Engineering Thermodynamics
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Lecture 33
Entropy, increase in entropy principle, isentropic process

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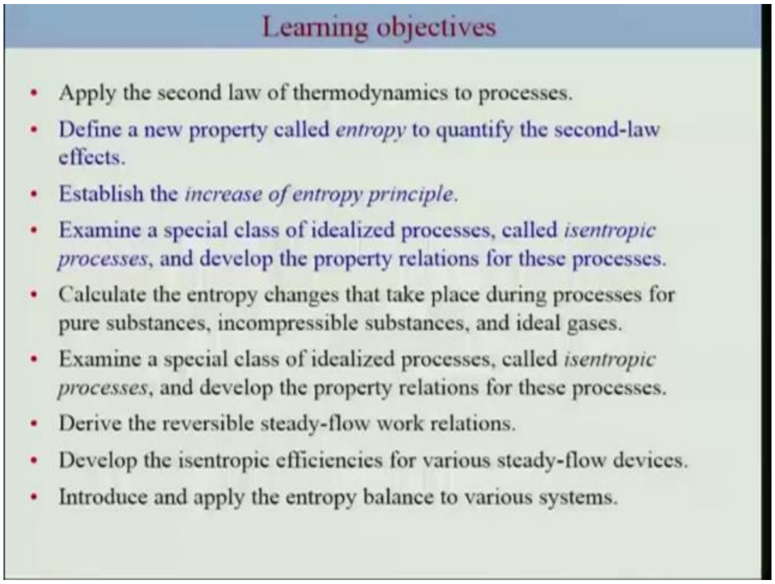

Learning objectives

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

Ok, welcome back in the last lecture we introduced very important quantitative way of finding out the feasibility of a certain process and that is inequality of process or process inequality expression ok. So now in this particular lecture we are going to make use of that defined a term called entropy and as well as we will specifically consider few particular process ok.

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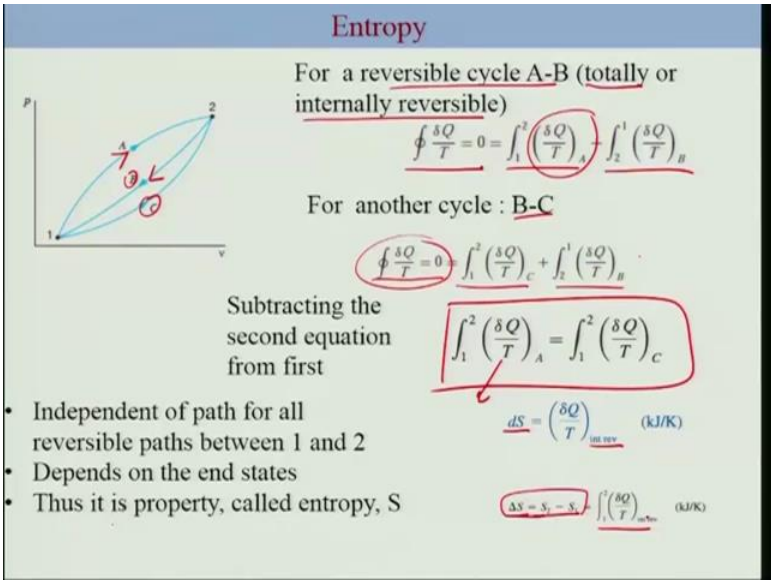

Entropy

For a reversible cycle A-B (totally or internally reversible)

$$\oint \frac{\delta Q}{T} = 0 = \int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_B$$

For another cycle : B-C

$$\oint \frac{\delta Q}{T} = 0 = \int_1^2 \left(\frac{\delta Q}{T}\right)_C + \int_2^1 \left(\frac{\delta Q}{T}\right)_B$$

Subtracting the second equation from first

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_A = \int_1^2 \left(\frac{\delta Q}{T}\right)_C$$

- Independent of path for all reversible paths between 1 and 2
- Depends on the end states
- Thus it is property, called entropy, S

$$dS = \left(\frac{\delta Q}{T}\right)_{int\ rev} \quad \text{(kJ/K)}$$

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{int\ rev} \quad \text{(kJ/K)}$$

So let me just first start with a specific reversible processes, here this is the illustration on a PV diagram you have state one and two and here you have two different ways to have a cycle. So let us consider a reversible cycle A and B, so you have A ok and B and in this reversible cycle this is reversibility would be either totally reversible which essentially means your system and surrounding and the internal reversible means just a system that means it having a very, it is undergoing a cause static process or does not have any finite temperature based heat transfer.

So for the case of A B reversible we can come up with the expression considering A B is a reversible and we know that reversible cycle the process expression is equal to zero ok, so that means the equality is there of for this del Q by del T is equal to zero which it can be simply the sum of this two path A followed by B ok for this 1 to 2 for the A plus 2 to 1 for this.

Now we can consider another cycle which will be your B and C ok, so this is for another cycle and the same expression can be written in this following form, so again it being a reversible cycle B C, this expression is zero which essentially can be calculated by this two paths B and C. Now you can subtract this from this ok, B being common which essentially means that you can write the following where 1 to 2 from path B should be same as for the case of C.

Or in other word this integral based on path A should be same as that of integral based on path C which means it basically, this particular expression is independent of path and thus this expression just depends on the final end point and this is what we call it as entropy ok because it is independent of path for all reversible paths from 1 to 2.

So we write this expression in terms of del S, so this becomes your del S, is nothing but del Q by T. We consider only at this point is internal reversible but this is as I said could be for overall total or internal reversible, so considering only the system here we are looking at only the internal reversible here and thus your del S is equal to simply your del Q by T. If you can integrate this by having a relation between Q and T, you can come up with this del S the change in entropy between state 2 and 1 ok.

Now this particular expression once you calculate ok, this will be fixed because this expression tells you only the difference between the two points 1 and 2, it doesn't depend on how this particular transformation from state 1 to 2 was achieved whether through your reversible or irreversible process, this is true for irreversible and irreversible processes as well with this particular expression. However in order to evaluate based on this interval you need to have a reversibility or internal reversibility then you can make use of this expression.

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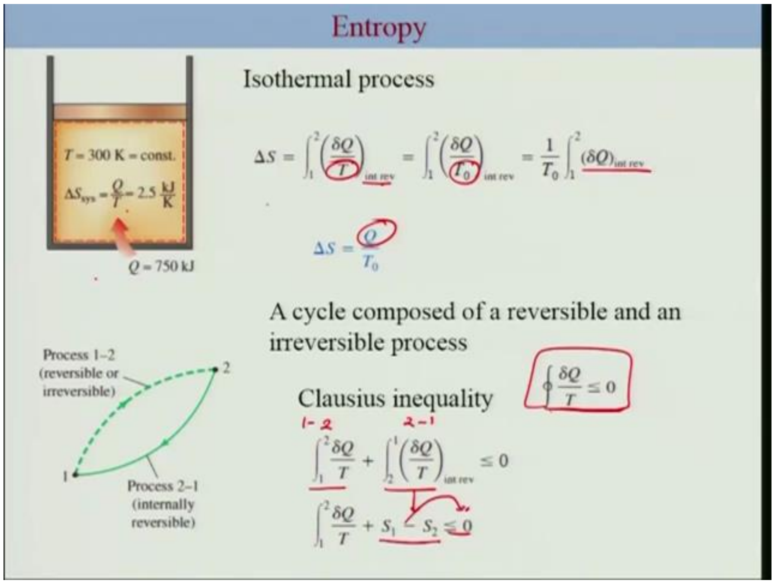

Entropy

Isothermal process

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} = \int_1^2 \left(\frac{\delta Q}{T_0}\right)_{\text{int rev}} = \frac{1}{T_0}\int_1^2 (\delta Q)_{\text{int rev}}$$

$$\Delta S = \frac{Q}{T_0}$$

A cycle composed of a reversible and an irreversible process

Clausius inequality $\boxed{\oint \frac{\delta Q}{T} \le 0}$

1-2 2-1

$$\int_1^2 \frac{\delta Q}{T} + \int_2^1 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \le 0$$

$$\int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \le 0$$

So now let us specifically consider an isothermal process ok, now for isothermal process consider this as a system and such that the heat is being supplied and the temperature is kept, now this delta S as I mentioned is already as I mentioned earlier is can be written as 1 by 2 del Q by T internal now considering the temperature is constant you can take out the temperature outside and this becomes simply the integral of 1 to 2 del Q ok and this essentially is nothing but Q which is supplied to the system considering is a internal reversible ok which essentially means that difference between the surrounding and this should not be very finite temperature difference ok.

So based on this you can calculate the entropy change of for the given example the Q is known temperature is known, so is nothing but 750 divided by 300 which essentially is 2.5 kilo juice per kelvin ok. Now let us consider a cycle which gets composed of reversible and irreversible process, so here you have state 1 and 2 the dotted line indicates the process which can be reversible or irreversible, essentially we are not aware of it but from 2 to 1 the process is internally reversible ok and what we are interested in this process inequality we write this expression.

So for this process to occur, you the process should satisfy this condition that is process inequality must hold.

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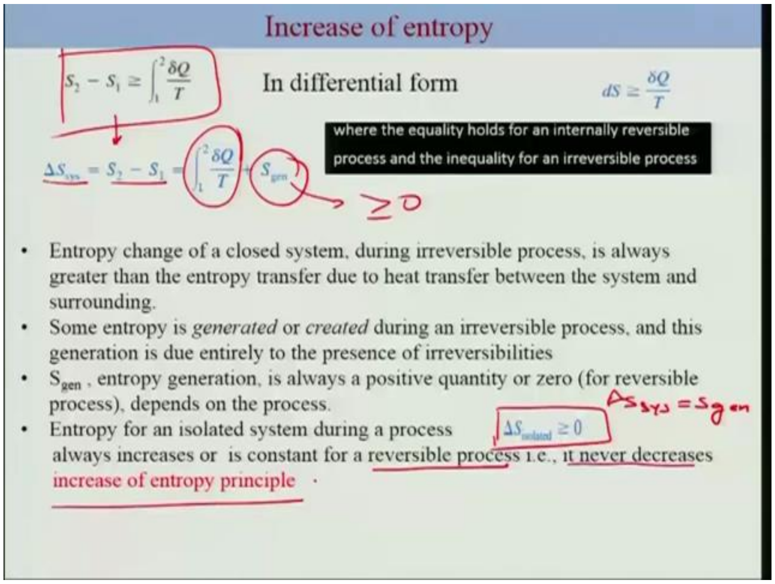

Increase of entropy

$$\boxed{S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}}$$

In differential form

$$dS \geq \frac{\delta Q}{T}$$

where the equality holds for an internally reversible process and the inequality for an irreversible process

$$\underline{\Delta S_{sys}} = \underline{S_2 - S_1} = \int_1^2 \frac{\delta Q}{T} + S_{gen} \rightarrow \geq 0$$

- Entropy change of a closed system, during irreversible process, is always greater than the entropy transfer due to heat transfer between the system and surrounding.
- Some entropy is *generated* or *created* during an irreversible process, and this generation is due entirely to the presence of irreversibilities
- S_{gen} , entropy generation, is always a positive quantity or zero (for reversible process), depends on the process.
- Entropy for an isolated system during a process $\boxed{\Delta S_{isolated} \geq 0}$ $\Delta S_{sys} = S_{gen}$ always increases or is constant for a reversible process i.e., it never decreases increase of entropy principle

Now we can write this expression by including the contribution from 2 1 process and 1 2 process, note that process equality is meant for the cyclic process ok but here we are talking contribution from individual process which is here 1 2 process and this is 2 1 process. Now 1 2 process is either irreversible or reversible which we are not aware of and this 2 1 process internal reversible thus we can write this in terms of simply the change in entropy ok and then this is less then equal to zero, now you can take it here, you can write this expression in terms of this ok.

So that means you would change in entropy between this two must be greater than equal to the integral of 1 to 2 del Q by T ok and this can be written as in differential form which we have already mentioned or in another word I can write this as that for a given change in entropy or the system S2 minus S1 that is your delta S system is the entropy due to the heat transfer or the entropy transfer due to the heat that is again due to the heat transfer plus the generation of the entropy within the system due to irreversible process or due to the presence of irreversibility, thus this is a positive value ok. This would be zero only when you have a reversible process, then you can simply write del S by is equal to 1 by 2 del Q by T ok.

So if you consider isolated system during a process which essentially means there is no heat transfer then this term is going to be zero and your delta S system will be just a simply S generation ok, considering S generation is greater than equal to zero your entropy change for an isolated system during a process will be always increasing or even constant for reversible process.

So in other word delta S isolated system will be always greater than or equal to zero, zero will be the case where for reversible process ok, that means if you have reversible process the entropy of an isolated system will never decrease or in another word or for normal process there is always an increase of entropy and this is what we call as increase of entropy principal ok.

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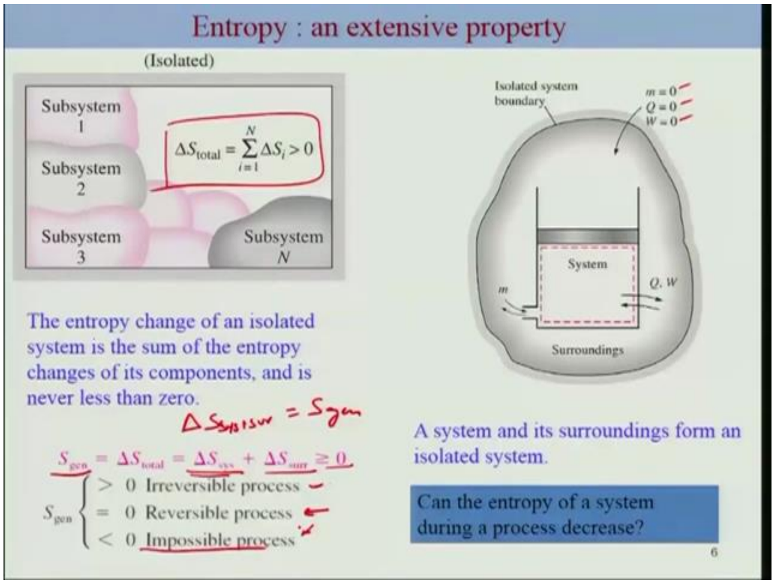

Entropy : an extensive property

(Isolated)

$$\Delta S_{\text{total}} = \sum_{i=1}^{N} \Delta S_i > 0$$

The entropy change of an isolated system is the sum of the entropy changes of its components, and is never less than zero.

$$\Delta S_{sys+sur} = S_{gen}$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

$$S_{\text{gen}} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$

A system and its surroundings form an isolated system.

Can the entropy of a system during a process decrease?

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So let me just get back to the fundamentals of entropy here, now entropy is an extensive property ok, which essentially means that the total entropy of the system is equal to the sum of the entropies of the part of the system, so we can consider an isolated system which is made of many sub systems and thus your change in the entropy of this particular isolated system is nothing but sum of the changes in the sub system as we represented here ok.

Now which essentially means this analogy can be taken to system plus surrounding conditions where you can isolate this system plus surrounding which essentially means there is no change in mass Q or heat and work transfer though there would be exchange of the mass heat and work between the surrounding and system but overall when we consider surrounding and system, we can consider that to be isolated system boundary. In that case the change in the entropy of this isolated system would be just a sum of your system change plus the surrounding change and that must be greater than zero ok.

Considering isolated system your delta system plus surrounding should be S generation ok, thus S generation is nothing but delta sis plus delta surrounding ok. So this tells you that universe can be considered as a isolated system and thus S generation of the universe which is an isolated system should be greater than equal to zero which means this entropy of the universe keeps increasing all the time.

Now S generation should be zero as we already mentioned for reversible process and for irreversible process should be greater than zero which essentially means it can never be less than zero which is a impossible process ok, so the question is can the entropy of a system during a process decrease.

As we already mentioned that for entropy of an isolated system should keep increasing all the time or should remain constant for our reversible process but what about the system here. Now system can have heat and work interaction ok, as well as mass flow from the surrounding or to the surrounding.

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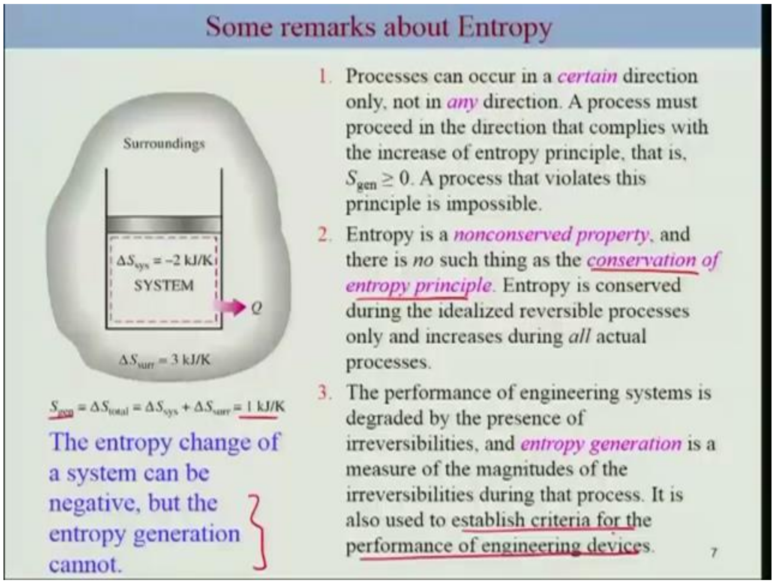

Some remarks about Entropy

$S_{gen} = \Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} = 1$ kJ/K

The entropy change of a system can be negative, but the entropy generation cannot.

1. Processes can occur in a *certain* direction only, not in *any* direction. A process must proceed in the direction that complies with the increase of entropy principle, that is, $S_{gen} \geq 0$. A process that violates this principle is impossible.
2. Entropy is a *nonconserved property*, and there is *no* such thing as the *conservation of entropy principle*. Entropy is conserved during the idealized reversible processes only and increases during *all* actual processes.
3. The performance of engineering systems is degraded by the presence of irreversibilities, and *entropy generation* is a measure of the magnitudes of the irreversibilities during that process. It is also used to establish criteria for the performance of engineering devices.

So in other word your entropy change for a system can be negative. This is an example, for example if you consider this is isolated system plus surrounding, the system loses heat to the surrounding ok and thus your system entropy decreases because your heat contribution to the change in the entropy is negative and thus your delta S system is negative. However the surrounding increases the overall S generation is positive, which is what the condition is.

So the entropy change of system can be negative but th e entropy generation cannot be ok, of the overall system and the surrounding ok. Let me just touch up on the key points here about the entropy what we know the process should occur in the direction where the S generation is greater than or equal to zero ok, for the reversible process it should be zero but otherwise the process should always increase, produce or create entropy.

Now the other important aspect is that entropy is a non-conserved property ok, there is nothing called conservation of a entropy principal, only for the reversible case entropy will remain constant ok. But otherwise the actual processes the entropy will keep increasing ok. The other aspect is considering that S generation is synonymous for irreversibility which essentially means the higher the irreversibility, higher the S generation is. So the performance of a device is can be analysed by looking at just S generation or the amount of entropy being generated ok.

So the higher the entropy is generated which means the performance is decreasing ok of the particular device. So this analysis can be used to establish criteria for the performance of engineering devices ok, that is based on entropy generation or based on our discussion what is evident is that entropy is a property ok and out interest is to find out the change in the entropy, the change in the entropy can be calculated from the measurable properties.

However it is very cumbersome and thus it is easy to derive the values from with certain reference and put it in a table as is done for steam and refrigerator fluids ok.

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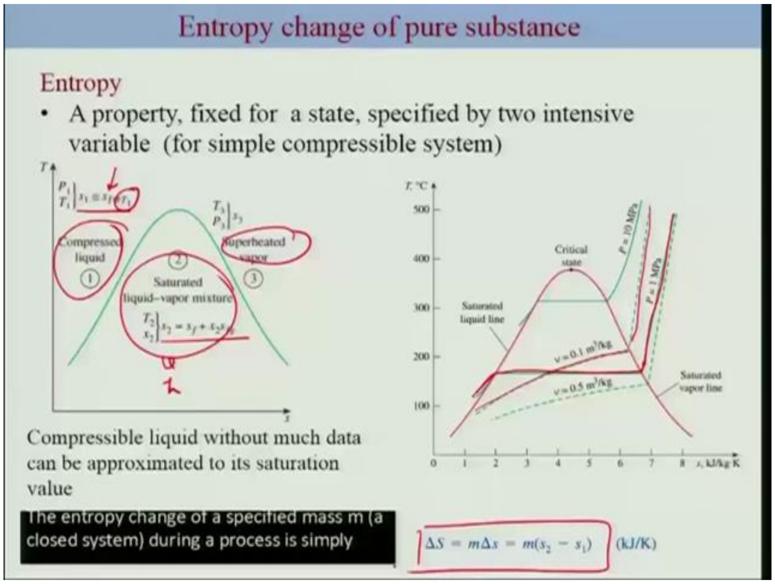

Entropy change of pure substance

Entropy

- A property, fixed for a state, specified by two intensive variable (for simple compressible system)

Compressible liquid without much data can be approximated to its saturation value

The entropy change of a specified mass m (a closed system) during a process is simply

$$\Delta S = m\Delta s = m(s_2 - s_1) \quad \text{(kJ/K)}$$

So typical TS diagram, temperature versus entropy diagram for your pure substance appears in a similar form as for the case of your PV diagram of TV diagram ok. So this is your just a schematic diagram for your pure substance and in a similar we have seen for your TV diagram of PV diagram, you have the left region which is a compressed liquid, the regions between the saturated liquid vapour dome is your saturate liquid mixture and this is your superheated vapour ok.

Now for the compressed liquid we will be making a assumption that the entropy is just approximated with the saturated liquid at the specific temperature for your P1 and T1, again remember that liquid properties are more dependent on the temperature than the pressure and thus we are going to make use of Sf at T1 ok, for the case of S1.

Now for the saturated liquid vapour mixtures we are going to just simply use similar expression as done for your Vo and Ho ok and for superheated vapour this would be the region which will have again based on the table, so the table contains just like your table A4 and A5 which we have used earlier has the datas for the entropy but this entropy calculations are with certain reference such as your triple point for water ok.

The other aspect if you look at your T and S diagram as we know this entropy of the vapour is going to be higher because note that the change in the entropy has a contribution to the heat transfer del Q by T and thus your certain contribution comes from the heat transfer. Now this is a typical your iso bars ok, that is here and ISO course are not constant within the dome, saturated liquid vapour region and it has a higher slope compared to that of ISO bars. So this is a typical diagram for T and S ok.

So for the case of entropy change of a close system and specified mass, the change can be given as in this form ok, so we will be making use of the tables having this entropy values for the case of your steam and refrigerator fluids and based on this diagram we can easily calculate in the same way as we have done for TV and your PV diagrams and this becomes very useful property diagram for calculation of processes for the enthalpy plays a big role and we will be making use of such diagram in our calculations shortly.

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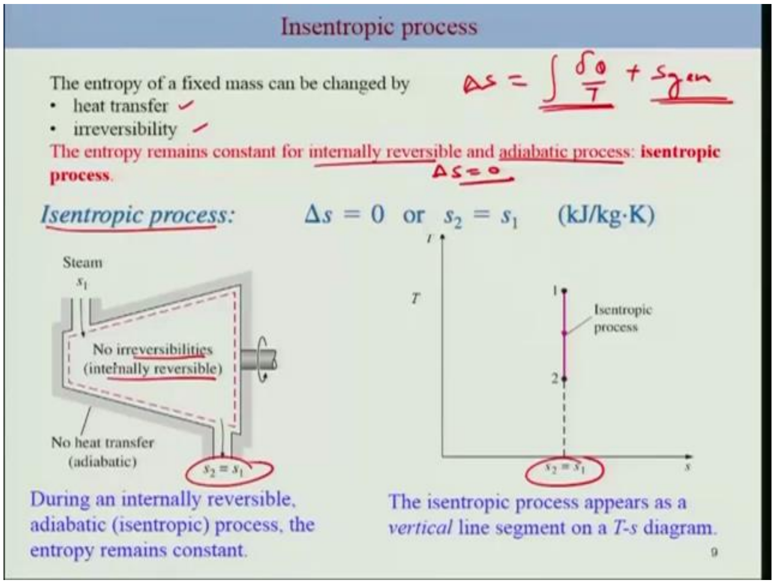

Insentropic process

The entropy of a fixed mass can be changed by
- heat transfer ✓
- irreversibility ✓

$$\underline{\Delta s = \int \frac{\delta Q}{T}} + \underline{S_{gen}}$$

The entropy remains constant for internally reversible and adiabatic process: **isentropic process**.

$\underline{\Delta S = 0}$

Isentropic process: $\Delta s = 0$ or $s_2 = s_1$ (kJ/kg·K)

During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.

The isentropic process appears as a *vertical* line segment on a *T-s* diagram.

Ok so let us take a specific process in this process you have, we call it as isentropic process, so let us just look at what does the change in entropy for a fixed mass constitutes of. So the entropy of a fixed mass can be changed by heat transfer ok and reversibility, ok as we already mentioned delta S is del Q by T plus S generation ok.

So this is for a process, so if you want to change entropy of a system, you can do that by heat transfer or for any other reversibility, so S generation is due to reversibility and this is your heat transfer. Ok for entropy if you consider to be internal reversible which essentially means this S generation is zero and if you consider your adiabatic then delta S should be zero or S should be constant or delta S should be zero.

So this would be the case for internal reversal or adiabatic process ok and if you plot your TS diagram this would be a vertical Y ok because at a specific constant. So this example of isentropic process where the entropy is constant could be given in the form of adiabatic turbine ok where essentially you have no irreversibility internally reversible and no heat transfer ok and then your S is it going to be constant in such a process ok. So this is an example of adiabatic turbine where the entropy is constant.

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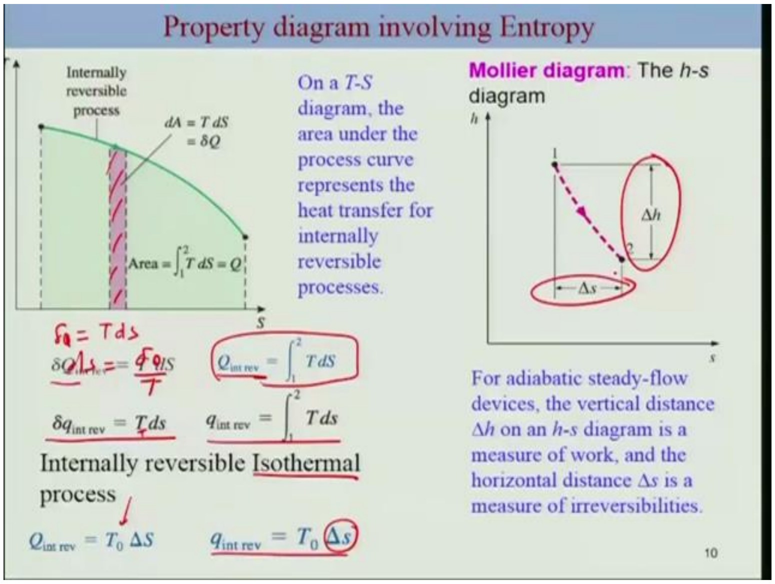

Property diagram involving Entropy

On a *T-S* diagram, the area under the process curve represents the heat transfer for internally reversible processes.

Mollier diagram: The *h-s* diagram

For adiabatic steady-flow devices, the vertical distance Δh on an *h-s* diagram is a measure of work, and the horizontal distance Δs is a measure of irreversibilities.

$\delta q = T ds$

$ds = \frac{\delta q}{T}$

$$\boxed{Q_{\text{int rev}} = \int_1^2 T\,dS}$$

$$\delta q_{\text{int rev}} = T\,ds \qquad q_{\text{int rev}} = \int_1^2 T\,ds$$

Internally reversible Isothermal process

$$Q_{\text{int rev}} = T_0\,\Delta S \qquad q_{\text{int rev}} = T_0\,\Delta s$$

Ok, so the, as I mentioned the property diagram becomes extremely useful to explain process and similar to your PV diagram or TV diagram where we looked into the processes related to first law of thermodynamics for a case of processes where you have to apply your cycle of thermodynamics the TS diagram or you HS diagram becomes quite useful diagram for explaining or analysing the process.

So this is an illustration of your TS diagram where this curve is of course the internal reversible process and this area here would be nothing but your Tds ok. So this Tds would be your delta Q considering your Ds is for the internally reversible is nothing but delta Q by T ok, you can analyse your differential heat supplied or heat transferred to the system. So on a TS diagram the area under the process curve represent the heat transform into the reversible processes, so this is what I mentioned here, del Q internal reversible is Tds or Q internal reversible would be simply the integral of this.

You can write this expression in terms of your per unit mass ok, so for the internally reversible as a thermal process, this T is going to be constant, T will become your T zero and if you integrate it, this becomes simply delta S for the unit mass bases, this can be written as simple T zero delta small s ok. So enthalpy entropy diagram is quite valuable in the analysis of study flow devices such as turbine compressor and nozzle ok.

So for the case of adiabatic steady flow devices this the change in the enthalpy is a major of a work output for example for turbine and the change in the entropy is a measure of your irreversibility associated with the process. So thus your property diagram such as your TS and HS becomes a great visual aids in the thermodynamic analysis of the processes where the second analysis have to be done ok.

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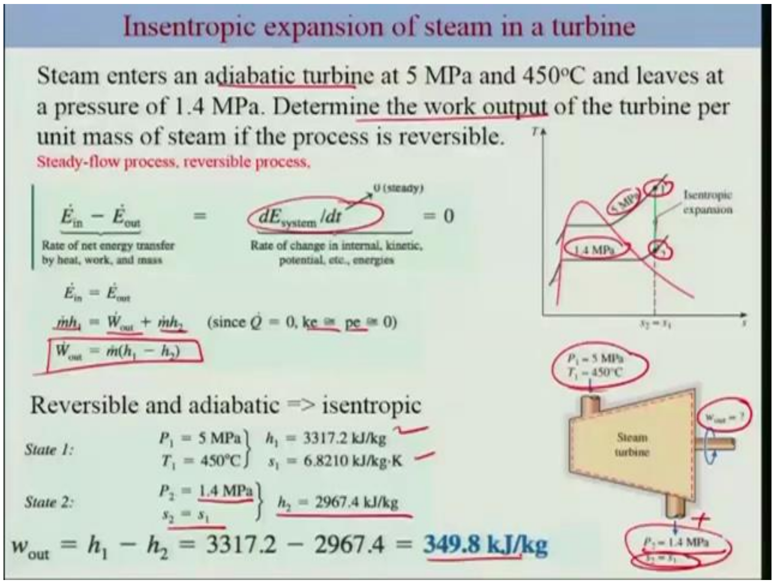

Insentropic expansion of steam in a turbine

Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at a pressure of 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible.

Steady-flow process, reversible process.

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{dE_{system}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{0\ (\text{steady})}{} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{out} + \dot{m}h_2 \quad (\text{since } \dot{Q} = 0,\ \text{ke} \cong \text{pe} \cong 0)$$

$$\boxed{\dot{W}_{out} = \dot{m}(h_1 - h_2)}$$

Reversible and adiabatic => isentropic

State 1: $\left.\begin{array}{l} P_1 = 5 \text{ MPa} \\ T_1 = 450°\text{C} \end{array}\right\} \begin{array}{l} h_1 = 3317.2 \text{ kJ/kg} \\ s_1 = 6.8210 \text{ kJ/kg·K} \end{array}$

State 2: $\left.\begin{array}{l} P_2 = 1.4 \text{ MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 2967.4 \text{ kJ/kg}$

$$w_{out} = h_1 - h_2 = 3317.2 - 2967.4 = 349.8 \text{ kJ/kg}$$

So let me just conclude this particular lecture with this example, this is a example of isentropic expansion of steam in a turbine, here the steam enters in adiabatic turbine at 5 mega Pascal, temperature is given here or 50 at least at 1.4 mega Pascal. Considering adiabatic turbine ok, so this entropy is constant what we need to do is determine the work flow this is your diagram TS diagram, so this is the iso bars 1.4 mega Pascal, 5 mega Pascal is here, these are all as superheated vapour, one is here and considering as a constant entropy is a vertical line drop here and this is your process path.

So we need to find out the work output, so one can start assuming first with a steady flow device is a reversible which especially means that your mass your entropy energy of this particular system or control volume here is going to be constant. The energy balances can be written in this form E in minus E out and considering this steady flow process, this is going to be zero ok.

So thus you can write your E in, E in is nothing but your contribution due to your mass flow, so M H1 considering your Ke and each prevents zero or the changes are zero ad Ke and Pe and E out is your W out plus the contribution due to mass flow out of the control volume. So this is going to be W out plus Ms, so you can write this W out in simple mass weight multiplied by the change in the enthalpy ok. So what you need to find out is simply this enthalpy of state 1 and state 2 ok.

Now the state 1 you can take out the table of a superheated vapour it is given here H1 and S1, the considering that that for the case of state 2 you have the information of 1.4 mega Pascal and the entropy again you make use of the table and find out where this condition meets that means P2 is this and S1 is this and you may have to do some interpolation to maintain the value of S2 ok.

So this is a generic process to solve such a problem based on the information of entropy which basically in this case we have considered entropy to be constant ok, so W out for this problem turn out to be your 349.8 kilo juice per kg.

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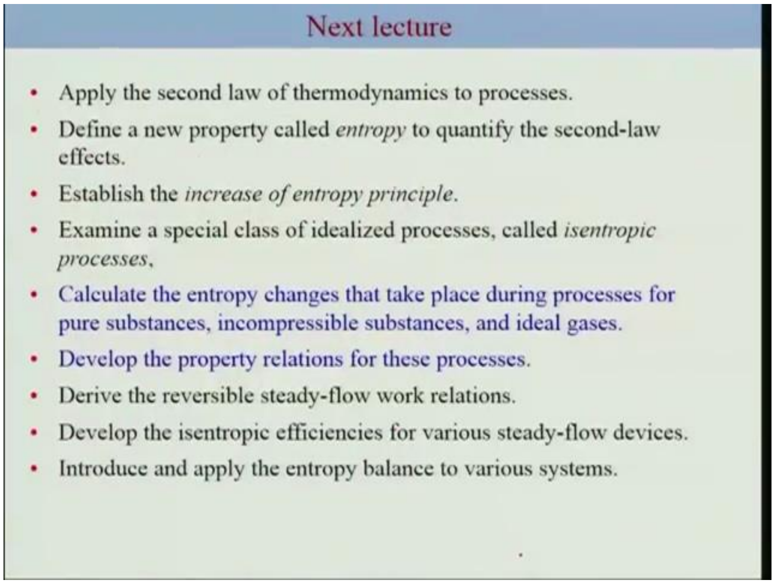

Next lecture

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Examine a special class of idealized processes, called *isentropic processes*.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

So that wold the end of this particular lecture, so hope it is clear the definition of entropy, making use of clauses in inequality, the process diagram for related to the entropy or particularly cycle of thermodynamics and how to make use of it in order to solve certain problem ok. In the next lecture we are going to explain this entropy change for ideal gas and we look into some other examples, ok so see you in the next lecture.