Engineering Thermodynamics
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Lecture 37
Examples related to entropy change in a system

Hello and welcome back. Today we will be discussing few problems related to entropy.

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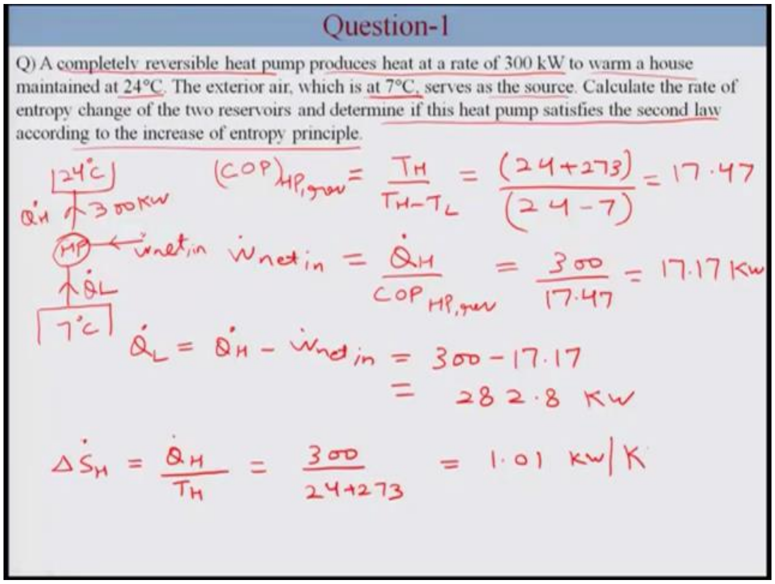

Question-1

Q) A completely reversible heat pump produces heat at a rate of 300 kW to warm a house maintained at 24°C. The exterior air, which is at 7°C, serves as the source. Calculate the rate of entropy change of the two reservoirs and determine if this heat pump satisfies the second law according to the increase of entropy principle.

$$(COP)_{HP,rev} = \frac{T_H}{T_H - T_L} = \frac{(24+273)}{(24-7)} = 17.47$$

$$\dot{W}_{net\,in} = \frac{\dot{Q}_H}{COP_{HP,rev}} = \frac{300}{17.47} = 17.17\ \text{KW}$$

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{net\,in} = 300 - 17.17$$
$$= 282.8\ \text{KW}$$

$$\Delta \dot{S}_H = \frac{\dot{Q}_H}{T_H} = \frac{300}{24+273} = 1.01\ \text{KW/K}$$

In the first problem a completely reversible heat pump produces heat at a rate of 300kW to warm a house maintained at 24 degrees Celsius, the exterior air which is at 7 degrees Celsius serves as the source. Calculate the rate of entropy change of the two reservoirs and determine if this heat pump satisfies the second law according to the increase of entropy principle.

So in this question there is a heat pump, which is reversible it produces heat at a rate of 300kW so QH dot is equal to 300kW. To warm a house with maintained at 24 degree Celsius, so this temperature is 24 degree Celsius. The exterior air which is at 7 degree Celsius serve as the source so it takes heat from the exterior air which is at 7 degree Celsius, so heat taken from rate of heat taken will be QL dot.

Calculate the rate of entropy change of the two reservoirs. Work input is, rate of work input is W dot net. So the COP for heat pump is TH divided by TH minus TL considering it as a reversible heat pump. So TH is 24 plus 73 divided by 24 minus 7 so this is equal to 17 point 47 this is the COP of reversible heat pump. Now the power required to drive this heat pump is W net and this is equal to QH dot divided by COP of this reversible heat pump and this is equal to 300 divided by COP which is 17 point 47 comes out to be 17 point 17kW.

Now the rate average heat is removed from the low temperature energy reservoir is QL dot is equal to QH dot minus W net input this is equal to 300 minus 17 point 17, so it gives 282 point 8 kW. Change in entropy of high temperature reservoir is delta S or its rate is Q dot H divided by the TH, so this TH is 24 degree Celsius so it is 300 divided by 24 plus 273 and it is 1 point 01 kW per Kelvin.

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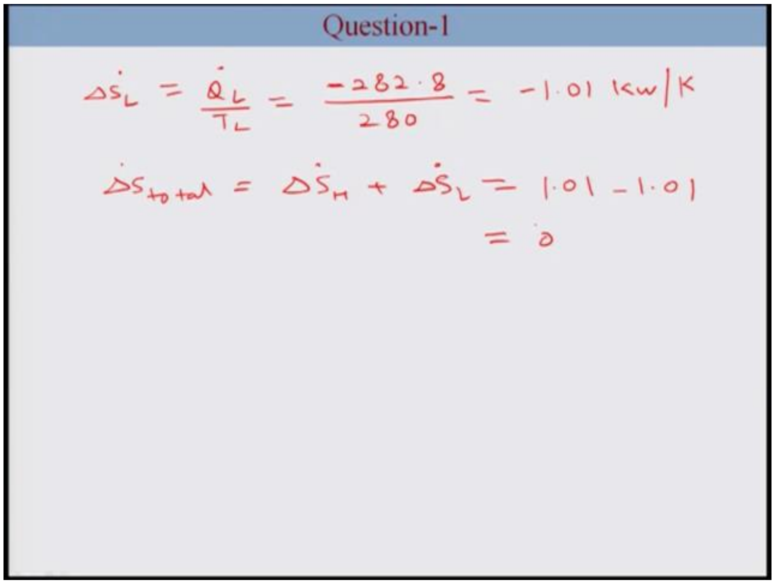

Question-1

$$\Delta \dot{S}_L = \frac{\dot{Q}_L}{T_L} = \frac{-282.8}{280} = -1.01 \text{ kw/K}$$

$$\Delta \dot{S}_{total} = \Delta \dot{S}_H + \Delta \dot{S}_L = 1.01 - 1.01$$

$$= 0$$

And rate of change in entropy of low temperature reservoir will be delta SL dot and it is equal to QL dot divide by TL. So it is equal to minus 282 point 8 divided by 280. This negative sign is because the heat is being removed from the low temperature reservoir that is why this negative sign is coming. So it will give a value of minus 1 point 01 kW per Kelvin. Now the net rate of entropy change of everything in the system is that is delta S dot total is equal to delta SH dot plus delta SL dot and this comes out to be 1 point 01 minus 1 point 01 and this is zero. It satisfies the second law according to the increase of entropy principle.

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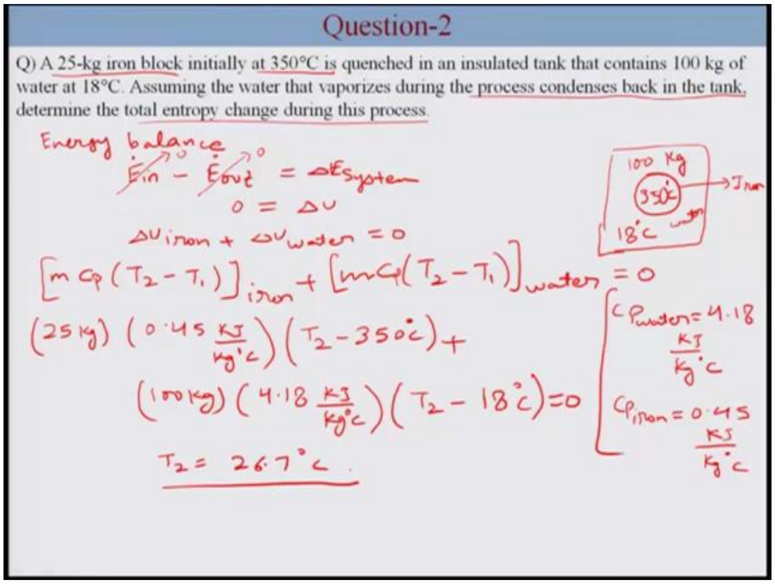

Question-2

Q) A 25-kg iron block initially at 350°C is quenched in an insulated tank that contains 100 kg of water at 18°C. Assuming the water that vaporizes during the process condenses back in the tank, determine the total entropy change during this process.

Energy balance

$$\cancel{\dot{E}_{in}}^{0} - \cancel{\dot{E}_{out}}^{0} = \Delta E_{system}$$

$$0 = \Delta U$$

$$\Delta U_{iron} + \Delta U_{water} = 0$$

$$\left[m\,C_p(T_2 - T_1)\right]_{iron} + \left[m\,C_p(T_2 - T_1)\right]_{water} = 0$$

$$(25\,kg)\left(0.45\,\frac{KJ}{kg^\circ C}\right)(T_2 - 350^\circ C) + (100\,kg)\left(4.18\,\frac{KJ}{kg^\circ C}\right)(T_2 - 18^\circ C) = 0$$

$$\underline{T_2 = 26.7^\circ C}$$

$C_{P_{water}} = 4.18\ \frac{KJ}{kg^\circ C}$

$C_{P_{iron}} = 0.45\ \frac{KJ}{kg^\circ C}$

Moving on to the next question here a 25 kg iron block initially at 350 degree Celsius. So there is an iron block at 350 degree Celsius is quenched in an insulated tank, so this is a tank which is insulated and it contains 100kg of water at 18 degree Celsius. So this is water and inside an iron block is quenched. Assuming that the water that vaporizes during the process condenses back in the tank. Determine the total entropy change during this process.

So taking all the contains of the tank as a system, So water and iron as a system and applying energy balance. E in dot minus E out is equal to delta E of the system. Now nothing is coming in and nothing is going out so both are zero. So zero is equal to change in energy of system will be equal to change in the internal energy of the system so that is delta U. So this can be written as delta U is change in internal energy of the iron plus change in internal energy of the water and this is equal to zero.

So the internal energy change for iron will be mass multiplied by the specific heat and the change in temperature, so this is for iron and similar expression will be there for water. Mass multiplied by specific heat to T2 minus T1 for water. T1 and T2 are different for iron and water so we will put the corresponding values and CP for water is equal to 4 point 18 kiloJoule per kg degree Celsius and CP for iron is 0 point 45 kiloJoule per kg degree Celsius.

T1 for iron and water will be different although the T2 will be same for both as it is the (())(07:22) temperature. So putting all the values in these expression mass of iron is 25 kg, multiplying this by C of water or Cp of water. Mass of iron is 25 kg Cp of iron is 0 point 45 kiloJoule per kg degree Celsius and T2 minus T1, so T2 is not known and T1 is 350 degree Celsius. Similarly for water mass is 100 kg, Cp is 4 point 18 kiloJoule per kg degree Celsius and T2 is unknown minus T1 is 18 degree Celsius and this whole is equal to 0. From this equation we get T2 is equal to 26 point 7 degree Celsius.

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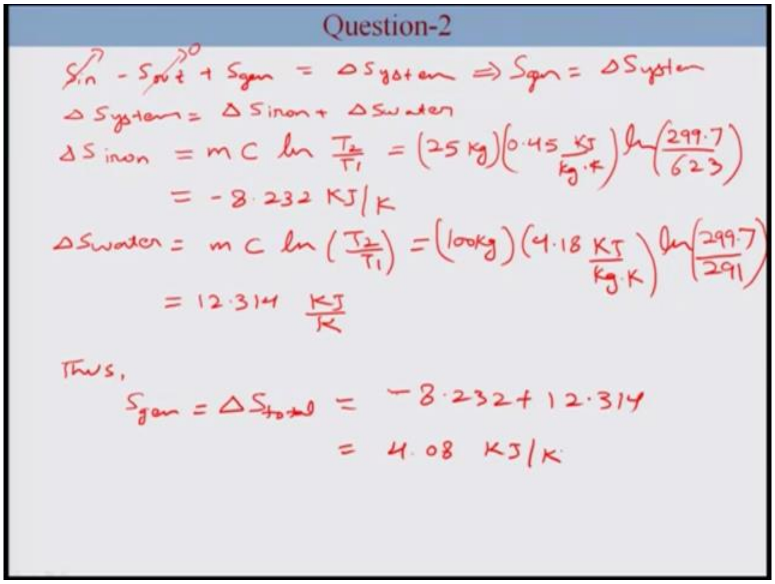

Question-2

$$\cancelto{}{S_{in}} - \cancelto{0}{S_{out}} + S_{gen} = \Delta S_{system} \Rightarrow S_{gen} = \Delta S_{system}$$

$$\Delta S_{system} = \Delta S_{iron} + \Delta S_{water}$$

$$\Delta S_{iron} = mC \ln \frac{T_2}{T_1} = (25\ kg)\left(0.45 \frac{KJ}{kg \cdot K}\right) \ln\left(\frac{299.7}{623}\right)$$

$$= -8.232\ KJ/K$$

$$\Delta S_{water} = mC \ln\left(\frac{T_2}{T_1}\right) = (100kg)\left(4.18 \frac{KJ}{kg \cdot K}\right) \ln\left(\frac{299.7}{291}\right)$$

$$= 12.314\ \frac{KJ}{K}$$

Thus,

$$S_{gen} = \Delta S_{total} = -8.232 + 12.314$$

$$= 4.08\ KJ/K$$

So we have to determine the entropy change during this process so applying entropy balance S in minus S out plus S gen entropy generated is equal to change in entropy of the system. Now S in and S out will be 0 as nothing is coming out and going. So S gen or entropy which is generated is equal to change in entropy of the system.

Now change I the entropy of the system is equal to change in entropy of iron plus change in entropy of water. The change in entropy of iron this is equal to mass multiplied by specific heat into LN T2 by T1 so this is equal to 25 which is the mass multiplied by the specific heat 0 point 45 kiloJoule per kg Kelvin into LN of T2 as we have calculated previously divided by T1, so this gives delta S for iron as minus 8 point 232 kiloJoule per Kelvin.

Similarly change in entropy of water is equal to mass of water multiplied by the specific heat and into LN T2 by T1 for water. So this is equal to 100 kg multiplied by 4 point 18 kiloJoule per kg Kelvin into LN 299 point 7 divided by 291. So this is equal to 12 point 314 kiloJoule per kg. So this will give S gen which is equal to change in entropy of the system which is equal to delta S iron plus delta S of water. So minus 8 point 232 plus 12 point 314 and this is equal to 4 point 08 kiloJoule per Kelvin.

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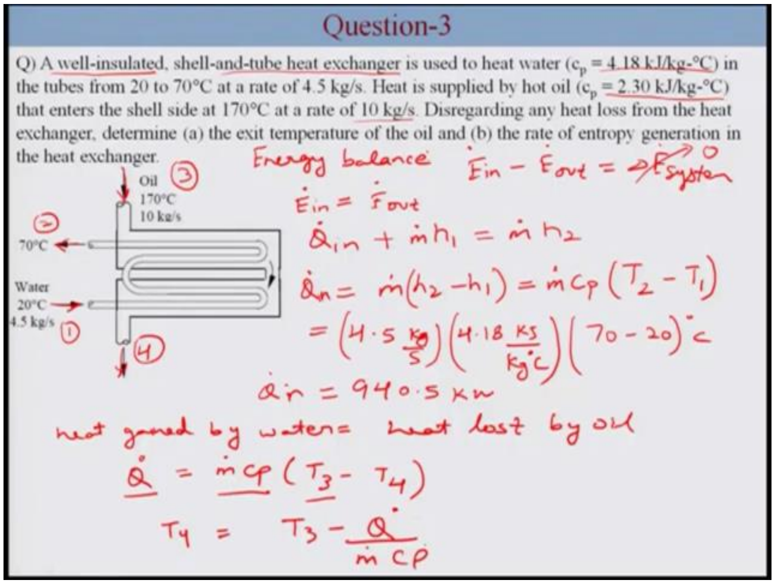

Question-3

Q) A well-insulated, shell-and-tube heat exchanger is used to heat water (c_p = 4.18 kJ/kg-°C) in the tubes from 20 to 70°C at a rate of 4.5 kg/s. Heat is supplied by hot oil (c_p = 2.30 kJ/kg-°C) that enters the shell side at 170°C at a rate of 10 kg/s. Disregarding any heat loss from the heat exchanger, determine (a) the exit temperature of the oil and (b) the rate of entropy generation in the heat exchanger.

Energy balance $\dot{E}_{in} - \dot{E}_{out} = \cancel{\Delta \dot{E}_{system}}^{\,0}$

$\dot{E}_{in} = \dot{E}_{out}$

$\dot{Q}_{in} + \dot{m} h_1 = \dot{m} h_2$

$\dot{Q}_{in} = \dot{m}(h_2 - h_1) = \dot{m} c_p (T_2 - T_1)$

$= \left(4.5 \frac{kg}{s}\right)\left(4.18 \frac{kJ}{kg\,^\circ C}\right)(70 - 20)^\circ C$

$\dot{Q}_{in} = 940.5\ kW$

heat gained by water = heat lost by oil

$\dot{Q} = \dot{m} c_p (T_3 - T_4)$

$T_4 = T_3 - \frac{\dot{Q}}{\dot{m} c_p}$

Moving onto the next question in this a well- insulated shell and the tube heat exchanger, so this heat exchanger is well-insulated is used to heat a water. So Cp is given of water in tubes from 20 to 70 degree Celsius, so water is entering at 20 degree Celsius and it is going out at 70 degree Celsius. So this point is taken as 1 this as 2.

Heat is supplied by hot oil and Cp is 2 point 30 that enters the shell 170 degree Celsius. So this enters at 170 degree Celsius from here so we take this as point 3 at a rate of 100kg per second. This regarding any heat loss from the heat exchanger. Determine the exit temperature of the oil. hh this temperature we need to find out and the rate of entropy generation in the heat exchanger. So this we mark as point 4.

WE take the cold water tubes as the system so if we apply the energy balance on the water tubes only so, E in rate of energy which is coming in minus rate of energy which is going out is equal to change rate of change of energy of the system. Now as it is a study system, so this will be zero.

E in dot is equal to E out now rate of energy which is entering is Q in dot the heat which is added to the water plus the enthalpy of the water m dot (())(12:58) mass floated of water into enthalpy of water is equal to the net energy which the water is taking with itself and it is moving out of the heat exchanger so that will be mass of water multiplied by the enthalpy of water at point 2.

Now the rate of heat transfer to the cold water and this heat exchanger becomes Q dot in is equal to m dot h2 minus h1. So this can be written in terms of specific heat and temperature difference so this is m dot Cp of water multiplied by T out of water is T2 minus T1, so putting all the values. Mass floated of water is 4 point 5 kg per second. Specific heat is 4 point 18 kiloJoule per kg degree Celsius and temperature difference is 70 minus 20 degree Celsius.

So this comes out to be 940 point 5 kW, now we know that the amount of heat gained by water will be equal to the heat loss by oil. So heat (gained by water is equal to heat lost by oil) so for oil Q dot is equal to m Cp T3 minus T4 so Q dot is known. We know T3, we know m dot and CP all for all. So T4 is equal to T3 minus Q dot divided by m dot into CP.

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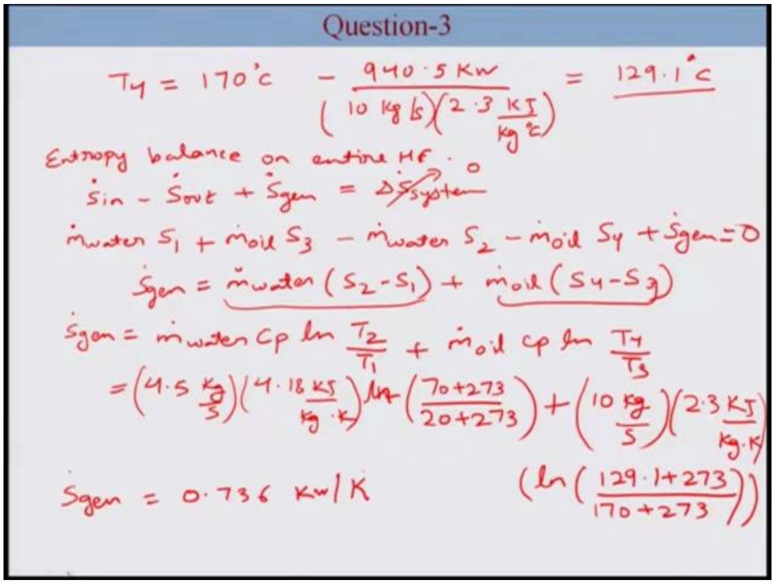

Question-3

$$T_4 = 170^\circ C - \frac{940.5\ \text{KW}}{(10\ \text{kg/s})\left(2.3\ \frac{\text{KJ}}{\text{kg}\,^\circ C}\right)} = \underline{129.1^\circ C}$$

Entropy balance on entire HF.

$$\dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen} = \cancel{\Delta \dot{S}_{system}}^{\,0}$$

$$\dot{m}_{water} S_1 + \dot{m}_{oil} S_3 - \dot{m}_{water} S_2 - \dot{m}_{oil} S_4 + \dot{S}_{gen} = 0$$

$$\dot{S}_{gen} = \underbrace{\dot{m}_{water}(S_2 - S_1)} + \underbrace{\dot{m}_{oil}(S_4 - S_3)}$$

$$\dot{S}_{gen} = \dot{m}_{water} C_p \ln \frac{T_2}{T_1} + \dot{m}_{oil} c_p \ln \frac{T_4}{T_3}$$

$$= \left(4.5\ \frac{\text{kg}}{\text{s}}\right)\left(4.18\ \frac{\text{KJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{70+273}{20+273}\right) + \left(10\ \frac{\text{kg}}{\text{s}}\right)\left(2.3\ \frac{\text{KJ}}{\text{kg}\cdot\text{K}}\right)\left(\ln\left(\frac{129.1+273}{170+273}\right)\right)$$

$$\dot{S}_{gen} = 0.736\ \text{Kw/K}$$

So this will give T4 is equal to 170 degree Celsius which is T3 minus 940 point 5 kW divided by 10 which is the mass floated and CP which is 2 point 3 kiloJoule per kg degree Celsius. So this gives a value of T for as 129 point 1 degree Celsius, so this is the exit temperature of the coil. Now the second problem is the rate of entropy generation in the heat exchanger, so if we apply entropy balance on the entire heat exchanger, so it is S in this is in the rate form S in minus S out dot plus rate of entropy generation is equal to rate of change of entropy of this system.

For a study flow system this term is zero, so the entropy which is entering is mass flow rate of water multiplied by the entropy at point 1 plus along with all the entropy which is entering is mass flow rate of oil multiplied by S3 streams which are leaving because of them the change in m dot water multiplied by S2 minus because of oil it is m dot oil into S4 plus S dot gen so this is equal to 0.

From here the rate of entropy generation is equal to m dot of water multiplied by S2 minus S1 plus mass of oil multiplied by S4 minus S3. I am noting that both fluids streams are liquid and are quite incompressible substances so rate of entropy generation is, so these expressions this and this expression can be written as m dot water so change in entropy will be Cp LN into T2 by T1 plus m dot of oil CP LN T4 by T3.

So putting all the values mass of water is 4 point 5 kg per second, Cp is 4 point 18 kiloJoule per kg Kelvin LN T2 is 70 plus 273 divided by T1 20 plus 273 plus mass of oil is 10 kg per second. Mass (())(18:44) of oil is 10 kg per second, specific heat is 2 point 3 kiloJoule per kg Kelvin and multiplying this by LN of so T4 is 129 point 1 plus 273 Kelvin divided by T3 which is 170 plus 273, so this gives a value of S gen which is equal to 0 point 736 kW per Kelvin. So this is the rate of entropy generation in the heat exchanger.

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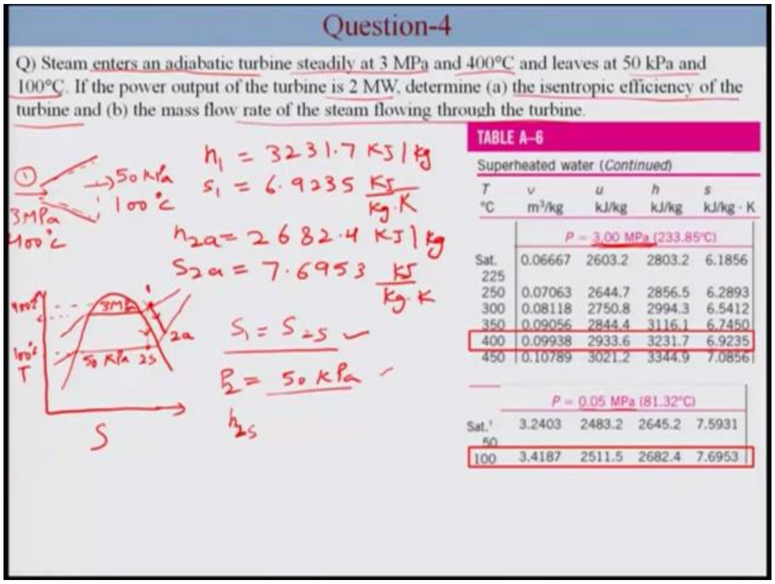

Question-4

Q) Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2 MW, determine (a) the isentropic efficiency of the turbine and (b) the mass flow rate of the steam flowing through the turbine.

$h_1 = 3231.7$ KJ/kg

$s_1 = 6.9235 \frac{KJ}{kg \cdot K}$

$h_{2a} = 2682.4$ KJ/kg

$S_{2a} = 7.6953 \frac{KJ}{kg \cdot K}$

$S_1 = S_{2s}$ ✓

$P_2 = 50$ kPa ✓

h_{2s}

TABLE A–6

Superheated water (*Continued*)

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 3.00 MPa (233.85°C)			
Sat.	0.06667	2603.2	2803.2	6.1856
225				
250	0.07063	2644.7	2856.5	6.2893
300	0.08118	2750.8	2994.3	6.5412
350	0.09056	2844.4	3116.1	6.7450
400	0.09938	2933.6	3231.7	6.9235
450	0.10789	3021.2	3344.9	7.0856
	P = 0.05 MPa (81.32°C)			
Sat.[†]	3.2403	2483.2	2645.2	7.5931
50				
100	3.4187	2511.5	2682.4	7.6953

Moving on to the next question so a steam enters an adiabatic turbine steadily at 3 MegaPascal and 400 degree Celsius and leaves at 50 kiloPascal and 100 degree Celsius. If the power output of the turbine is 2 MW. Determine the isentropic efficiency of the turbine and the mass flow rate of the steam flowing through the turbine. So there is a turbine at point 1 here at 3 MegaPascal and 400 degree Celsius and then stream is moving out at 50 kiloPascal and 100 degree Celsius.

Now state one is superheated stage so if we see from the steam rebel for superheated water at 400 degree Celsius and 3 MegaPascal, so h1 is 3231 pint 7 and S1 is 6 point 9235. So h1 is equal to 3231 point 7 kiloJoule per kg and S1 is equal to 6 point 9235 kiloJoule per kg Kelvin. At state two it is at 50 kiloPascal and 100 degree Celsius. This is also a superheated state so at a pressure of 50 kiloPascal which is 0 point 05 MegaPascal and 100 degree Celsius so values are h 2682 point 4 kiloJoule per kg and S2a is 7 point 6953 kiloJoule per Kelvin.

Now we are representing this as h2 and S2a because this is the actual process but in the question we need to calculate the isentropic efficiency of the turbine, so we see in the TS diagram so this is T S so corresponding to 400 degree Celsius so this is the line and this is so this is for 3 MegaPascal. This is 400 degree Celsius, this temperature is 100 degree Celsius. So this is point one 3MegaPascal and so this is 3 MegaPascal and 400 degree Celsius.

Now in the actual process it is moving like this , so this is point 2a but for a isentropic process it the, entropy will remain constant and it will go something like this in a vertical. So this is represented as point 2s, so here the s1 is equal to S2S so this represent an isentropic process. To calculate the properties at this point 2S we know that this pressure is 50 kiloPascal, so P is 50 kiloPascal and SI is equal to S2S.

At point 2S the pressure is 50kiloPascal and the entropy is equal to S1 for getting enthalpy at point 2S so point h2s, we know pressure and the entropy. So hh if we see from the TS diagram we see that this is moving into the vapor and liquid region where there is a mixture of liquid and vapor.

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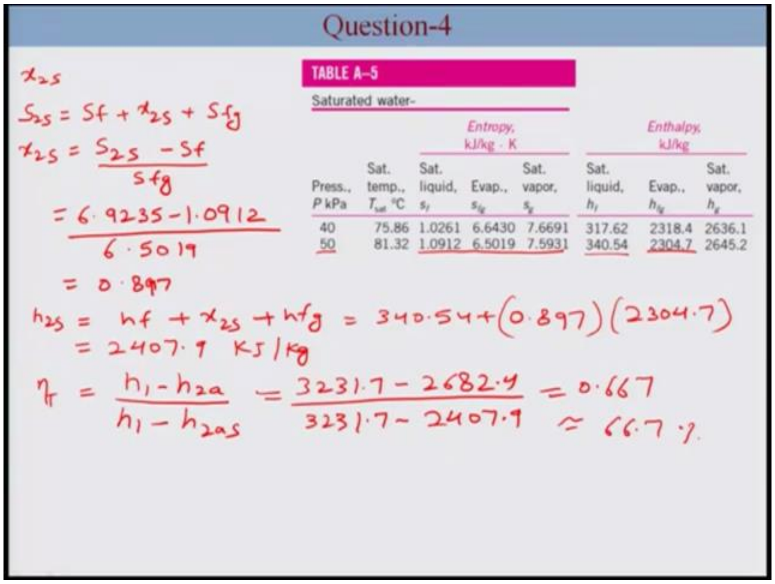

Question-4

x_{2s}

$S_{2s} = S_f + x_{2s} + S_{fg}$

$x_{2s} = \frac{S_{2s} - S_f}{S_{fg}}$

$= \frac{6.9235 - 1.0912}{6.5019}$

$= 0.897$

$h_{2s} = h_f + x_{2s} + h_{fg} = 340.54 + (0.897)(2304.7)$

$= 2407.9$ KJ/Kg

$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2as}} = \frac{3231.7 - 2682.4}{3231.7 - 2407.9} = 0.667 \simeq 66.7\%$

TABLE A–5

Saturated water-

		Entropy, kJ/kg · K			Enthalpy, kJ/kg		
Press., P kPa	Sat. temp., T_{sat} °C	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g
40	75.86	1.0261	6.6430	7.6691	317.62	2318.4	2636.1
50	81.32	1.0912	6.5019	7.5931	340.54	2304.7	2645.2

And also if we see at 50 kiloPascal from the table of saturated water so 50 kiloPascal if we see the values of the entropy for state 2S lies between these two values, so it means that it will be a mixture of liquid and vapor so for this we can calculate the value of X at 2S so this is equal to S2S is equal to Sf plus X2s plus Sfg.

So from here X2s is equal to minus Sf divided by Sfg , so these values are known, 1 point 0912 divided by Sfg 6 point 5019 and this comes out to be 0 point 897. Now as we know the value of X2s so we can also calculate the value of h2s which is equal to hf plus X2s plus hfg and this is equal to hf is 340 point 54 plus X2s which is 0 point 897 multiplied by hfg which is 2304 point 7. So this gives the value of h2s which is 2407 point 9 kiloJoule per kg.

Now the isentropic efficiency of the turbine is given as h1 minus h2a this is for the actual process divided by h1 minus for the isentropic process, so all the values are known so this will give 3231 point 7 minus 2682 point 4 divided by 3231 point 7 minus 2407 point 9 and this is equal to 0 point 667or 66 point 7 percent. So the isentropic efficiency of the turbine is 66 point 7 percent.

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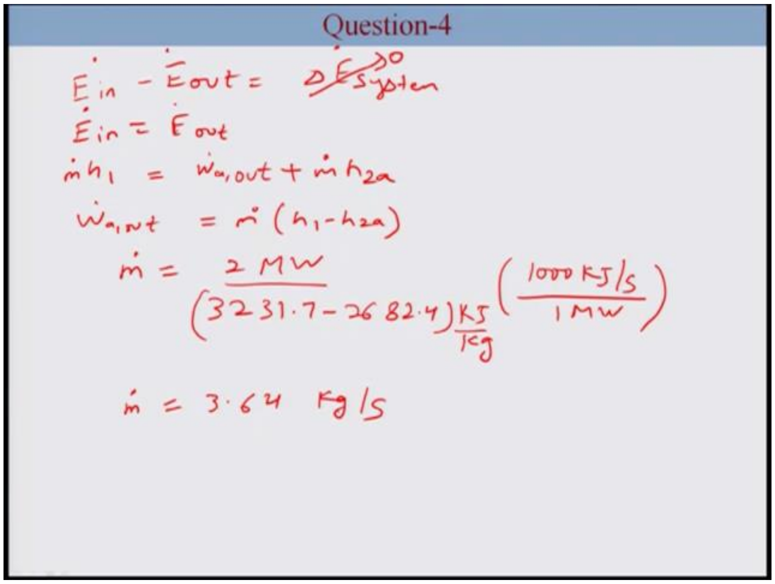

Question-4

$$\dot{E}_{in} - \dot{E}_{out} = \cancelto{0}{\Delta E_{system}}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m} h_1 = \dot{W}_{a,out} + \dot{m} h_{2a}$$

$$\dot{W}_{a,out} = \dot{m}(h_1 - h_{2a})$$

$$\dot{m} = \frac{2\,MW}{(3231.7 - 2682.4)\frac{KJ}{Kg}} \left(\frac{1000\,KJ/s}{1\,MW}\right)$$

$$\dot{m} = 3.64\ Kg/s$$

In part B we have to determine the mass flow rate of stream flowing through the turbine, so mass flow rate of stream flowing through the turbine can be determined from the energy balance of this steady flow system so E in minus E out is equal to delta E of the system for this study system this is equal to 0. So E in dot is equal to E out dot and E in is equal to m dot h1 is equal to E out which is work out out plus m dot h2a.

So work output is equal to m dot h1 minus h2a and from this m dot is equal to 2MW which is work output divided by 3231 point 7 minus 2682 point 4 and this is in kiloJoule per kg and also for making units consisting, so 1000 kiloJoule per second is equal to 1 MW. So this gives a mass flow rate which is equal to 3 point 64 kiloJoule per second so this is the flow rate of the stream flowing through the turbine. So we will end this lecture here. Thank you.