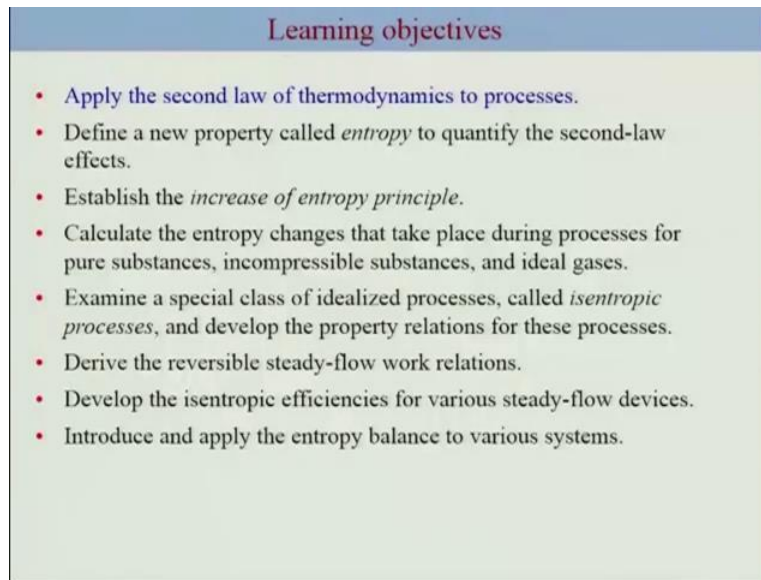


**Engineering Thermodynamics**  
**Professor Jayant K Singh**  
**Department of Chemical Engineering**  
**Indian Institute of Technology Kanpur**  
**Lecture 32**  
**Clausius inequality, application of second law**

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**Learning objectives**

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

Welcome back, we are going to now start a new topic isentropic and this is learning objective of this particular topic. We will start with the application of second law of thermodynamic processes. So till date what we have learned in second law of thermodynamic is related to thermodynamic cycle ok, or in general second law of thermodynamics was applied to us cycling process.

However our interest is to understand the application of the second law of thermodynamics to processes which we deal with our daily life or for example in a automobile or compression of a certain gas or in general cooling of a sub substance in a room temperature for example, so in this direction the Clausius came up with a inequality and this is something which we are going to discuss in today's lecture which will be called Clausius inequality which is true for all reversible or irreversible processes ok.

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**The inequality of Clausius**

*Clausius in equality*  $\oint \frac{\delta Q}{T} \leq 0$  *valid for all rev/irrev cycle*

*TH = const.*

↓  $Q_H$

*H.E* →  $W_{rev}$

↓  $Q_L$

*TL = const.*

*consider rev. H.E Carnot H.E*

$\oint \delta Q = Q_H - Q_L > 0$

$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}$

$\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$

$\oint \frac{\delta Q}{T} = 0$

*Limiting case  $\delta Q \rightarrow 0$*

*rev. H.E*

$\oint \delta Q \geq 0$

$\oint \frac{\delta Q}{T} = 0$

So let me just write down this, in the previous lectures what we have gone through is your Kelvin Planck statement or Clausius statement which was related to thermodynamic cycle in particular dealing with the high temperature source low temperature sink and the devices were operating at under cyclic conditions but what is needed is more of a qualitative information in order to come up with the explanation of a certain process to be valid or not ok or feasible or not feasible.

So in this regard the Clausius inequality is a quite a useful criteria and it is valid for all reversible or irreversible ok, process or since we are dealing with here we will still say cycle, so this circle represents cycle ok. So let us try to derive this using heat engine and refrigerator, so let us consider first a reversible heat engine ok.

So consider reversible heat engine which essentially is nothing but a Carnot heat engine ok, so you have heat engine which takes a certain amount of heat,  $Q_H$  from a high temperature source and rigid  $Q_L$  amount of heat to a low temperature sink and delivers a work which we are going to say  $W$  reversible ok.

Now let us look at the change a total heat for this particular cyclic process and this would be nothing but your  $Q_H$  minus  $Q_L$  for a given cycle ok and considering  $Q_H$  is going to be more than  $Q_L$  this is going to be greater than a zero.

Now since these are source and sink, so the temperature  $T_H$  is constant and  $T_L$  is constant, let us look at this expression  $\frac{dQ}{dT}$  which is going to be your  $Q_H$  by  $T_H$  minus  $Q_L$  by  $T_L$ . Now we know for our reversible heat engine or Carnot heat engine based on your absolute temperature the ratio of the  $Q$  is proportional temperature as done by Kelvin, so which essentially means your  $Q_H$  by  $Q_L$  is nothing but  $T_H$  by  $T_L$  for this reversible heat engine and thus your expression  $\frac{dQ}{dT}$  is nothing but zero ok.

So this is one expression for the further reversible heat engine, now if you consider limiting case when your  $W$  reversible approaches zero or in other word your  $T_H$  approaches  $T_L$  in that case your for limiting case, this will approach to zero ok.

So thus for reversible heat engine your this expression is greater than as well as equal to because this condition will arrive when your  $T_H$  approaches  $T_L$  or in in a another word your work output is zero ok and other condition would be ok. So even in the limiting case, you will have this particular condition matter. This is the for the case of a reversible heat engine ok, as your live statement.

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**The inequality of Clausius**

IRREV. cycle (HE) between the same  $T_H$  &  $T_L$ ,  $Q_H$

2nd law  $W_{irrev} < W_{rev}$

1st law  $Q_H - Q_L = W$  (rev or irrev cycle)

$Q_H - Q_{L,irrev} < Q_H - Q_{L,rev}$   
 $\Rightarrow Q_{L,irrev} > Q_{L,rev}$

$\oint \delta Q = Q_H - Q_{L,irrev} > 0$   $\rightarrow$  for increasing irreversibility

$\oint \frac{\delta Q}{T} = \frac{Q_H}{T} - \frac{Q_{L,irrev}}{T} < 0$   $-ve$

In the limit  $W_{irrev} \rightarrow 0$  for increasing irreversibility  
 $\oint \delta Q \rightarrow 0$  &  $\oint \frac{\delta Q}{T} < 0$ .

So now let us look at irreversible heat engine which is being operated at the same conditions  $T_H T_L$  ok and it takes same amount of  $Q_H$ . And let us now look at reversible heat engine operating on those conditions ok. So this is our reversible irreversible cycle or heat engine ok, between the same  $T_H$  and  $T_L$  ok and it takes the same amount of  $Q_H$  ok.

So as we know from the second law, second law tells you that for or the reversible heat engine produces maximum amount of work. So if you considering irreversible heat engine, essentially tells you that the work cross point irreversible heat engine would be less than that of a reversible heat engine, which essentially means  $W_{\text{irreversible}}$  is less than  $W_{\text{reversible}}$  ok.

Now you can make use of the first law, simple energy balance, you know that for a system containing cross point to the heat engine, your  $Q_h$  minus  $Q_l$  is nothing but work ok. this is true for reversible or irreversible cycle ok. Now you can substitute this expression here, so now we have an expression  $Q_h$  which is same for irreversible and reversible as we considered here and  $Q_l$  will change for the reversible or this should be less than  $Q_h$  minus  $Q_l$  reversible ok. Now this will cancel out which essentially means your heat loss of a heat loss during a irreversible cycle is more than that of a reversible cycle ok.

Now based on this now you can look at specifically the expression which there is a expression of  $\Delta Q$  for the cycle ok, now for the case of irreversible this would be replaced by,  $Q_l$  would be replaced by  $Q_l$  irreversible, now this is more ok but even though this is more than the  $Q_l$  reversible, the difference still will be greater than zero considering  $Q_h$  is more and what about your  $\Delta Q$  by  $\Delta T$ , so  $\Delta Q$  by  $T$  is going to be your  $Q_h$  by  $T$  minus  $Q_l$  irreversible by  $T$ . Now considering for the reversible this was zero, now this is more than  $Q_l$  reversible and considering here negative, this should be less than zero ok.

So this is expression which comes out from this analysis, now if you have a system where the irreversibility  $A$  can be increased by let us say due to the finite difference in temperature between the system and surrounding for example or fictional laws if you have more irreversibility, this term will become more negative ok. For increasing irreversibility ok and what about this, if we have more reversibility this will approach to zero for increasing irreversibility ok.

So in specific limit, in the limit let us say when your  $W$  irreversibility is zero which essentially means it approaches zero, your  $\Delta Q$  should be zero and  $\Delta Q$  by  $\Delta T$  is less than zero, so that is what exactly what we meant by increasing irreversibility ok which essentially means that the work output is also reducing and eventually it be approaching towards zero which essentially means your this conditions are going to be the case for highly irreversible process ok.

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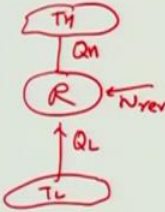
**The inequality of Clausius**

For all IRREV. HE

$$\oint \delta Q \geq 0$$

$$\oint \frac{\delta Q}{T} < 0$$

Consider REV. Refrig. cycle.



$$\oint \delta Q = Q_L - Q_H < 0$$

$$\oint \frac{\delta Q}{T} = \frac{Q_L}{T_L} - \frac{Q_H}{T_H} = 0$$

In the limit  $T_H \rightarrow T_L$   $\oint \delta Q \rightarrow 0$

$$\oint \frac{\delta Q}{T} = 0.$$

So this was the case where we discussed irreversible heat engine, so that means for all irreversible heat engine, the following condition would be true ok. Ok, so this was analysis based on the heat engine, first we did irreversible and then we did irreversible. Let us now look at the refrigerator particularly to complete the demonstration here for the any of the Clausius inequality.

So now consider a reversible refrigerator cycle ok. So which essentially means that you have a high temperature here sync and this is R it has to be some work considering reversible, this is W reversible and some heat is extracted from a low temperature reservoir ok. So let us look at this terms of delta Q for this cycle, this is going to be your Ql into the system and then this is your Qh minus Qh and considering Qh is going to be more than Ql, this because of the work supplied or work done on the system here on the refrigerator or the work input to the refrigerator, thus Ql minus Qh is going to be zero ok.

And what about delta Q by T ok, for the reversible as we already discussed this based on the Kelvin's temperature cycle, this for the reversible cycles this should be equal to zero ok. So in the limit , in the limit Th approaching Tl, your del Q should be approaching zero and your del Q by T too should remain equal to zero, ok so this would be the limit for the case ok.

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**The inequality of Clausius**

(irrev. refrigeration cycle)  
Same  $Q_L, T_L, T_H$

2nd law  $W_{irrev} > W_{rev}$  ←  $W$  required will be greater for irrev R

1st law  $Q_H - Q_L = W$

$$Q_H - Q_L > Q_{H,rev} - Q_L$$

$$\Rightarrow Q_{H,irrev} > Q_{H,rev}$$

∴ Heat rejected, for irrev is more than that of rev R.

$$\oint \delta Q = Q_L - Q_{H,irrev}$$

Now let us look at so this was a for the reversible refrigerator cycle, now let us look at for the case of irreversible ok refrigeration cycle ok. Ok and then we are considering that it receives the same considering same  $Q_L$  and the rest of the conditions such  $T_L$  and  $T_H$  are same ok. So from the second law of thermodynamics we know that for the refrigeration or reversible refrigeration cycle, the amount of work required is minimal in order to deliver the task ok, so when you have a irreversible cycle, it requires more amount of work in order to do the same operation.

So in another word we are saying that  $W_{irreversible}$  is going to be more than  $W_{reversible}$  for the case of refrigerator cycle. So input is more for the refrigerator cycle compared to the reversible cycle ok. So we know is  $W_{reversible}$  should be more than reversible, that is  $W$  required will be greater for irreversible refrigerator ok. So let us now look at first law, first law again tells you that  $Q_H$  minus  $Q_L$  is nothing but  $W$ .

You apply this for reversible and irreversible because it is true for both, so  $Q_H$  minus  $Q_L$  you apply it here, irreversible should be greater than  $Q_H$  reversible minus  $Q_L$  ok. So this essentially means  $Q_H$  irreversible is greater than  $Q_H$  reversible. So this essentially tells you that the heat rejected for the case of irreversible is more than that of reversible refrigerator cycle ok. So now let us look at your this expression of the change overall heat for the cycle, this will be your  $Q_L$  minus  $Q_H$ , now for the case of irreversible, we will put irreversible ok.

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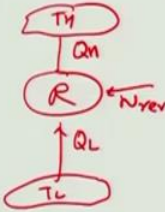
**The inequality of Clausius**

For all irrev. HE

$$\oint \delta Q \geq 0$$

$$\oint \frac{\delta Q}{T} < 0$$

consider Rev. Refrig. cycle.



$$\oint \delta Q = Q_L - Q_H < 0$$

$$\oint \frac{\delta Q}{T} = \frac{Q_L}{T_L} - \frac{Q_H}{T_H} = 0$$

In the limit  $T_H \rightarrow T_L$   $\oint \delta Q \rightarrow 0$

$$\oint \frac{\delta Q}{T} = 0$$

Now irreversible as we already mentioned here earlier that this for the reversible refrigerator cycle, this expression is always less than zero or equal to zero depending on the limiting case.

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**The inequality of Clausius**

irrev. refrigeration cycle

Same  $Q_L, T_L, T_H$

2nd law  $W_{\text{irrev}} > W_{\text{rev}}$   $\leftarrow$   $W$  required will be greater for irrev R

1st law  $Q_H - Q_L = W$

$$Q_H - Q_L > Q_{H,\text{rev}} - Q_L$$

$$\Rightarrow Q_{H,\text{irrev}} > Q_{H,\text{rev}}$$

$\therefore$  Heat rejected, for irrev is more than that of rev. R.

$$\oint \delta Q = Q_L - Q_{H,\text{irrev}} < 0$$

$$\oint \frac{\delta Q}{T} = \frac{Q_L}{T_L} - \frac{Q_{H,\text{irrev}}}{T_H} < 0$$

} increasing irrev. this becomes larger in negative

But now when you have irreversibility, this term is more which essentially this is going to be further negative and the other term  $\oint \delta Q$  by  $T$  is going to be more negative ok. So this also tells you that if you increase the irreversibility, this will become more and more negative, so this also

tells you that increasing irreversibility, this becomes larger in negative value ok or negative direction.

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**The inequality of Clausius**

Summary

For all possible rev. cycle  $\delta a \geq 0$   
 $\oint \frac{\delta Q}{T} = 0$

For possible irrev cycle  $\delta a \geq 0$   
 $\oint \frac{\delta Q}{T}$

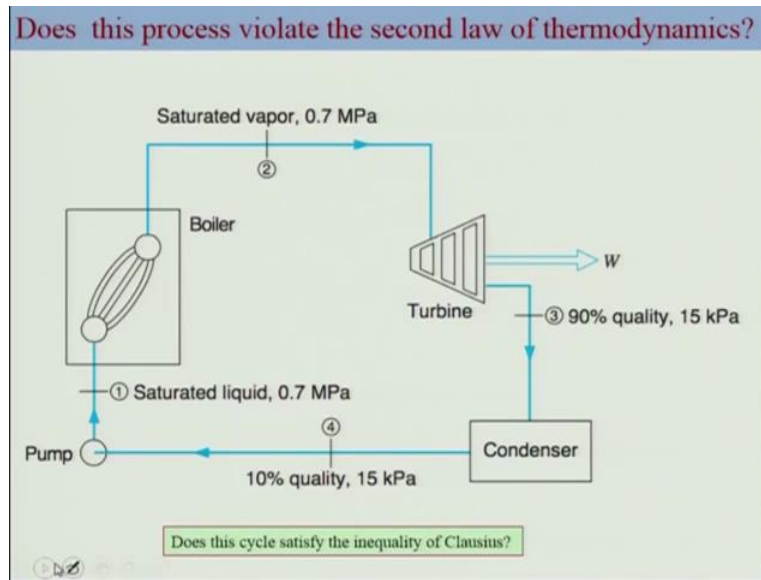
For all cycle  $\oint \frac{\delta Q}{T} \leq 0$

So let me just summarise, so this is a summary here for all possible reversible cycle, which essentially means your delta Q can be greater than zero or less than zero because it can also have your refrigerator cycle or heat cycle, your del Q by T over cycle will be zero. So this is for all possible reversible cycle and for all possible irreversible cycle again including the heat engine and as well as refrigerator where you have considered different sign of delta Q ok. Your Del Q by D is going to be less than zero.

So thus we can now conclude that for all cycle whether it is reversible or irreversible, your del Q by del T (u) should be less than or equal to zero and this is nothing but your Clausius inequality where the equality is true for reversible cycle and inequality indicates your irreversibility ok. So this is what would identify your processes ok.



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So let us look at a specific example here and ask the question that if a certain process is being defined whether this process violates the second law of thermodynamics, so now we can analyse such a problem using quantitative measure ok and that's what we are going to make use of it, we are going to try to find out what is the Clausius expression or inequality of Clausius ok. So let us look at specifically this process, here you have two particular device which either takes a heat or rejects, so that is going to be your boiler and condenser.

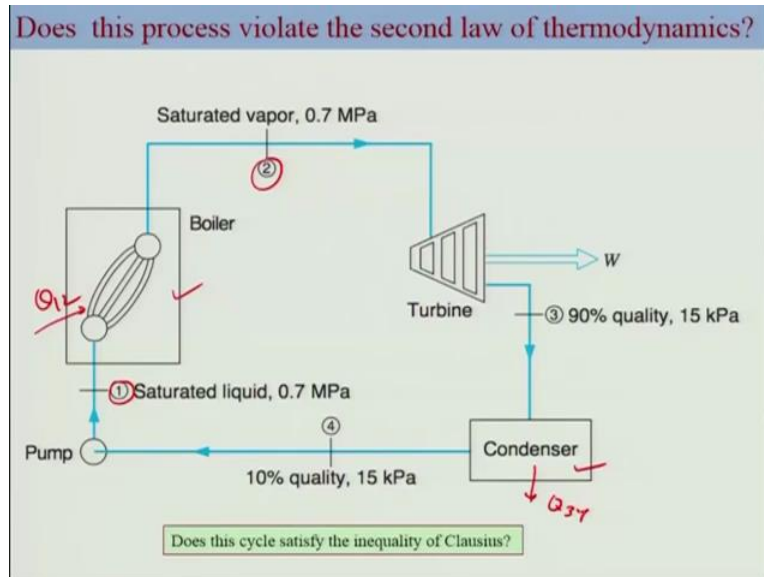
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Does this process violate the second law of thermodynamics?

$$\oint \frac{\delta Q}{T} = \int \left( \frac{\delta Q}{T} \right)_{\text{boiler}} + \int \left( \frac{\delta Q}{T} \right)_{\text{condenser}}$$

So overall your  $\frac{dQ}{dT}$  for this cycle can be simply sum of your  $\frac{dQ}{dT}$  for the boiler and that for the condenser. So without going and solving numerically all aspect of it, let us me just go through a bit of how to solve this problem.

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So let us assume this heat corresponding to the boiler is going to be your  $Q$ , so this is nothing but your 1 to 2, if you look at your process this is 1 and this is 2 so this is going to be the heat supplied to the boiler ok, so this is  $Q_{12}$  to and this is going to be your  $Q$  let us say  $Q_{34}$ , heat rejected from the condenser ok.

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Does this process violate the second law of thermodynamics?

$$\oint \frac{\delta Q}{T} = \int \left( \frac{\delta Q}{T} \right)_{\text{boiler}} + \int \left( \frac{\delta Q}{T} \right)_{\text{condenser}}$$

$$= \frac{1}{T_1} \int_1^2 \delta Q + \frac{1}{T_3} \int_3^4 \delta Q = \frac{Q_{12}}{T_1} + \frac{Q_{34}}{T_3}$$

1 kg mass basis

$$q_{12} = h_2 - h_1, \quad q_{34} = h_4 - h_3$$

$$T_1 = T_{\text{sat}} @ 0.7 \text{ MPa}, \quad T_3 = T_{\text{sat}} @ 15 \text{ kPa}$$

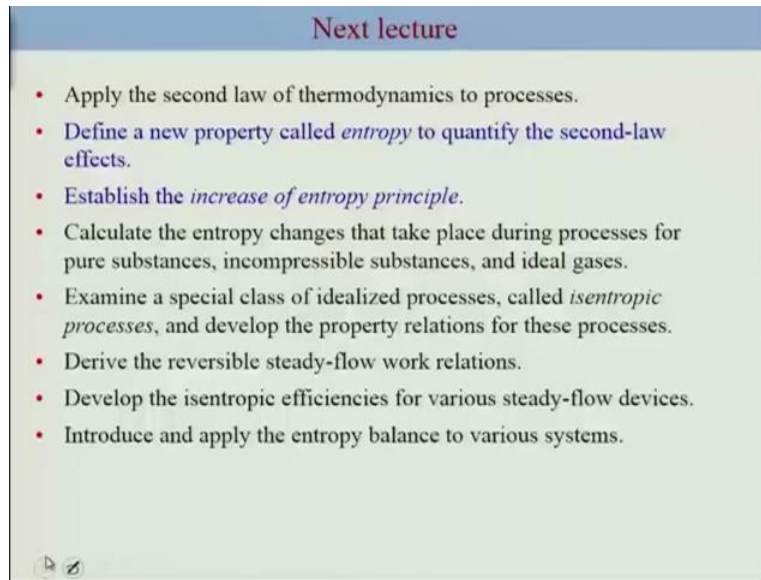
$$\oint \frac{\delta Q}{T} = \frac{q_{12}}{T_1} + \frac{q_{34}}{T_3} = -1.087 \text{ kJ/kg K}$$

So this is nothing but del Q, now the temperature for the boiler and the condenser are constant, so this is T1, this is going to be 1 by T3, 3 to 4 del Q ok. Or in other words we can write is simple Q 12 by T1 plus Q34 by T4 ok.

Now considering 1 kg mass bases ok, then we can replace this then we can write Q 12 is simply because this is saturated liquid and this is saturated vapour it is simple the vaporization change in the enthalpy between the vapour and the liquid because this is a steady flow device, so in that case is we have to make use of the control volume expression, so this is (not) nothing but going to be Q2 minus H1 ok, similarly for Q 34 this is going to be H4 minus H3 and the T1 would be your T sat ok at the pressure which is already given to you point 7 mega Pascal and T 3 is again if you look back here, this again is given the pressure 15 kilo Pascal is T sat at 15 kilo Pascal ok.

So thus you can simply plug this information and calculate your value from based on the steam table, it turns out that this for 1 kg mass of the steam, this value is minus 1.087 kilo joule per kg Kelvin. And considering is negative, this satisfies the inequality of Clausius, that means this particular process do not violate the second law of thermodynamics. So this is a very valuable tool to analyse the validity of a certain process or the feasibility of a certain process and thus it is quite useful for engineering applications ok.

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Next lecture

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

So with that I am going to end this particular lecture and in the next lecture we are going to define the term called entropy and we will analyse the increase in entropy principle and other aspect of entropy ok, so see you in the next lecture.