

**Engineering Thermodynamics**  
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**Lecture 16**  
**Polytropic process**

Ok! So welcome back. As we were discussing the boundary work, we will continue with a little more analysis.

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**Polytropic process**

During actual expansion/compression process of gases P and V often are related to  $P = CV^{-n}$

Polytropic process: C, n (polytropic exponent) constants

$$W_b = \int_1^2 P dV = \int_1^2 C V^{-n} dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1}$$

$$= \frac{P_2 V_2 - P_1 V_1}{-n+1} = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

I.G:  $PV = mRT$

$$W_b = \frac{mR(T_2 - T_1)}{1-n} \quad n \neq 1$$

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I would consider a which is normally used for this expansion and contraction in a piston gas cylinders. So this is a process called Polytropic process. In such case your P and V have a relation with such that P V to the power n is constant. So this is a common actual process expansion compression. This relation is called Polytropic process.

So let us consider such a process in a quasi static equilibrium conditions and our interest is to find out the boundary work for such a system when the gas expands and contracts having this relation of P and V. So we will consider again W b boundary work and we will make use of this expression. Ok? Again this is true the P when assuming it to quasi static equilibrium so (( ))(1.10) make use of the pressure of the gas in the system.

So we can put P as C V minus n dv, this can be written as C V 2 minus n plus 1 V 1 minus n plus 1 and this is minus n plus 1 or we can write this as because C is constant here and we can take out V 2 to the power minus n together it becomes P 2 and then the remaining V 2 will be here.

Similarly we can write P 1 V 1 and minus n plus 1. So we can simplify this in this form. Ok? Now if you are considering an ideal gas then we can make use of the ideal gas equation of state and replace P V by m R T in terms of temperatures. So you can write for ideal gas, P V equal to m R T so thus your Wb is m R (T 2 minus T 1) 1 minus n. So note that n should not be equal to same.

So this is true for n not equal to 1. Okay so this is the typical P V diagram process path for such a system. Of course it depends on the value of n and we are going to discuss these aspect the effect of n on the work by considering the values of n. I will illustrate in an example.

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**Polytropic process**

$PV=C$

When  $n = 1$  (isothermal process)  
 Ideal gas:  $PV = mRT = C$

$$W_b = \int_1^2 P dV = \int_1^2 C V^{-1} dV = PV \ln\left(\frac{V_2}{V_1}\right)$$

$W_b = \int_1^2 P dV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$  Constant pressure process

**What is the boundary work for a constant-volume process?**

Ok so let us consider starting with n equal to 1. Now if you consider n equal to 1 and you have a system P V is equal to C, ok? Now (if) for an ideal gas n equal turns out to an isothermal process because for ideal gas P V is equal to m R T when you say this to be C here, a constant which essentially means temperature is constant and in that case you can come up with an expression so you can replace P in the boundary work expression by C V to power minus 1 and this will lead to this expression for the case of n equal to 1.

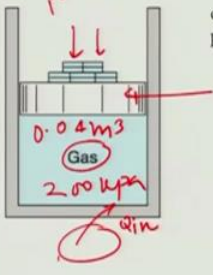
Similarly you can also find out the boundary work for constant pressure process and this would be your simply  $P_0 V_2$  minus  $V_1$ . what about the boundary work for constant volume process. Now considering the boundary work where  $dV$  has not changing. It should be 0.

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**Example**

Consider as a system the gas in the cylinder shown in Figure. The cylinder is fitted with a piston on which a number of small weights are placed. The initial pressure is 200 kPa, and the initial volume of the gas is 0.04 m<sup>3</sup>.

a) Let a Bunsen burner be placed under the cylinder, and let the volume of the gas increase to 0.1 m<sup>3</sup> while the pressure remains constant. Calculate the work done by the system during this process.



$$\begin{aligned}
 W_b &= \int P dV \\
 &= P(V_2 - V_1) \\
 &= 200 \text{ kPa} \times (0.1 - 0.04) \text{ m}^3 \\
 &= 12.0 \text{ kJ}
 \end{aligned}$$

Ok, so let us try this Polytropic process with different values of N and understand by an example. So consider this as a system. So this is a system of gas in a cylinder, ok? This is a piston. Ok? Piston contains small weights which are represented here. These are the weights. Initial pressure is 200 kiloPascal and the initial volume of this system is 0.04 meter cube. Now so there are many different parts we are going to do that, the first part is where we are going to heat up the system, using a Bunsen burner, so placed under the cylinder here. Ok?

So we are going to provide Q in and let the volume of the gas increases, so we want to increase to 0.1 meter cube, while the pressure remains constant. Keeping the pressure constant because the way it is constant. So the  $P_0$  external and other aspects are constant. So hence the pressure is constant, only the volume is going to change. So what we have to do is we have to find out the work done by the system on the surrounding.

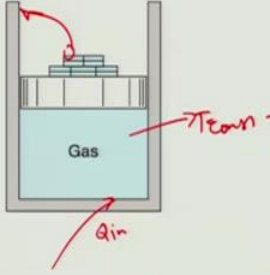
So we are going to simply put  $W_b$  as simply  $P dV$ . Now in this particular case, the pressure is constant, so this is going to be simply  $P(V_2 - V_1)$ , pressure we know is 200 kiloPascal,

final volume is we know, minus 1 and the initial volume is 0.04 meter cube. This is going to be 12.0 kilo Joules. So that was the first part of this example.

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**Example**

B) Consider the same system and initial conditions, but at the same time that the Bunsen burner is under the cylinder and the piston is rising, remove weights from the piston at such a rate that, during the process, the temperature of the gas remains constant.



$$pV = mRT = C$$

$$W_b = \int p dv = P_1 V_1 \ln \frac{V_2}{V_1}$$

$$= \frac{200 \text{ kPa} \times 0.04 \text{ m}^3}{\times \ln \frac{0.1}{0.04}}$$

$$= \underline{7.33 \text{ kJ}}$$

Now let us consider this same system and the same initial conditions. So you have 200 kiloPascal, you have 0.04 meter cube as initial volume but at the same time when we place the Bunsen burner under the cylinder and the piston is rising, what we are doing is we are removing the weights. So while we providing the heat, you also removing the weight in such a way that the temperature is constant.

So we are keeping the temperature constant. So now the question is what would be the work for such a system. So again we are going to make use of the ideal gas here, for the gas we are going to make use of the ideal gas equation of state. So  $P V$  is equal to  $m R T$ . Ok, now temperature is constant which means this is  $C$ . So  $W_b$  is  $P dv$  and we have already done calculated or derived the expression for the constant temperature expansion, so isothermal process and that would be  $P_1 V_1 \ln \frac{V_2}{V_1}$ .

So this we can obtain the values by putting the values for each variables.  $P_1$  is 200 kiloPascal,  $V_1$  initial volume is 0.04 meter cube and then  $\ln \frac{0.1}{0.04}$ . Ok, so this turns out to be 7.33 kilo Joules. Note this is less than the previous one which was constant pressure work. Ok? So this is a

constant temperature work, and this previous one was a constant pressure work and the final state in terms of volume is same.

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**Example**

C) Consider the same system, but during the heat transfer remove the weights at such a rate that the expression  $PV^{1.3} = \text{constant}$  describes the relation between pressure and volume during the process. Again, the final volume is  $0.1 \text{ m}^3$ . Calculate the work.

$$P_1 V_1^{1.3} = P_2 V_2^{1.3} = C$$

$$\text{or } P_2 = P_1 \left( \frac{V_1}{V_2} \right)^{1.3} = 60.77 \text{ kPa} \times \frac{200 \times 10^{-6} \text{ m}^3}{0.1 \text{ m}^3}^{1.3}$$

$$W_b = \int P dV = \frac{P_2 V_2 - P_1 V_1}{1 - 1.3}$$

$$= \frac{60.77 \times 0.1 - 200 \times 0.04}{1 - 1.3} \text{ kPa} \cdot \text{m}^3$$

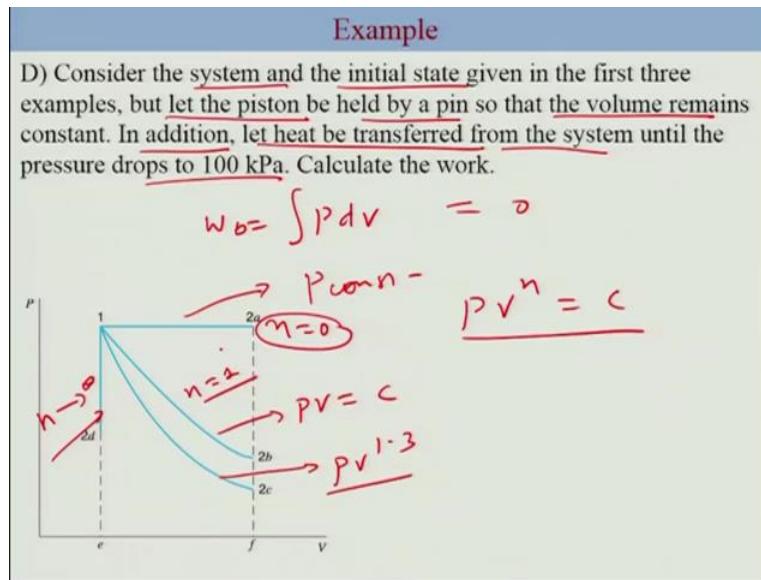
$$= 6.41 \text{ kJ}$$

So now let us consider the third case, now in this case you have the same system but now doing the heat transfer, we remove the weight at such a weight that this expression is constant. So this a Polytropic process, which describe the relation between pressure volume during the process. The final volume is still the same which is 0.1 meter cube. So we are going to calculate the work for such a system, make use of the direct expression of the Polytropic process. So this means that we have  $P_1 V_1^{1.3}$  is same as  $P_2 V_2^{1.3}$  is constant, ok?

Or we can write  $P_2$  is  $P_1 V_1^{1.3} / V_2^{1.3}$  and  $W_b$  we have already calculated the expression, this was  $P_2 V_2 - P_1 V_1 / 1 - n$ .  $n$  here is 1.3 and you know  $P_1$  which was 200 kilo Pascal ok? You know initially this is your 0.04 meter cube.  $P_2$  you can calculate from this expression, so this turns out to be 60.77 kiloPascal and then you plug in these values in this expression. So this to comes out to be 60.77 into 0.1 minus 200 into 0.04. 1 minus 1.3 kiloPascal meter cube. The answer is 6.41 kilo Joules.

So this is work done during this process having a relation between pressure and volume in this form, Polytropic process.

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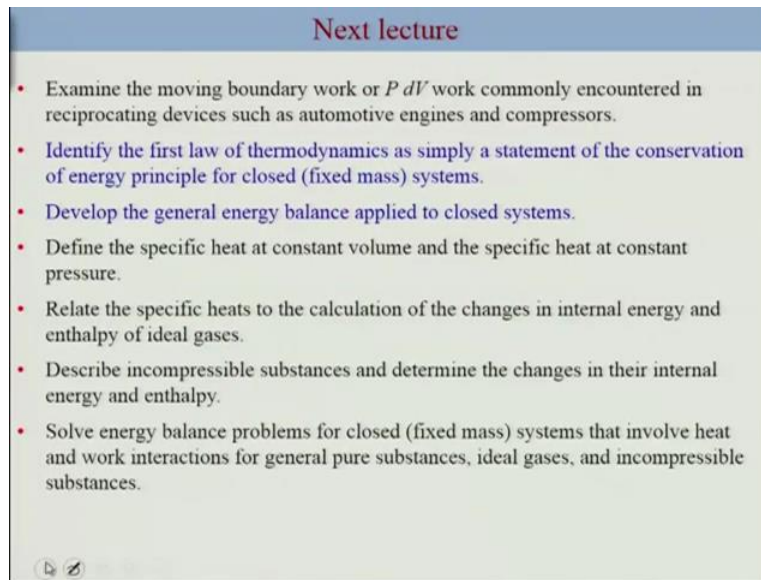
Ok, so let us take a last case for this example. So in this case you have the same system, same initial condition but let the piston be held by a pin which essentially means that the volume will remain constant in this process. Now the, so volume remains constant in addition that heat be transferred from the system until the pressure drops to 100 kiloPascal.

So so you have a system now which you do not allow the volume to move but you let, you allow heat from the system to surrounding. So what will be the work? Work is depended on the volume, the boundary work particular in this case depends on the volume change as we know from this expression. Now what is a volume change? Volume change is 0 and thus this is going to be 0.

So let us summarise the rougher part of this example in a graph. So this is the pressure volume and this represent different processes which we have considered. This is the constant pressure process, ok? This was the case where we consider isothermal, this was the case which we consider 1.3 Polytropic and this is case where the volume was constant so this is where n is 0 if you look at  $P V^n$ , constant process.

Then this was the case where n was 0 and this was the case where n approaches to infinity. Ok? And this is where n is equal to 1. Ok, so you can clearly see how the work varies depending on the variable n here for the Polytropic process.

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### Next lecture

- Examine the moving boundary work or  $P dV$  work commonly encountered in reciprocating devices such as automotive engines and compressors.
- Identify the first law of thermodynamics as simply a statement of the conservation of energy principle for closed (fixed mass) systems.
- Develop the general energy balance applied to closed systems.
- Define the specific heat at constant volume and the specific heat at constant pressure.
- Relate the specific heats to the calculation of the changes in internal energy and enthalpy of ideal gases.
- Describe incompressible substances and determine the changes in their internal energy and enthalpy.
- Solve energy balance problems for closed (fixed mass) systems that involve heat and work interactions for general pure substances, ideal gases, and incompressible substances.

Ok so with that we are going to end this lecture. Ok? In the next very lecture we will take up the generalised energy balance which is applied to the closed system with few examples. So see you in the next lecture.