Engineering Thermodynamics
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Lecture 21
Conservation of mass and steady flow processes

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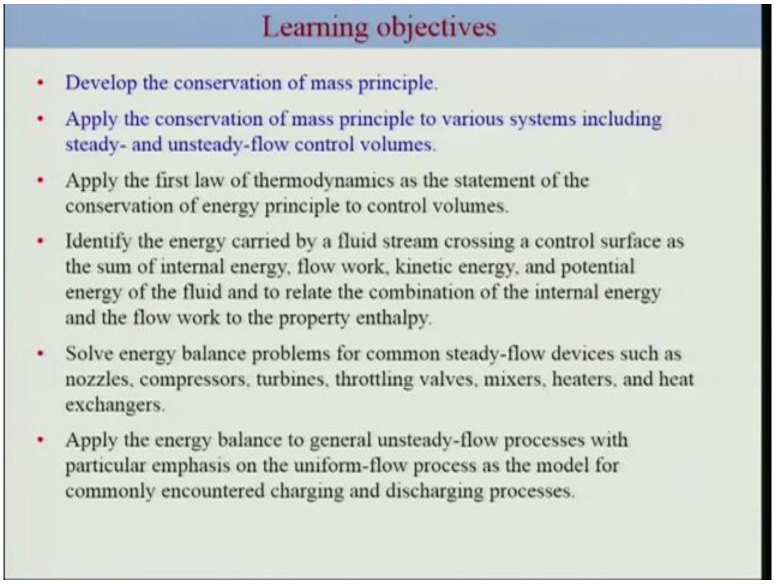

Welcome back we are going to start a new topic is called mass and energy analysis. So in earlier we already looked into the energy analysis of closed system. In this case we are going to take mass and energy analysis for they have control of in a systems. So we are going work on a control body.

So this is the learning objective of this particular lecture. We going to develop the conservation of mass principle and specifically apply to various system that include steady and unsteady flow control volumes okay.

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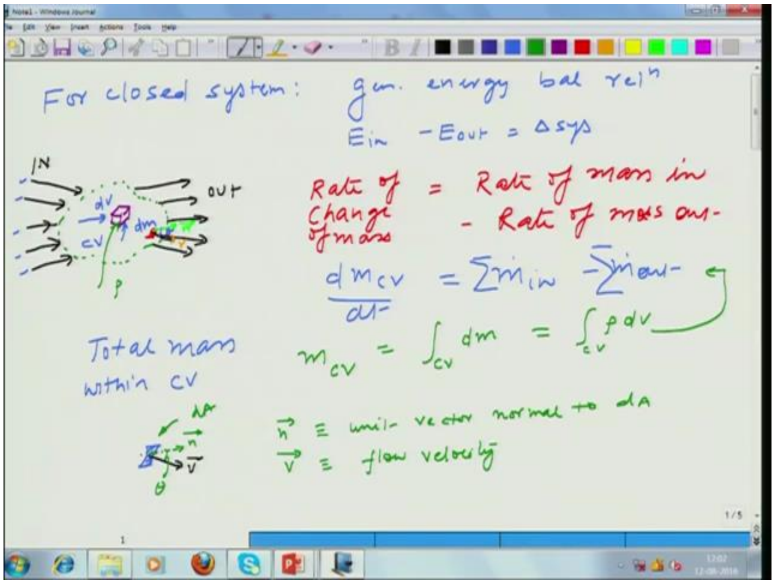

So let me summarize what we have done earlier for the case of closed system. So for closed system the general energy balance can be written as relation okay. So this is a specifically the general energy balance for the closed system. Now what we are going to do is to extend this analysis for the case of a system where flow (())(1:19) there is entering of the flow and as well as exit of the flow from the system and hence the energy analysis will be for the case of control volume okay.

So let us try to develop conservation of first mass principle, so we are going to control volume so this is let us say our control volume okay which is inside your the dash line okay. So this is my control volume and then you have series of streams which enters from one side or in generally it enters an indicated by this arrow.

So, this would be your in and this would represent out stream. So in general we are going to write your energy for your conservation of mass by evaluating a general expression which would be that rate of change of mass okay. Within the control volume should be equal to rate of mass in okay minus rate of mass out okay.

This is of course there is no term on the generation because mass cannot be created or destroyed that would be the system which you are going to consider would be true. So in general we can write this expression in mathematical form by considering the mass of the control volume as mcv okay. So this will be your change in the mass of the control volume and the rate of change of that they should be equal to effectively whatever comes in, in terms of dot m dot that will be rate of mass in minus write of mass out.

This would be your specific relation but if they are many streams then we can write this as a summation of all the rate of mass and due to different streams. So there should be sum over this this and so forth as similarly for the case of mass out okay. Now how we are going to find the total mass of the control volume, so total mass within cv okay.

So we can consider small volume here okay, so this let us say volume and the mass of this particular volume we can consider at to be dm okay and this would be volume would be cross (())(4:06) volume of dv.

So we can write the expression of the total mass of volume would be simple integral of this small box over the the control volume which essential means we are integrating over dm over cv which essentially can be rewritten as in terms of the density and volume, so let us assume that the density here is rho okay of this particular small volume within the control volume so this would be your rho times the the dv volume of the small box here.

Okay so this will be the expression of the mass of the control volume and thus you can plug this expression that in this equation. Okay so now let us also consider small differential area component on this particular control volume okay so this could be a small differential component here now which would be area okay.

lets say the flow which comes out from this particular area is represented by this velocity here v and the normal vector could be let us say your n here which is n vector, the angle between then would be your theta. So another word we are considering a small differential area within this control volume and the flow is represented by this v dash which is a velocity in this direction and the corresponding your normal vector to this differential area would be this n vector here so the angle between them is represented by theta okay.

So n vector is this is a unit vector normal to lets say this area is dA so this is dA okay and your velocity is the flow velocity.

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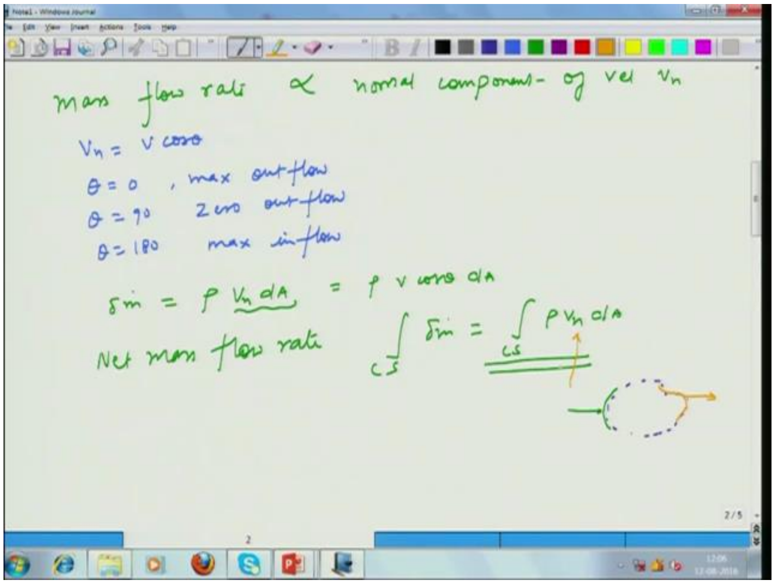

Okay so what about the mass flow rate for this particular system so the mass flow rate would be nothing but rate should proportional to the normal component of velocity okay. So let us define this as Vn so Vn would be your v cos theta. So now depends on the theta itself like whether if you have a component which is perpendicular to the differential area then it will be the maximum flow rate.

If it is parallel then they would be zero flow. If it is one eighty then it is actually invert okay so depending on the theta one can consider to be outflow or inflow. So this theta could be zero so this would be your maximum outflow okay, if it is ninety then it is the zero outflow okay, if it is one eighty then it is a maximum inflow.

Okay so now we can consider the differential mass rate so the differential mass rate essential means what the amount of mass which comes out from that differential area would be represented let say by dot here, which will be rho times the normal velocity times (())(7:48) the differential area. So another reason is that we are considering the density here and this a would be your the volume so this can be further as rho times Vn can be written as v cos theta dA okay.

So if you integrate this differential area over the surface of the control volume you would get effective net mass flow rate. So which essentially means that you can calculate your net mass flow rate by integrating over the control surface which means on the surface of this control volume okay which essentially be your rho Vn dA okay.

Now this essentially has also the component inflow and as well the outflow okay because you are what you have done is you have you are consider let say the regions which are inflow and as well as the regions which are outflow so when you are integrating over this you also considering that different thetas Associated this Vn component and hence you have the net mass flow rate which includes the inward and as well as outward flow okay.

So the direction is already accounted in this expression so now we can use this information to come of with the final expression for the mass conservation.

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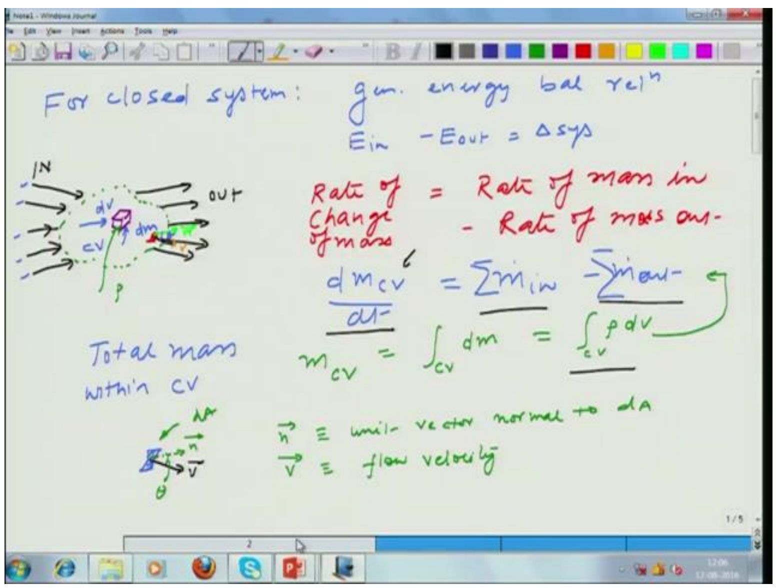

So let us write that so we already written this this is our expression so we can replace this mcm by this expression and m dot in by this expression okay.

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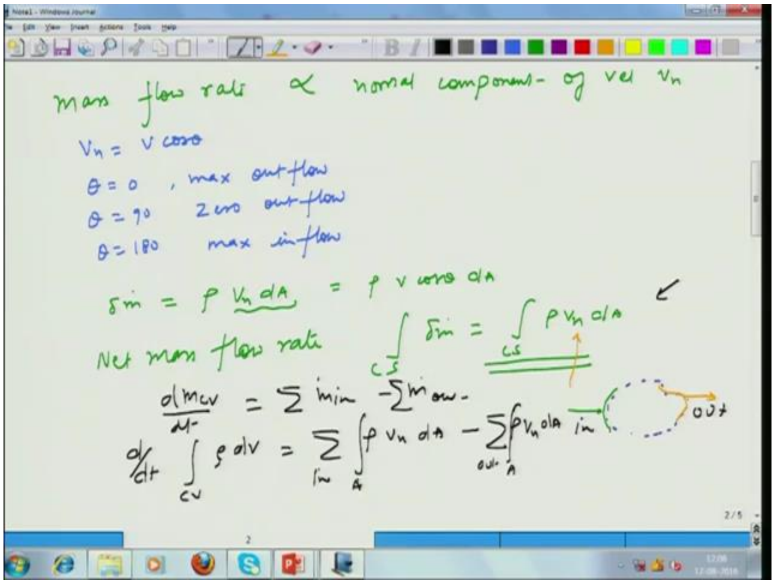

Now instead of taking the overall control we can say this is effective in and this is a effective out surface area dm or this would be the direction so hence we can separate these two directions and this divide into this two direction such that this can be written as separately in this following form.

So you have your dmcv by dt okay. Sum of m in minus m out okay dot here so now we can write it as d by dt the control volume okay rho dv and this would be your sum of rho Vn dA minus sum of rho they should integral all, so this will be four specific area and we are saying this to be if you use is expression this would be your in and this will be your Vn dA for the specific case of out okay.

Okay so this is my the complete balance based on the expression what we have written rate of mass in minus rate of mass out okay that be the rate of change of mass within this system. Let us consider the fact that your velocity need not be constant across the cross section and this may also change with time.

Now we will just consider a case of a pipe and then you will consider like we aware of the flow profile of the fluid with in the pipe, so we will try to understand that can be replace this variable velocity across cross section by a average value and this is what we are going to consider now.

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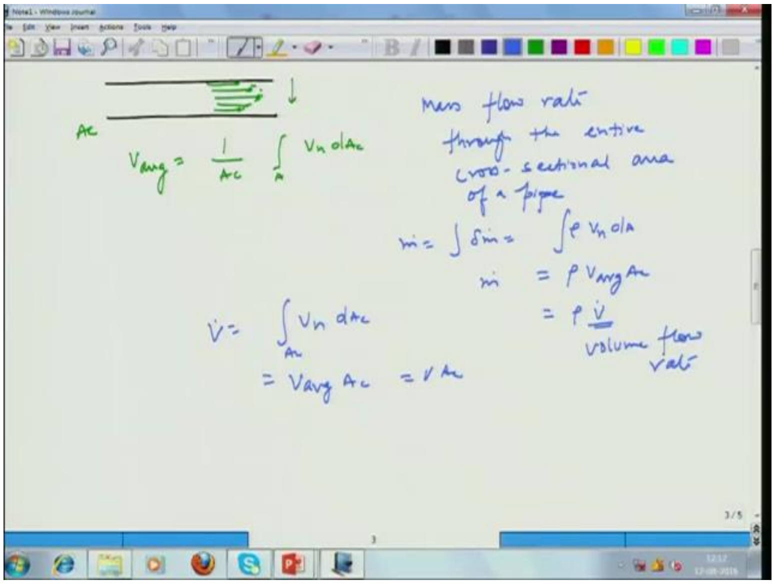

So a typical pipe can be consider as this parallel lines here and the flow profile would be something like this okay. So here if it is so which essentially means that the velocity in the middle be highest so this is a level of profile now this is more cumbersome to make use of it because you need to know the profile expression across this cross section okay and in this case what would be more useful is to find out the average velocity so this average velocity would be, so if we say this cross sectional area is Ac this would be your okay.

So this is your average velocity okay, so what about the mass flow rate through this entire cross section mass flow rate would be through the entire cross section of a pipe okay. This would be your m dot dm which now we can write in terms of if we write the general expression as we have done earlier then this would be these but now we can replace this expression by rho V average Ac.

So this would be your mass flow rate you can also rewrite in terms of the volume flow rate this expression can be written as your rho times V dot where this is your volumetric flow rate okay. Where V dot can be written as it is the by definition the the velocity multiply by the differential area across the cross section we integrate and this is what we can write as V average Ac okay or V times Ac okay.

So this was the general mass balance we want to apply this mass balance for the state process so let us look at how what are the conditions which you are going to make use of it.

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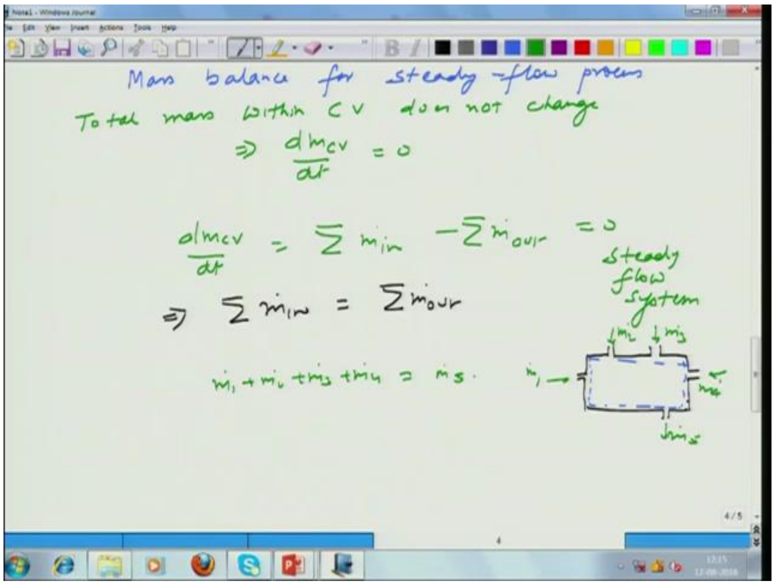

So what we are going to done up now is your mass balance for steady flow process okay. So considering the steady flow conditions the total properties within the control volume is going to be constant over time. So thus your mass of the control volume should not change which essentially means the derivative of the mass with respect to time should be zero.

So total mass within cv does not change okay which essentially means your dmcv by dt should be zero okay. Now this is a straight is obvious now we can simply make use this particular expression in our mass conservation principle. So what we get is the following so your dmcv by dt we know this is now written as m in minus m out which should be zero considering that this is for steady flow system.

So which means your m in so that is net mass flow rate in should be equal to net mass flow rate out okay. So for the case of lets say we can illustrate this using an example so you have a system with let say four inlet. Okay so this is your control volume okay now this are the inlet and this is outlet so this could be your m1m2 dot, m3, m4 and this is m5.

So based on this expression considering this to be your steady a flow system can be written as m1 the inlet are summation of this m dot in is m1 plus m2 plus m3 plus m4 they should be equal to the outlet summation of outlet mas flow rate streams and this would be m dot 5. So we have now developed this expression for the steady a process.

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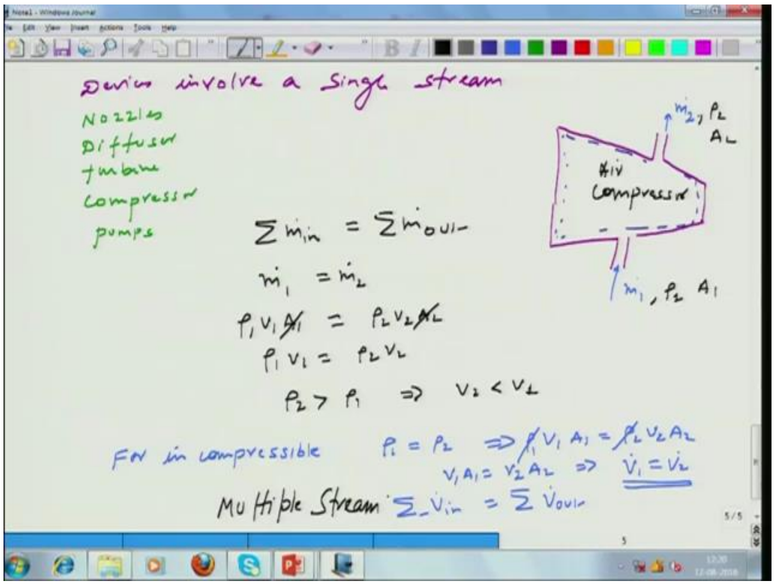

Now many applications just involves single streams okay. The examples are nozzles, diffuser, turbine okay compressor and pumps okay. So these are the devices which involve a single stream. So we can consider a case so for such a device, so this is an example of example of your air compressor okay and this is your control volume this would be your outlet stream let us say m2 air and this is your inlet stream with m1 dot okay.

So based on the single stream and steady flow process we can write summation m dot in is equal to m dot out as simply in this case m1 dot is equal to m2 dot okay. Now we can consider this is having a density rho this will have different density because its air compressor is going to compress it and the density at the exit would be different compare to what is at the inlet okay and the cross sectional area will be let say A1 and A2 okay.

So in that case if we consider the velocity here to be average velocity so this would be your rho1 V1 A1 is equal to rho2 V2 A2 okay. So if you consider cross sectional area is to be same in that case your, this would get cancel and rho1 V1 should be equal to rho2 V2. Now being a compressor your rho2 density will at exit should be higher than rho1. This would indicate with the velocity at the exit be lower than at inlet condition.

So we can consider specific case for this single stream study flow process and this would be your case of incompressible fluid. So for the case of incompressible fluid your rho1 is same as rho2 okay which essentially means for the case of single fluid your V1 times A1 so this would mean that your V1 times A1 should be equal to V2 times A2.

In case of same cross sectional area this would (())(19:36) your V1 should be equal to V2 or if it is a different cross sectional area which means this is nothing but your volumetric flow right should be same for such incompressible fluid okay.

So this would be your generic expression for incompressible single systems okay. For the case of a multiple streams incompressible fluid would be the expression would lead us to summation of V1 (())(20:04) for inlet is equal to summation of volumetric flow rate of outlet okay for different stream. So this would be the expression for the case of multiple stream.

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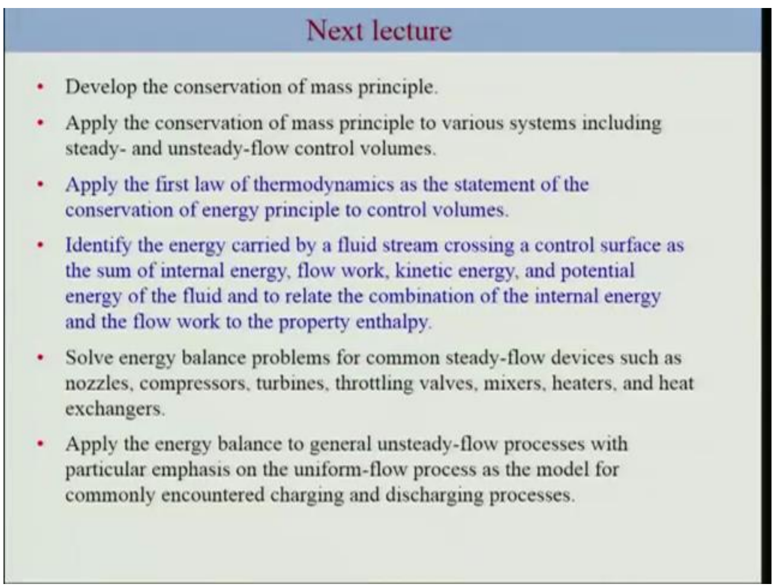

So with that we will come to the end of this lecture and in the next lecture we are going to apply the energy principle for the case of control volume and we will develop the expression. I will discuss specifically the total energy of the flowing systems okay. So we will see you in the next lecture.