. So this is a formal definition which we have used many times in simple mechanics, however in thermodynamics it is more convenient if we can make or connect this definition into something related to process, properties, thermodynamic definition of work which is defined in this form that work is done by a system if the sole effect on the surrounding that is the sounding means everything outside to the system could be the raising of a weight.

So essentially what it means that the effective work done by a system to the surrounding can be represented as effective raising of weight even though we may not directly use any weight. So the definition says that only the effect could be raising of a weight. But does not say specifically that we have to physically take a weight, ok? So we will try to explain this excerpts by taking an example.

We already know that work is a form of energy in transit, that means it is recognised at the boundary, ok? And the system do not contain work. And that is for the true for the heat as well. It only recognises the boundary.

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**Work**

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*Work is done by a system if the sole effect on the surroundings (everything external to the system) could be the raising of a weight.*

Work is a form of energy in transit, recognized at the boundary.

(a) (b)

System boundary

Now just take an example, ok? this is an example of a battery motor which drives the fan. Now if you look at it carefully at the boundary, if you ask the question whether the work crosses the boundary or for that matter, is there any work done for such a system?

And how do you explain? Because you cannot even see that the force effectively has acted through a distance. So what you can do is you can replace this fan by a system, replace with pulley and weight. And then whence the motor drives, it is essentially raise this particular weight. So the sole effect of this particular system work acting on the surrounding is the raising of this particular weight. Ok so essentially this is how we are going to analyse the thermodynamic work.

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**Work**

*ideal frictionless piston*

*The general differential defn*

$$\delta W = F_{ext} dl$$

$$= P_{ext} A dl$$

$$= P_{ext} dV$$

*$P_{ext} = P_{int}?$*

$P_{ext}$  need not be equal to  $P_{int} = P = F/A$

Limiting case:  $P_{ext}$  and  $P_{int}$  are nearly equal, but one is infinitesimally larger to accomplish a net change in volume: quasi-static equilibrium

Only for such case  $P = P_{ext}$  and  $\delta W = P dV$

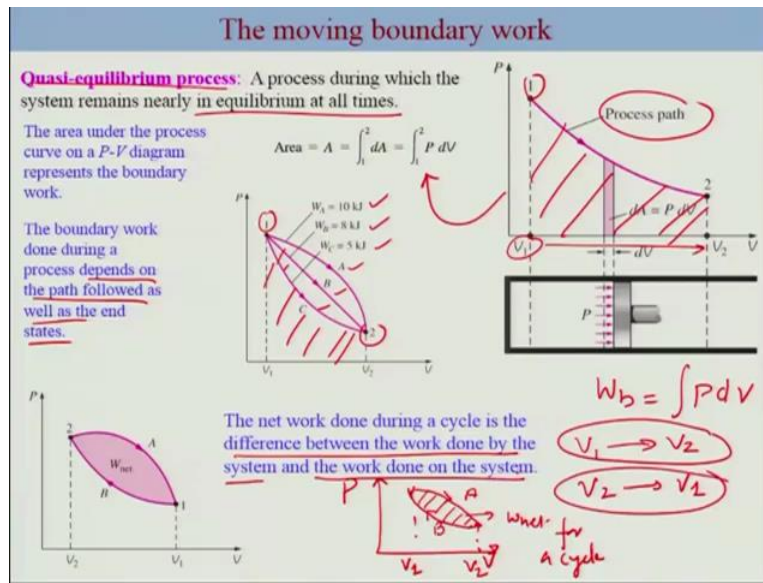
So let us take a specifically a system here, this is a system where you have a  $F$  external applied to ideal frictionless piston. Ok, having an area  $A$ . So this  $F$  external is basically the force which would be needed by the surrounding in order to compress let us say the gas and reduce the volume by half or this particular force will be felt by the surrounding as the gas expands.

Now we can write the work done by the system by writing this the general differential definition. This is a general differential definition of this mechanical work which we write in this form. Ok? This external force, that is the force which this posing the resistance for this particular boundary to move. And the  $dl$  is a change in the height of the container or rather the gas volume. So  $dl$  would be distance typically raised here, for example this could be a  $dl$ . So in general we defined the work in this form.

The considering the area of the piston, the pressure you can relate this work in terms of pressure. This could be related by in this form,  $P_{ext}$  into  $A dl$  or in other words this  $A$  is constant so this is simply  $P_{ext} dV$ . Now we also have this  $P_{int}$ , so is this  $P_{ext}$  equal to  $P_{int}$ ? And the answer is not necessary, ok? There is only specific case when  $P_{ext}$  is equal to  $P_{int}$  when the system is extremely slow or that is what we call it quasi static equilibrium.

In that case, your P external and P internal are nearly equal but one of them should be infinitesimally larger so that there is a change in the volume in the specific direction. So this is what the approximation is quasi static equilibrium. In such case, your P internal would be same as P external. Ok? And then we can write this particular work expression in this form.

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Now for the case of a quasi static equilibrium, we can refer this expression more formally in terms of boundary work. Ok? So this we are using the same expression. Now this is again the same piston cylinder, ok? And the work associated with the movement of the gas which does a differential boundary work is referred to be  $W$  subscript b here. Ok, which essentially means is a boundary work.

This is nothing but the same as a force into distance, ok as seen here. Now for the case of a quasi static equilibrium we can write  $F$  as simply  $P$  into  $A$  because  $P$  external is same as  $P$  internal in this case and this is the expression for the boundary work which we can make use of it. Now for the case of expansion, that means when the gas expands, the boundary work will be considered as a positive as our convention and then it (( ))(6.19) that means when the system does a work on the surrounding, our  $W_b$  or the boundary work will be positive.

And when the surrounding does work on the system that means a compression in this case, the boundary work would be negative. So this is what the sign convention would be for such a case.

Ok, so as I mentioned already that for quasi equilibrium process the system will remain almost or nearly in equilibrium at all times. In that case you can represent the process in this by a process path represented by this line here.

Ok, this is an example where volume is expanding, going from  $V_1$  till  $V_2$  in a very slow process, such that the equilibrium is maintained and thus the  $W_b$ , in this case can be written as in this form which is nothing but the area under the curve, ok? And that is what it is mentioned here. Now we have mentioned it earlier during the introduction of the particular course that the boundary work done during a process depends on the path followed as well as the end point.

So in this case, if you follow, through different path, which is represented by path A, B, C, they will have different work, basically the area under the curve would be different as you can clearly see from here. So this area will be less than the other area which is covered by the path B. So thus you have, A will be the having the maximum work, B would be lesser and C would be least among the three possible paths for this particular example.

Now if you consider process where you have expanded the system, in the piston gas cylinder case from  $V_1$  to  $V_2$ . So this would be the work done on the surrounding, that is work done by the system on surrounding. So and then you can also bring back the system to the original state by doing by performing a work done on the system. That means surrounding is going to do a work on the system in the reverse process. So that would be from  $V_2$  to  $V_1$ .

Ok. So this is a one path and this is a another path. So in this case if we consider a system which we have described (( ))(8.39) process, so the one let us say we are starting from, these are the final state represented by  $V_2$  here is larger volume, so this is  $V_2$  and this is  $V_1$ . So initially you have gone, the first process or the path is from  $V_1$  to  $V_2$ , so that let us say this is our system A, ok?

And then you compress the system back to  $V_1$  that the surrounding has worked on the system and this is B. So the net work could be the work done by the system minus work done on the system which essentially will simply mean the area under this particular true curve. So that is what would be the net work for a cyclic process of a cycle because we have brought back the system using two paths.

First is you went from one to two, you on path A and then you have returned back to again on using path B, but then the surrounding has worked on the system. So essentially the total work or the net work could be simply the area the difference between the work done by the system and the work done on the system.

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**Example**

A piston–cylinder device initially contains  $0.4 \text{ m}^3$  of air at  $100 \text{ kPa}$  and  $80^\circ\text{C}$ . The air is now compressed to  $0.1 \text{ m}^3$  in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process

- Quasi-equilibrium process
- Air is an ideal gas  $PV = mRT_0 = c = P_1 v_1$

$$W_b = \int_1^2 P dv = \int_1^2 \frac{c}{v} dv = c \ln \frac{v_2}{v_1} = P_1 v_1 \ln \frac{v_2}{v_1}$$

$$= (100 \text{ kPa})(0.4 \text{ m}^3) \ln \frac{0.1}{0.4} = -55.5 \text{ kJ}$$

work done on the syst

Ok, so let us try to do some example here, so this is an example of a piston cylinder device, ok? Which initially contains this volume of Air at 100 kiloPascal, the temperature is already given and the air is now compressed to 0.1 meter cubed. So it is compressed such that the volume decreases in such a way that the temperature inside the cylinder remain constant (( ))(10.20) to determine the work done during this process.

Ok, so this is a process path so you have P and essentially you are at 1 at higher volume and you have compressed it keeping the T 0 or the temperature constant to T 2. So naturally if you think about the boundary work, this should be the simply the work of done on this systems. So but let us do thorough calculation here, so we will assume this quasi equilibrium process, ok? And we are assuming air to be ideal gas because we have to make use of some equation here, equation of state in this case.

And we can write ideal gas in this form, the equation is written as  $m R T$ , ok? Since the temperature is constant that means T is equal to T 0, this is nothing as constant, mass does not



change. In this case your R is constant and T 0 is constant. So P V is nothing but some constant. So we can write P as C by V, right?

Now what this boundary work? Boundary work is  $\int_1^2 P dV$  and this we can write as  $C \int_1^2 \frac{1}{V} dV$  or  $C \ln \frac{V_2}{V_1}$ . Now what is C? C can be written as  $P_1 V_1$ , so this is nothing but  $P_1 V_1 \ln \frac{V_2}{V_1}$ . Now you can plug in the values which are given already to you because you know the pressure, you know the initial volume, you know also the final volume.

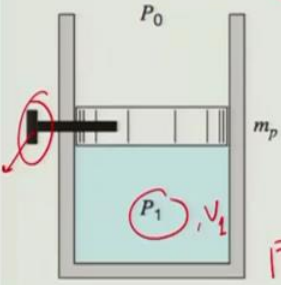
So let us try to do that, so this is 100 kiloPascal, 0.4 meter cubed  $\ln \frac{0.1}{0.4}$  and this is minus 55.5 kilo Joules. Now what you have obtained is the value but there is a negative sign here. So what does it mean? It means that the work because the expression which we have used is basically the work done by the system. So essentially the negative sign represents that the opposite effect is there. That means there is a work done on the system.

So our assumption is not right and hence with a negative sign indicates there is a work done on the system. Ok, so this is how we are going to solve a simple problem, so this should be a simple compression which is a means negative sign represents work done on the system.

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**The moving boundary work**

Consider the system in which the piston of mass  $m_p$  is initially held in place by a pin. The gas inside the cylinder is initially at pressure  $P_1$  and volume  $V_1$ . When the pin is released, calculate the work done by the system when the piston has come to rest.



$$P_{ext} = \frac{F_{ext}}{A} = \frac{P_0 A + m_p g}{A}$$

$P_1 > P_{ext} \Rightarrow$  Piston moves upward at finite rate

$$W = \int P_{ext} dV = P_{ext}(V_2 - V_1)$$

= work done by the sys against the force resisting the boundary movement.

$P_1 < P_{ext}$

Ok so let us try another example this example is again related to moving boundary but it will provide important relevance of considering the external force. so here is a system in which the



system of mass  $m$   $P$  is initially held in place by a pin. The gas inside the cylinder is initially at  $P_1$  and the volume is  $V_1$ . Ok?

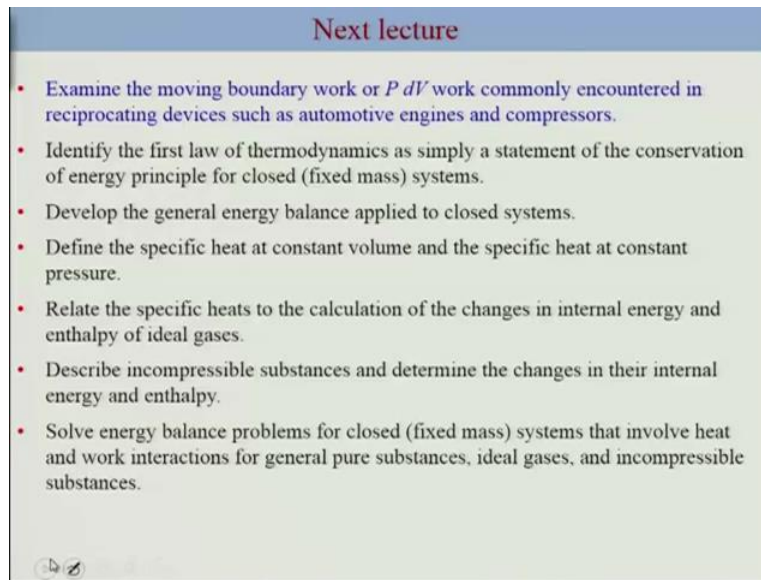
Now the pin is released and what we have to calculate is the work done by the system. Once you take out the pin, the forces are not balanced and effectively the piston will accelerate until it reach to the mechanical equilibrium with the pressure outside and insides are same; or the forces outside and inside are same.

What is external force on the systems? So external force is  $F$  and so the  $P_{\text{external}}$  in this case is going to be  $F_{\text{external}}$  divided by  $A$ , so that will be your  $P_0$  plus the weight of the piston  $m P g$  by  $A$ . Now, so this  $P_{\text{external}}$  will not be same as  $P_1$ . So if  $P_1$  is greater than  $P_{\text{external}}$  for which let us say we assume that then the piston will move upward. Ok? The piston will move upward at a finite rate. Ok?

Ok so in this case the work done would be against the force which is resisting the movement of the boundary, ok? So what is the force which is resisting the movement of the boundary? It is a basically  $F_{\text{external}}$ . Ok, so so we would be writing this  $W$  in terms of  $P_{\text{external}} dV$ . Ok? So this is the effective work, ok I will. So the this would be the work done by the system against the force resisting the boundary movement, ok?

Now since  $P_{\text{external}}$  is fixed because your  $P_0$  is fixed and the weight is fixed, the value in this case is going to be simply  $P_{\text{external}} V_2$  minus  $V_1$ . Now if  $P_1$  was lower than  $P_{\text{external}}$  then of course the piston would move downward.

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Next lecture

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So that was the exercise which we wanted to do in order to in order to understand the moving boundary concept. in the next lecture we will continue with this, (( ))(15.40) little more with certain example. So see you in the next lecture.