Engineering Thermodynamics
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Lecture 26
Examples on energy & mass balance for open systems

Welcome to this tutorial. Today we will go through few examples based on energy and mass balance for open systems. So let us get started.

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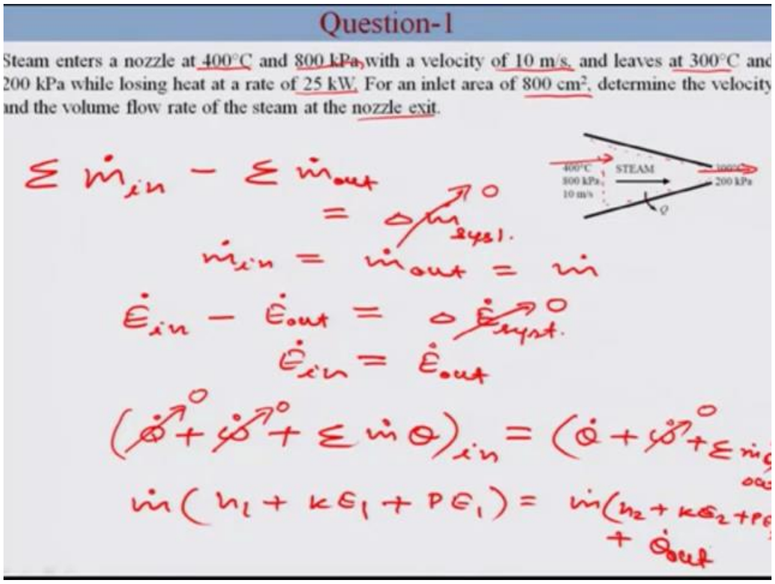

Question-1

Steam enters a nozzle at 400°C and 800 kPa, with a velocity of 10 m/s, and leaves at 300°C an
200 kPa while losing heat at a rate of 25 kW. For an inlet area of 800 cm², determine the velocit
nd the volume flow rate of the steam at the nozzle exit.

$$\sum \dot{m}_{in} - \sum \dot{m}_{out} = \Delta \cancelto{0}{m_{syst.}}$$

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \cancelto{0}{\dot{E}_{syst.}}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$(\cancelto{0}{\dot{Q}} + \cancelto{0}{\dot{W}} + \sum \dot{m}\theta)_{in} = (\dot{Q} + \cancelto{0}{\dot{W}} + \sum \dot{m}$$

$$\dot{m}(h_1 + KE_1 + PE_1) = \dot{m}(h_2 + KE_2 + P$$
$$+ \dot{Q}_{out}$$

Steam enters a nozzle at 400 degrees centigrade and 800 kiloPascal, with a velocity of 10 metre per second and leaves at 300 degrees Centigrade and 200 kiloPascal while losing heat at a rate of 25 kW. For an inlet area of 800 centimetre square, determine the velocity and the volume flow rate of the steam at the nozzle exit.

So this is the sketch of the system and this is the control volume. We have assumed that the steam is the system and since it is h control volume mass is entering here and exiting at the other end. So we need to find the exit velocity and volume flow rate at the nozzle exit. So from the mass balance mass in rate of mass in minus total rate of mass out of the system will be equal to total change in mass of the system. Ok.

This is this is steady step process, ok so nothing will change with time. So delta m will be zero of the system that means mass flow rate coming in will be equal to mass flow rate coming out of the system and let us say it is equal to m dot ok. Now from the energy balance on this system we get rate of energy in minus rate of energy out will be equal to change in energy of the system ok.

So this is this is h steady state process so it will be zero and from here we get rate of energy in will be equal to rate of energy out. So total energy coming in this system will be heat coming in plus (())(02:31). Total energy of the flowing fluid in will be equal to heat coming out of the system plus work coming out of the system plus again total flow energy of the fluid coming out of the system.

Since nothing is going in, no energy is since no energy is going in the system no shaft work is there in the system so these two terms will be zero and same there is no work done by the system. So this double dot will also be zero. So from here we get mass flow rate times flow energy of fluid coming in the system will be equal to mass flow rate times flow energy of fluid coming out of the system plus heat coming out of the system.

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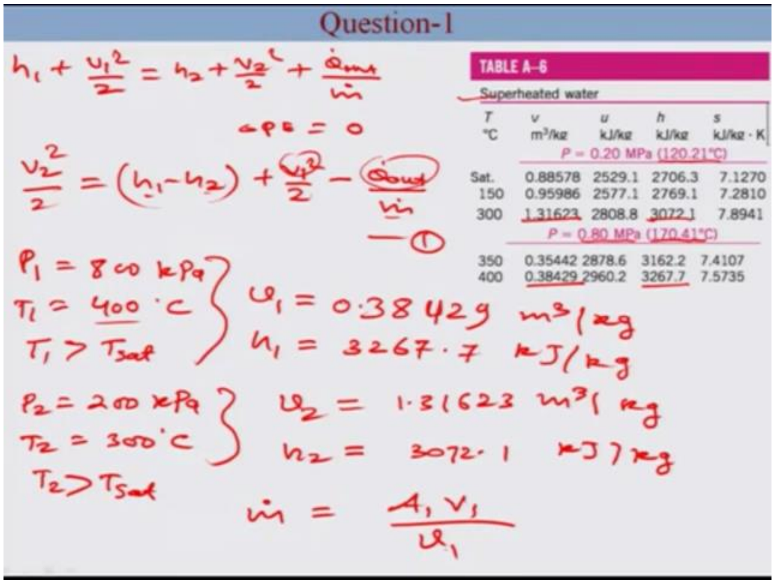

Question-1

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} + \frac{\dot{Q}_{out}}{\dot{m}}$$

$$\Delta PE = 0$$

$$\frac{V_2^2}{2} = (h_1 - h_2) + \frac{V_1^2}{2} - \frac{\dot{Q}_{out}}{\dot{m}} \quad \text{—①}$$

$P_1 = 800$ kPa
$T_1 = 400$ °C
$T_1 > T_{sat}$

$v_1 = 0.38429$ m³/kg
$h_1 = 3267.7$ kJ/kg

$P_2 = 200$ kPa
$T_2 = 300$ °C
$T_2 > T_{sat}$

$v_2 = 1.31623$ m³/kg
$h_2 = 3072.1$ kJ/kg

$$\dot{m} = \frac{A_1 V_1}{v_1}$$

TABLE A–6

Superheated water

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 0.20 MPa (120.21°C)			
Sat.	0.88578	2529.1	2706.3	7.1270
150	0.95986	2577.1	2769.1	7.2810
300	1.31623	2808.8	3072.1	7.8941
	P = 0.80 MPa (170.41°C)			
350	0.35442	2878.6	3162.2	7.4107
400	0.38429	2960.2	3267.7	7.5735

So after solving this we get h1 plus v1 square by 2 equal to h2 plus v2 square by 2 plus Q out upon m dot. Assuming here that delta p equal to zero ok. So from here we need to find exit velocity that is v2, so after solving this we get h1 minus h2 plus v1 square by 2 minus Q dot out upon m dot. So this is our first expression so in order to get v2 we need to know h1, h2 and mass flow rate. Since v1 we know and Q out we also know ok, so it is given that that inlet condition p1 equal to 800 kiloPascal and T1 equal to 400 degree centigrade.

At this condition saturation temperature should be 170.41 degree centigrade, but here this T1 is greater than T saturated that means this condition is super-heated ok. So we need to look into super-heated water table to get these h1 and h2, so from here we get specific volume at 400 degree centigrade and 800 kiloPascal which is nothing but.8 MPa. So v1 will be 0.38429 which is this value meter cube per Kg.

Similarly inlet enthalpy 3267.7 kiloJoule per kg at this 400 degree centigrade and 800 kiloPascal which is this value. So at the exit it is also given that P2 is 200 kiloPascal and T2 is 300 degrees centigrade. So corresponding to these conditions this condition is also at super-heated condition because T2 is great than T sat because T sat should be 120.1 degree centigrade at this 200 kiloPascal but this temperature is above this T saturation temperature.

So corresponding specific volume v2 will be equal to 1.31623 meter cube per kg which is this value and corresponding enthalpy h2 will be 3072.1 kiloJoule per Kg, ok so this value. Now we need to find mass flow rate ok, so the mass flow rate can be calculated as inlet area in an velocity divided by initial specific volume, ok.

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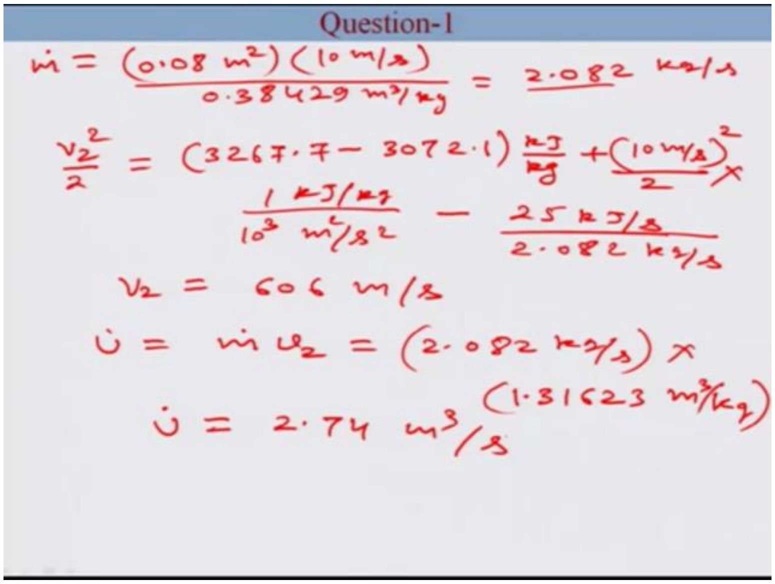

Question-1

$$\dot{m} = \frac{(0.08\ m^2)(10\ m/s)}{0.38429\ m^3/kg} = \underline{2.082}\ kg/s$$

$$\frac{V_2^2}{2} = (3267.7 - 3072.1)\frac{kJ}{kg} + \frac{(10\ m/s)^2}{2} \times \frac{1\ kJ/kg}{10^3\ m^2/s^2} - \frac{25\ kJ/s}{2.082\ kg/s}$$

$$V_2 = 606\ m/s$$

$$\dot{V} = \dot{m}\, v_2 = (2.082\ kg/s) \times (1.31623\ m^3/kg)$$

$$\dot{V} = 2.74\ m^3/s$$

So after plug in these values here we get m dot equal to 0.08 meter square it is given in the problem. Velocity is also given 10 meter per second and specific volume that we have calculated right now 0.38429 meter cube per kg.

So this gives us 2.082 kg per second, ok. Substituting these values in equation 1 we get v2 square by 2 equal to h1 minus h2 which is 3267.7 minus h2 is 3072.1 kiloJoule per kg plus v1 square by 2 which is 10 metres per second by 2. We have to convert it into energy unit. So multiplied by 10 to power 3 meter square per second square is equal to 1 kiloJoule per kg. Okay minus Q out upon m so Q out is given in the problem which is 25 kiloJoule per second and m dot that we have calculated here 2.082 kg per second ok.

So after solving it we get v2 equal to 606 meter per second, so further we need to calculate volumetric flow rate at the exit ok. So that will be equal to mass flow rate times specific volume at exit. So after putting these values here 2.082 kg per second into 1.3162 meter cube per kg so from here we get volumetric flow rate v dot equal to 2.74 meter cube per second ok.

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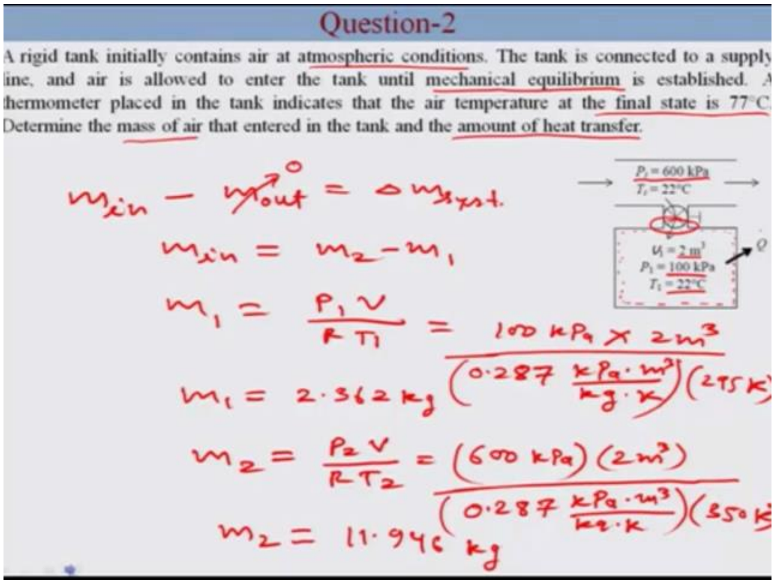

Question-2

A rigid tank initially contains air at atmospheric conditions. The tank is connected to a supply ine, and air is allowed to enter the tank until mechanical equilibrium is established. hermometer placed in the tank indicates that the air temperature at the final state is 77°C. Determine the mass of air that entered in the tank and the amount of heat transfer.

$$m_{in} - \cancel{m_{out}}^{0} = \Delta m_{syst.}$$

$$m_{in} = m_2 - m_1$$

$$m_1 = \frac{P_1 V}{R T_1} = \frac{100\,kPa \times 2\,m^3}{\left(0.287\,\frac{kPa\cdot m^3}{kg\cdot K}\right)(295\,K}$$

$$m_1 = 2.362\,kg$$

$$m_2 = \frac{P_2 V}{R T_2} = \frac{(600\,kPa)(2\,m^3)}{\left(0.287\,\frac{kPa\cdot m^3}{kg\cdot K}\right)(350\,K}$$

$$m_2 = 11.946\,kg$$

So moving to the next question. A rigid tank initially contains air at atmospheric conditions. The tank is connected to a supply line, and air is allowed to enter the tank until mechanical equilibrium is established. A thermometer placed in the tank indicated that air temperature at the final state is 77 degrees centigrade. Determine the mass of air that enter in the tank and amount of heat transfer.

So there is the sketch of the problem and here we assume that this tank as the system which is a control volume since mass is crossing the boundary and here we need to find the amount of mass that has entered in the tank and total heat coming out of the system. So from the mass balance for this system, we get mass coming in minus total mass coming out of the system will be equal to delta m of the system.

This is an unsteady state process but we can analyse as a uniform full process since the state of the fluid at this inlet condition remains constant, ok. So we will analyse it as a uniform flow process, nothing is coming out of the system so the mass out will be zero. So from here mass in will be equal to change in mass of the system which is m2 minus m1 ok. And now here we assume that air as an ideal gas initial and final mass of the air can be calculated as m1 equal to p1 v upon RT1 ok.

So p1 is given to us which is 100 kiloPascal, volume of the tank is 2 meter cube and gas constant for air is 0.28 kiloPascal meter cube per kg and temperature is 295 Kelvin correspond to 22 degree centigrade ok. So m1you will get here 2.362 kg so similarly m2 will be p2 v upon RT2. Since tank is rigid so volume will be same. We need to find p2 we do not know p2 but it is given that tank is in mechanical equilibrium with the supply line that means the pressure of the tank will be equal to pressure of the supply line which is 600 kiloPascal.

So putting this value here 600 kiloPascal, volume is 2 meter cube upon R is 0.287 kiloPascal meter cube kg Kelvin into 350 Kelvin. So from here m2 would be equal to 11.946 kg ok.

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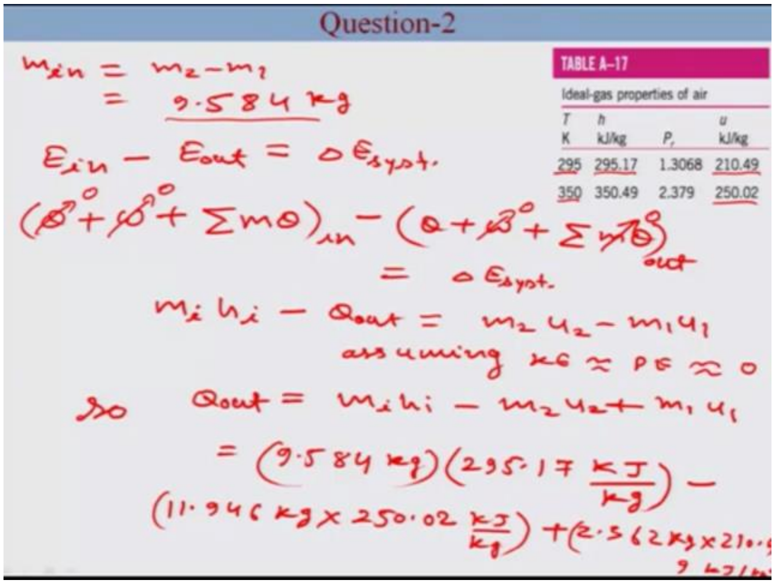

Question-2

$$m_{in} = m_2 - m_1$$
$$= \underline{9.584\ kg}$$

$$E_{in} - E_{out} = \Delta E_{syst.}$$

$$(\cancel{Q}^0 + \cancel{W}^0 + \sum m\theta)_{in} - (Q + \cancel{W}^0 + \sum \cancel{m\theta}^0)_{out} = \Delta E_{syst.}$$

$$m_i h_i - Q_{out} = m_2 u_2 - m_1 u_1$$

assuming $KE \approx PE \approx 0$

so $Q_{out} = m_i h_i - m_2 u_2 + m_1 u_1$

$$= (9.584\ kg)\left(295.17\ \frac{kJ}{kg}\right) - \left(11.946\ kg \times 250.02\ \frac{kJ}{kg}\right) + (2.562\ kg \times 210.$$

TABLE A–17

Ideal-gas properties of air

T K	h kJ/kg	P_r	u kJ/kg
295	295.17	1.3068	210.49
350	350.49	2.379	250.02

Now further from the mass balance equation we get amount of mass coming in the system will be equal to m2 minus m1 which is 9.584. So this much of air will come into the system. Next we have to calculate amount of heat coming out of the system ok.

So from the energy balance in that total energy in minus total energy out and equal to delta E of the system, ok so total energy is heat coming in plus total shaft work plus total flow energy of the fluid coming in the system minus total energy coming out of the system which is Q plus shaft work plus total flow work of the fluid. This is equal to delta E of the system ok. So we know that there is no heat coming in the system no shaft work involved here and there is no work done by the system, so this will also be zero and there is no flow work coming out of the system so this will also be zero.

So from here we get mass coming in the system times enthalpy minus heat coming out of the system will be equal to change in internal energy of the system ok. Assuming here K p to be zero. So Q out will be mass coming in times enthalpy coming in minus m2 u2 plus m1 times u1. So we have already calculated mass coming into the system, so after putting this value here mass in is 9.584 Kg, initial enthalpy is enthalpy at 295 kelvin because temperature is given initially.

So initial enthalpy would be 295.17 kiloJoule per kg minus m2 that we have already calculated which is 11.946 kg times u2. U2 is the internal energy at 350 degree centigrade because the final temperature is 77 degree centigrade. So u2 will be corresponding to 350 Kelvin which is 250.02, 250.02 kiloJoule per kg and m1 plus u1. M1 is 2.362 kg times u1, u1 will be the internal energy at 295 Kelven so 210.49 kiloJoule per kg ok.

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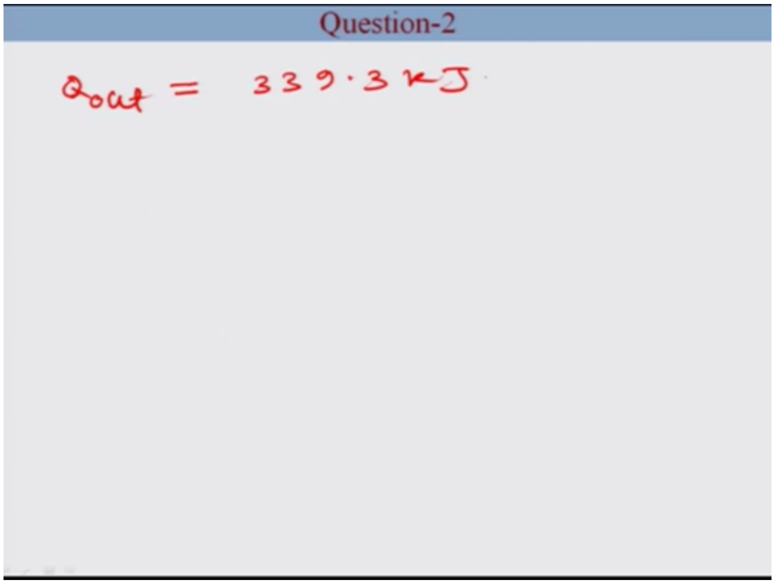

Question-2

$Q_{out} = 339.3\ kJ$

So after solving this we get heat coming out of the system equal to 339.3 kiloJoule.

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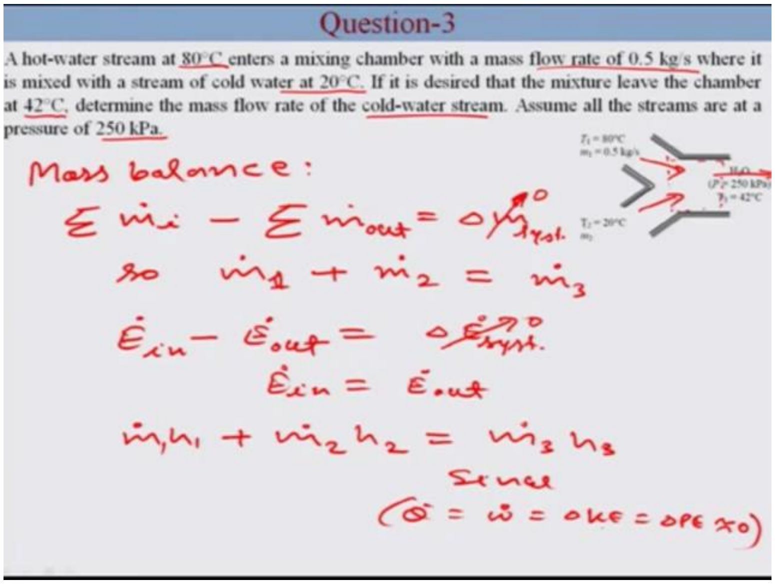

Question-3

A hot-water stream at 80°C enters a mixing chamber with a mass flow rate of 0.5 kg/s where it is mixed with a stream of cold water at 20°C. If it is desired that the mixture leave the chamber at 42°C, determine the mass flow rate of the cold-water stream. Assume all the streams are at a pressure of 250 kPa.

Mass balance:

$$\sum \dot{m}_i - \sum \dot{m}_{out} = \overset{0}{\cancel{\Delta m_{syst.}}}$$

so $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

$$\dot{E}_{in} - \dot{E}_{out} = \overset{0}{\cancel{\Delta \dot{E}_{syst.}}}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Since

$(\dot{Q} = \dot{W} = \Delta KE = \Delta PE \approx 0)$

So moving to the next question. A hot water stream at 80 degree centigrade enters a mixing chamber with a mass flow rate of.5 kg per second well it is mixed with a stream of cold water at 20 degree centigrade. It is desired that the mixture leave the chamber at 42 degree centigrade, determine the mass flow rate of the cold water stream. Assuming all the streams are at pressure of 250 kiloPascal.

So here we assumed the mixing chamber as the system and this is a control volume since two streams are coming into the system and one stream is going it of the system ok. So from the mass balance of this system total mass flow rate coming into the system minus total mass flow rate coming out of the system will be equal to change of mass flow rate of the system ok. This is the steady state process so nothing will change with time so delta m dot will be zero, so from here we get m1 dot plus m2 dot will be equal to m3 dot because this is the only stream which is coming out of the system.

From the energy balance we get rate of energy in minus rate of energy out equal to change in energy of the system. So this will also be zero so from here we get rate of energy in will be equal to rate of energy out. Total energy in will be nothing but enthalpy of this first stream plus enthalpy of the second stream which is going into the control volume may be equal to enthalpy of this stream going out of the system ok. Since Q dot, w dot and delta k equal to delta p we assume that it to be zero.

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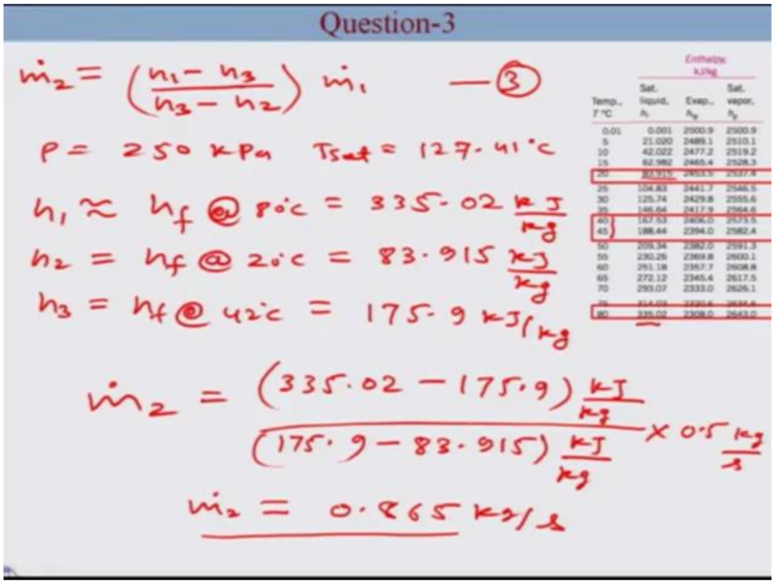

Question-3

$$\dot{m}_2 = \left(\frac{h_1 - h_3}{h_3 - h_2}\right)\dot{m}_1 \quad \text{—③}$$

$P = 250$ kPa $T_{sat} = 127.41\,^\circ C$

$h_1 \approx h_f\ @\ 80^\circ C = 335.02\ \frac{kJ}{kg}$

$h_2 = h_f\ @\ 20^\circ C = 83.915\ \frac{kJ}{kg}$

$h_3 = h_f\ @\ 42^\circ C = 175.9\ kJ/kg$

$$\dot{m}_2 = \frac{(335.02 - 175.9)\,\frac{kJ}{kg}}{(175.9 - 83.915)\,\frac{kJ}{kg}} \times 0.5\ \frac{kg}{s}$$

$$\underline{\dot{m}_2 = 0.865\ kg/s}$$

Temp., T °C	Enthalpy, kJ/kg Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g
0.01	0.001	2500.9	2500.9
5	21.020	2489.1	2510.1
10	42.022	2477.2	2519.2
15	62.982	2465.4	2528.3
20	83.915	2453.5	2537.4
25	104.83	2441.7	2546.5
30	125.74	2429.8	2555.6
35	146.64	2417.9	2564.6
40	167.53	2406.0	2573.5
45	188.44	2394.0	2582.4
50	209.34	2382.0	2591.3
55	230.26	2369.8	2600.1
60	251.18	2357.7	2608.8
65	272.12	2345.4	2617.5
70	293.07	2333.0	2626.1
75	314.03	2320.6	2634.6
80	335.02	2308.0	2643.0

So after combining this equation and this equation we get m2 dot equal to h1 minus h3 upon h3 minus h2 times, initial mass flow rate ok. Now to get the value of m2 dot we need to know all these variables ok. Since it is given that all these streams are at 250 kiloPascal so at this 250 kiloPascal of pressure T saturation temperature should be 127.41 degree centigrade but it is given at this pressure temperature of all the streams are way below than 127.41 degree centigrade but it is given that at this pressure temperature of all these streams are way below than 127.41 degree centigrade which 80 degree centigrade, 20 degree centigrade and 42 degree centigrade.

That means all these streams are compressed streams which can be approximated as a saturated liquid at the given temperature, so in order to get enthalpy data. We need to look into the saturated water table so h1 will be equal to enthalpy of fluid at 80 degree centigrade from the saturated water table which is listed here. So enthalpy would be 335.02 kiloJoule per kg, h2 will be enthalpy of the fluid at 20 degree centigrade which is 83.915 and similarly h3 so we do not have exact data at 42 degree centigrade.

So we have to interpolate between 40 and 45 degree centigrade temperature, so corresponding to 42 we get 175.9 kiloJoule per kg. So after putting these values in the third expression we get m2 dot equal to 335.02 minus 175.9 k J per kg upon 175.9 minus 83.915 kiloJoule per kg times m1 dot which is 0.5 kg per second. So from here we get m2 dot equal to 0.865 kg per second so this is the mass flow rate of the cold stream coming into the mixing chamber. So we will stop here we will meet you in the next tutorial. Thank you.