Engineering Thermodynamics
Professor Jayant K Singh
Indian Institute of Technology Kanpur
Department of Chemical Engineering
Module 07
Lecture No 46
Analysis of Brayton Cycle

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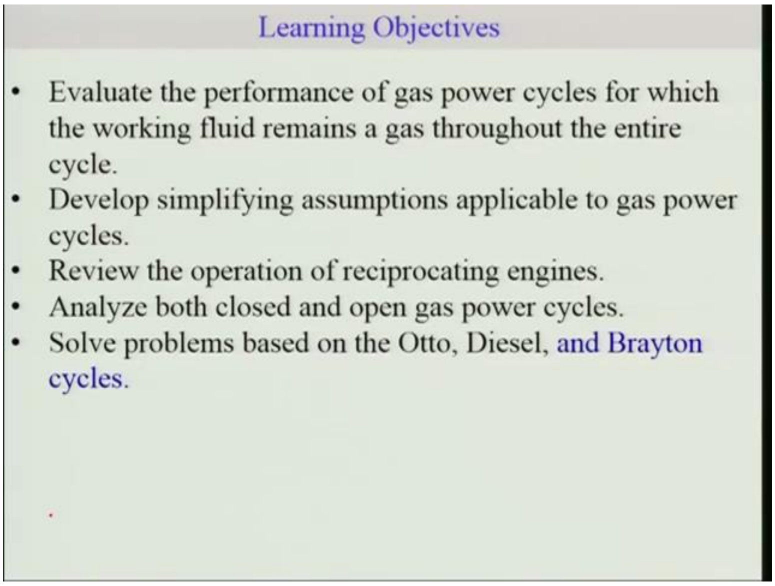

Learning Objectives

- Evaluate the performance of gas power cycles for which the working fluid remains a gas throughout the entire cycle.
- Develop simplifying assumptions applicable to gas power cycles.
- Review the operation of reciprocating engines.
- Analyze both closed and open gas power cycles.
- Solve problems based on the Otto, Diesel, and Brayton cycles.

Welcome back, we were discussing the gas power cycles, in the last few lectures, we looked into otto cycle which is ideal cycle for spark ignition engines and diesel cycle which is ideal cycle for compression ignition engines. In this particular lecture, we will look at Brayton cycles which are ideal cycles for gas turbines okay.

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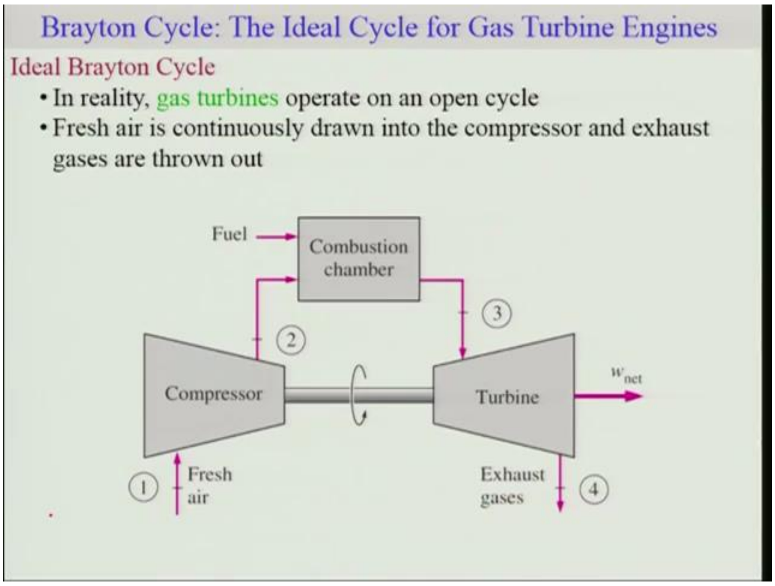

Brayton Cycle: The Ideal Cycle for Gas Turbine Engines

Ideal Brayton Cycle

- In reality, gas turbines operate on an open cycle
- Fresh air is continuously drawn into the compressor and exhaust gases are thrown out

So in reality, the gas turbines operate on open cycle as presented in this schematic diagram, where of course this fresh air is compressed to the conditions of combustion chamber and it is after the combustion, the gas at high temperature and pressure pass through turbine and the exhaust gas is thrown out okay.

Now you can model this particular actual gas turbine operation as a closed cycle okay and what we can do is, you can consider the combustion chamber and replace that combustion chamber by heat exchanger, which just takes Q in and as well as the exhaust process can be replaced by heat exchanger again which basically rejects the Q out. So we can consider combustion process as a constant pressure heat addition and exhaust process as a constant pressure heat rejection process okay and that, this will make the complete cycle okay, which includes compressor, heat exchanger, turbine and another heat exchanger for rejecting the heat okay.

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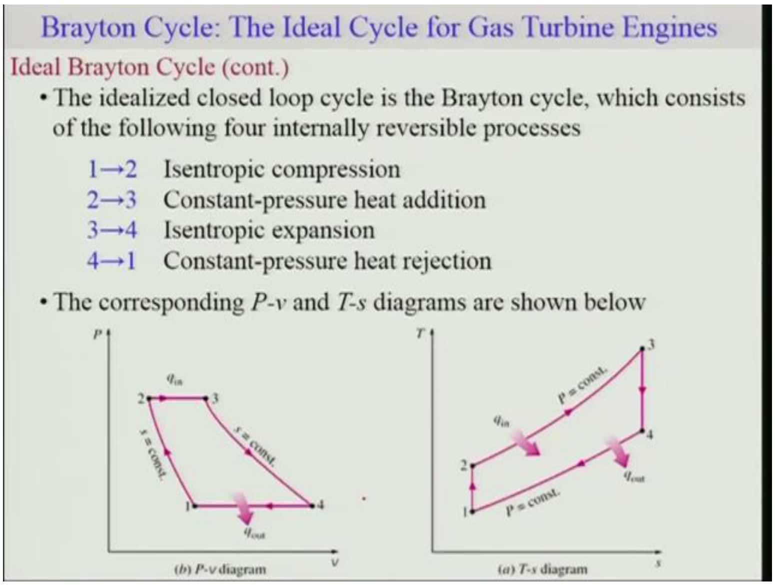

Brayton Cycle: The Ideal Cycle for Gas Turbine Engines

Ideal Brayton Cycle (cont.)

- The idealized closed loop cycle is the Brayton cycle, which consists of the following four internally reversible processes

1→2 Isentropic compression
2→3 Constant-pressure heat addition
3→4 Isentropic expansion
4→1 Constant-pressure heat rejection

- The corresponding *P-v* and *T-s* diagrams are shown below

Now, one can represent this on PV diagram and TS diagram. So let us try to represent this Brayton cycle which consists of 4 internally reversible processes, so one is the of course the compressor which works on isentropic condition, so this would be isentropic compression so that would be this, 1 to 2 on a PV diagram. Then you have this heat exchanger or basically the combustion which is at constant pressure heat addition so that is going to be 2 and 3 followed by 3 to 4, which is an isentropic expansion, the turbine and followed by your constant pressure heat rejection which is this, the exhaust gas which is thrown out or represented by heat exchanger again.

So this becomes the complete cycle on a PV diagram. TS diagram you can clearly represent as 1 and 2 as isentropic followed by constant pressure or by your again isentropic expansion followed constant pressure heat rejection. So we can see how we change this PV diagram from otto cycle to diesel cycle to Brayton cycle, it is a beautiful way of making use of the different conditions okay. Now earlier the otto cycle was both these process 2-3 and 4-1 at constant volume whereas the diesel cycle, the 2-3, one of the particular cycle was at constant pressure, the other one was at constant volume, and in this case the Brayton cycle was completely constant pressure heat addition.

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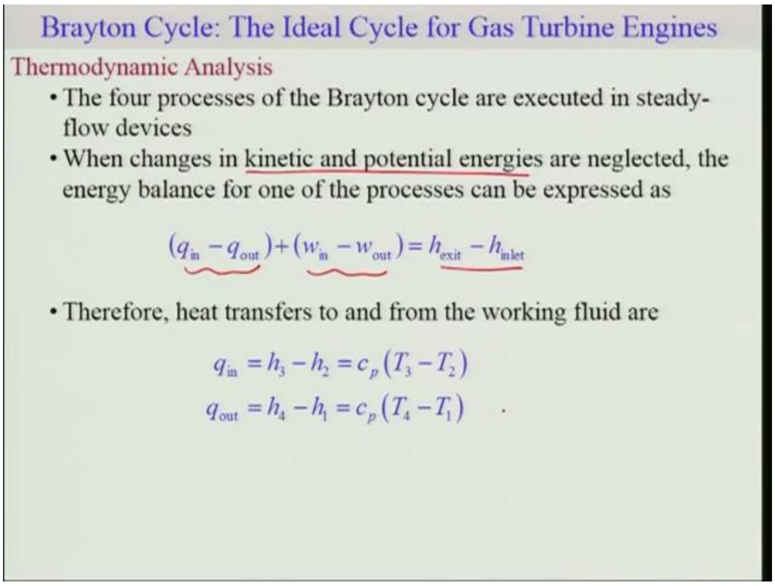

Brayton Cycle: The Ideal Cycle for Gas Turbine Engines

Thermodynamic Analysis

- The four processes of the Brayton cycle are executed in steady-flow devices
- When changes in kinetic and potential energies are neglected, the energy balance for one of the processes can be expressed as

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

- Therefore, heat transfers to and from the working fluid are

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$$
$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1)$$

Okay, so considering that we understand the idolized closed look for the Brayton cycle, which consists of these 4 internal reversible processes where the heat additions and rejections are at constant pressure. We can do the thermodynamic analysis of these 4 processes, we can apply as usual our energy balance, we will consider the changes in the kinetic and potential energy to be negligible, so what you have is here Q in - Q out, net heat in and then you have this W in - W out, that will be = H exit - H in.

This is energy balance which we are going to consider, but you can consider effectively for each of them separately. For example, in this case, Q in is at constant pressure and this is going to be a simply the change in enthalpy from 3 to 2 okay and say because this work for this particular heat exchanger is 0. Similarly, for heat exchanger corresponding to heat rejection process is the work is of course is going to be 0 and hence you can consider simply Qin as change in the enthalpy, Q out is change in enthalpy and this can be represented for the working fluid simply as CP delta T okay considering CP is constant for this change in temperature, so this is a simple energy balance.

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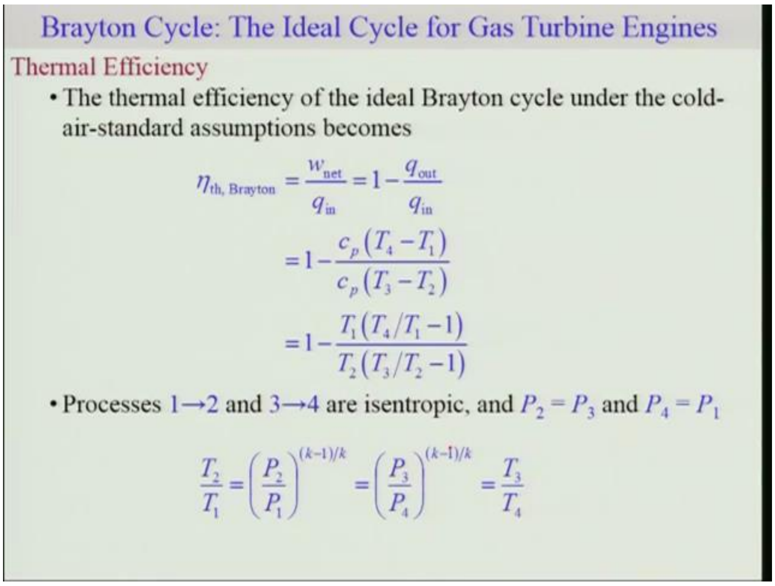

Brayton Cycle: The Ideal Cycle for Gas Turbine Engines

Thermal Efficiency

- The thermal efficiency of the ideal Brayton cycle under the cold-air-standard assumptions becomes

$$\eta_{\text{th, Brayton}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}}$$

$$= 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

- Processes 1→2 and 3→4 are isentropic, and $P_2 = P_3$ and $P_4 = P_1$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_4}$$

We can also evaluate the thermal efficiency of this ideal Brayton cycle and we will consider the cold air standard assumption which essentially here means that the air in this case is going to be considered ideal gas and the cold means we are going to consider CP at typical value of 25 degree Celsius.

Okay, so by definition of course the efficiency is given by UW net by Q in. You can rewrite this as W net is nothing but your Q in - Q out okay and Q out in this case is going to be Q out in this case is nothing but your H4 - H1, Q in is H3 - H2 okay and you can replace this in terms of your CP value, CP delta T and CP will get canceled as we have considered this to be constant and thus, you can write your Braynot efficiency in the in this form okay.

Now for the processes of expansion and compressions which we are going to consider in the Brayton cycles are isentropic and of course, we have this constant pressure heat addition which essentially means your P2 is P3 and P1 is P4. This particular process of expansion and compressions are isentropic okay, so making use of this expression okay which means the ratio T2 by T1 is P2 by P1 to the power K - 1 by K and of course K is your nothing but CP by CV okay.

And considering that of course the pressures are constant, for the case of 2 to 3 and 4 to 1 condition that means that at heat rejection and heat addition processes okay, you can write in this expression. Now this particular expression okay can be replaced in this particular efficiency in order to obtain the expression in terms of ratios of the P okay. So what we have done here is that T1 by T2 is replaced by the ratio and as well as T3 by T4 is also replaced by the ratio in terms of the pressures and considering these ratios P2 by P3 is same as P4 by P1.

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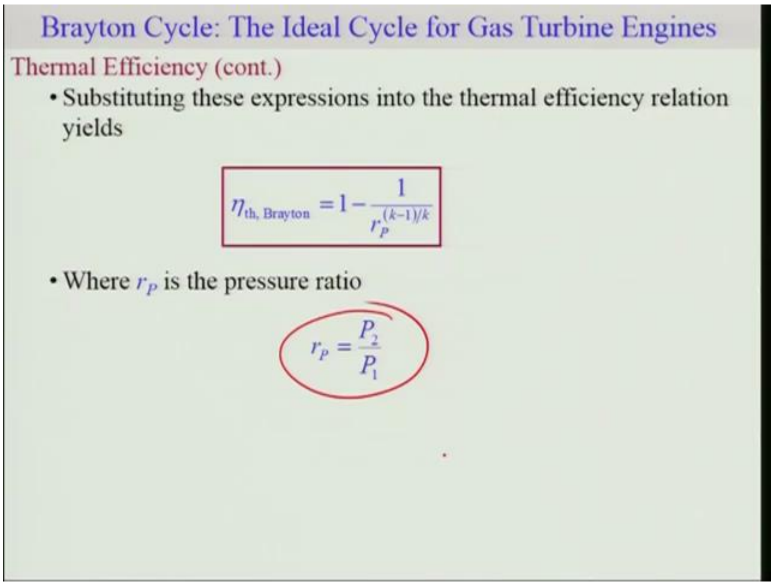


Brayton Cycle: The Ideal Cycle for Gas Turbine Engines

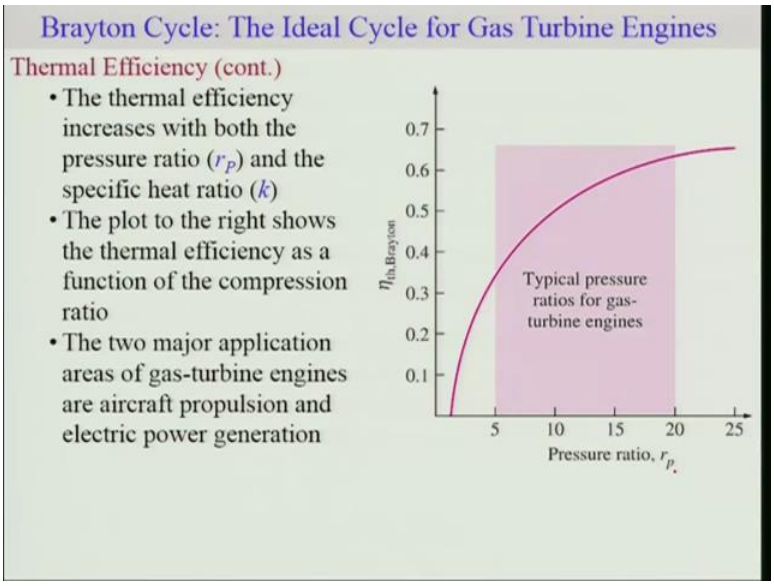
Thermal Efficiency (cont.)

- Substituting these expressions into the thermal efficiency relation yields

$$\boxed{\eta_{\text{th, Brayton}} = 1 - \frac{1}{r_P^{(k-1)/k}}}$$

- Where r_P is the pressure ratio

$$r_P = \frac{P_2}{P_1}$$



Brayton Cycle: The Ideal Cycle for Gas Turbine Engines

Thermal Efficiency (cont.)

- The thermal efficiency increases with both the pressure ratio (r_P) and the specific heat ratio (k)
- The plot to the right shows the thermal efficiency as a function of the compression ratio
- The two major application areas of gas-turbine engines are aircraft propulsion and electric power generation

We easily can express this expression in terms of the ratios of the pressures okay and then if you can you can plot this efficiency as a function of pressure ratio and you clearly can see that as you increase this pressure ratio okay, you can clearly see that the efficiency increases and as well as it also increase with specific heat ratio okay which is K, so you can increase the efficiency by increasing the pressure ratio as well as your K.

Now however, for the typical gas turbine so that pressure ratios are in this region okay and other aspect is that the highest temperature in the cycle which we have seen here is at 3 okay when you add the heat okay through heat exchanger or in the sense through combustion chamber. This high temperature is actually limited by the material properties because material should sustain this high temperature or particularly the turbine blade and thus this aspect of the material limits the pressure ratio which can be used by the cycle okay. Even though the theoretically it can increase further if you increase the pressure ratio but typically it is limited by the material properties and hence typical pressure ratios are in this shaded region, the 2 major applications for gas turbine, aircraft propulsion and electric power generation.

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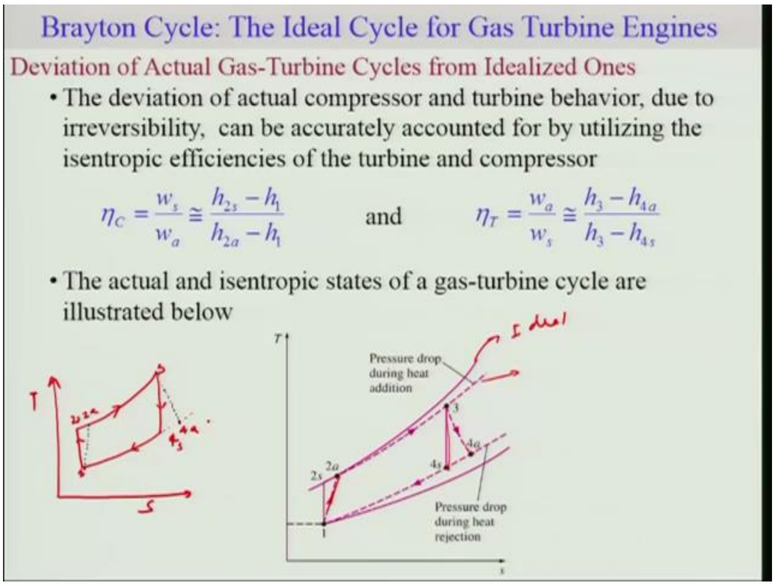

Brayton Cycle: The Ideal Cycle for Gas Turbine Engines

Deviation of Actual Gas-Turbine Cycles from Idealized Ones

- The deviation of actual compressor and turbine behavior, due to irreversibility, can be accurately accounted for by utilizing the isentropic efficiencies of the turbine and compressor

$$\eta_C = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \qquad \text{and} \qquad \eta_T = \frac{w_a}{w_s} \cong \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

- The actual and isentropic states of a gas-turbine cycle are illustrated below

Now as we know the fact that irreversibility is present due to fluid friction or heat losses and thus the actual gas turbine cycle will deviate from the idealized one okay, the deviation can be characterized in this form on this TS diagram. So as we know this we have already discussed this 1 and 2, this is going to be isentropic process or the compression for the case of reversible process because of irreversibility, there will be pressure drop okay across the pump and hence there will be some irreversibility, due to irreversibility the actual path of the this compression would follow in this along this line until this pressure okay. And there would be a certain pressure drop during the heat addition as well so this is the ideal case okay and this dash is basically nothing but your actual process.

Now if we consider the turbine, the turbine also undergoes the similar operation so this is your isentropic process and this will be the case for the actual one and there would be certain pressure drop as well and thus your line, this particular process will deviate from the actual one okay. So you can represent this is more simpler form and what we are going to consider is in this following way, we can write this TS okay. This is essentially isentropic compression, this is your isentropic expansion and you can show this dash line as your actual process so this constant pressure would be here.

This would be nothing but U to A and this is nothing but 2S and this is your 4S and this is would be 4A okay, so we can represent this actual isentropic process in the on the TS diagram in the following way. One can calculate the isentropic efficiency of the compressor and the turbine by this formulation, so the isentropic as we have already discussed it earlier, the isentropic efficiency of compressor is nothing but the work under isentropic condition divided by the actual work.

Okay and on the other hand, for the case of your turbine, this will be the ratio of the actual work and isentropic work okay and these works are clearly related to the changes in the enthalpy as we have discussed earlier okay. So we can demonstrate this understanding of the Brayton cycle by doing an example and this is an example of air which is used as a working fluid okay in a simple ideal Brayton cycle that has a pressure ratio of 12.

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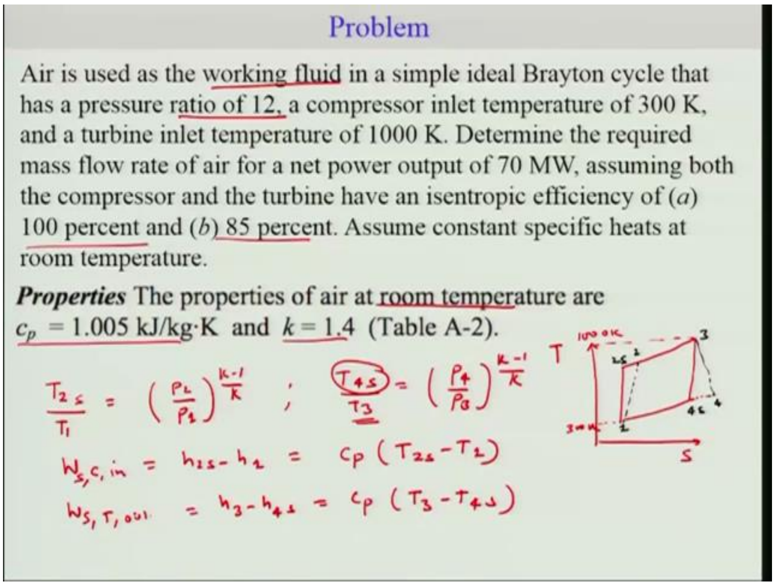

Problem

Air is used as the working fluid in a simple ideal Brayton cycle that has a pressure ratio of 12, a compressor inlet temperature of 300 K, and a turbine inlet temperature of 1000 K. Determine the required mass flow rate of air for a net power output of 70 MW, assuming both the compressor and the turbine have an isentropic efficiency of (*a*) 100 percent and (*b*) 85 percent. Assume constant specific heats at room temperature.

Properties The properties of air at room temperature are $c_p = 1.005$ kJ/kg·K and $k = 1.4$ (Table A-2).

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad ; \quad \frac{T_{4s}}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}}$$

$$W_{s,c,in} = h_{2s} - h_1 = c_p\,(T_{2s} - T_1)$$

$$W_{s,T,out} = h_3 - h_{4s} = c_p\,(T_3 - T_{4s})$$

Now compressor inlet temperature of 300 Kelvin and the turbine inlet temperature of 1000 Kelvin, so I can first draw TS diagram okay and so let me draw this okay so this is nothing but your this is your 4, 4S, 2S, 2 and this is your 3 and this is your 1 okay, so what we know is that the turbine inlet temperature is 1000 Kelvin. Now, the turbine process is basically 3 to 4 okay so inlet will be at 3 so this is your turbine at 1000 Kelvin and the compressor inlet temperature is 300 so 300 is 1 because this is inlet of a compressor.

Now what we have is we have to determine the required mass flow rate of air for net power output of 70 megawatt assuming both the compressor and turbine are having an isentropic process. In the first case, when we are considering an ideal condition that is 100% and then you have the actual condition which is where the efficiency is 85% okay. We will be considering constant heat capacity, heat capacity from the table A2 is given here okay.

Okay so we can start with the simple analysis and the first thing which we are going to do is make use of isentropic process property that means your T2S and T1 is going to be your P2 by P1, K - 1 by K so from here, we can evaluate the other properties because we know T1, okay we do not T2S but we are aware of the pressure ratio so because this pressure is constant so P1 and P2 is 12 so we are aware of the pressure ratio hence we can calculate T2S, similarly we can calculate T4S because we know T3 okay.

Now so we use this information to evaluate these temperatures. Now what we need to find out is the W compressor for the isentropic consider so let me say in here, this is going to be of course only the change in enthalpy okay, which is H2S - H1 and this is can be written in terms of CP delta T. We are considering the cold air condition so hence we are going to take the CP value at room temperature. Similarly, we can find out the turbine work and that is going to be simply your H3 - H4S based on your simple energy balance here and this is going to be T3 - T4S okay.

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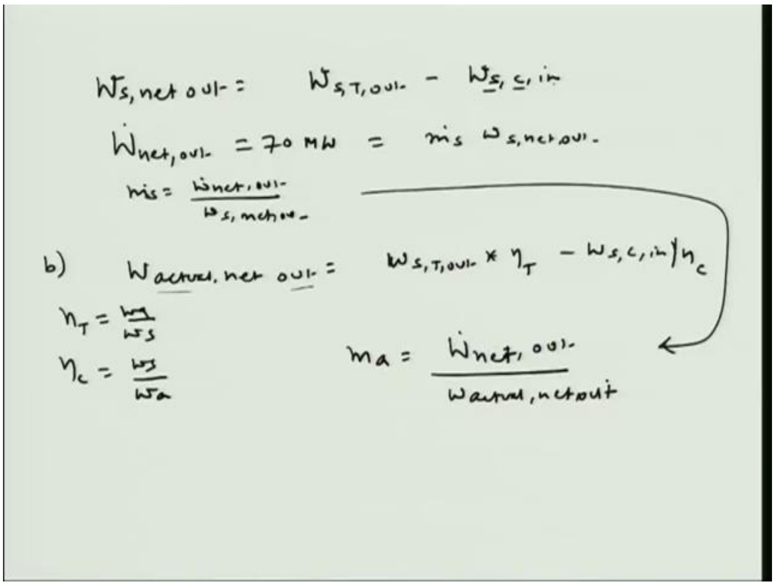

$$w_{s,net\,out} = w_{s,T,out} - w_{\underline{s},\underline{c},in}$$

$$\dot{W}_{net,out} = 70\text{ MW} = \dot{m}_s\, w_{s,net\,out}$$

$$\dot{m}_s = \frac{\dot{W}_{net,out}}{w_{s,net\,out}}$$

b) $$W_{\underline{actual},net\,\underline{out}} = w_{s,T,out} \times \eta_T - w_{s,c,in}/\eta_c$$

$$\eta_T = \frac{w_a}{w_s}$$

$$\eta_c = \frac{w_s}{w_a}$$

$$\dot{m}_a = \frac{\dot{W}_{net,out}}{w_{actual,net\,out}}$$

So we can calculate the work in this way, so the net work is the turbine out okay - for the compressor in so isentropic compressor in and, now this is of course what we have done is, we have considered all in per unit mass as a basis and we are being given the net power output that is W ., net out is given as 70 megawatt, this going to be your mass flow rate multiplied by WS net out okay and from there we can calculate your MS W net out divided by WS net out okay.

So this is how we are going to calculate the mass flow rate. In the second part, we have to consider the isentropic efficiency of compressor and turbine that is 85%, so we can calculate the W actual net out okay is going to be WST out multiplied by your eta of turbine - WS of compressor in divided by eta of isentropic of C because remember that we have discussed this eta of T is WA by WS and eta of C is WS by WN, hence your this expression will come.

Okay and after you calculate this, because your etas are given, you can use this expression here to calculate your MA which is going to be M net out okay divided by your W actual net out okay and this is how you make use of this specific simple analysis for the Brayton cycle okay, so this is the simple ideal Brayton cycle and of course we have calculated, we have made use of the isentropic efficiency in order to analyze the problem.

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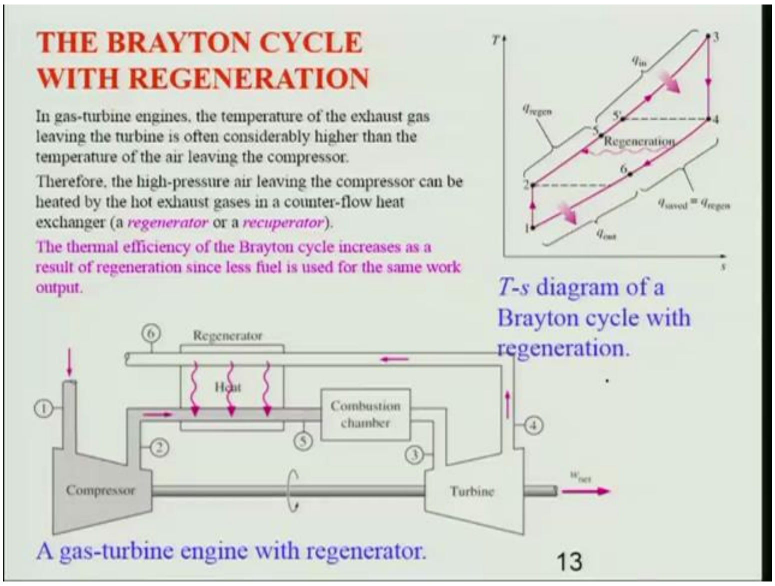

THE BRAYTON CYCLE WITH REGENERATION

In gas-turbine engines, the temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor.

Therefore, the high-pressure air leaving the compressor can be heated by the hot exhaust gases in a counter-flow heat exchanger (a *regenerator* or a *recuperator*).

The thermal efficiency of the Brayton cycle increases as a result of regeneration since less fuel is used for the same work output.

T-s diagram of a Brayton cycle with regeneration.

A gas-turbine engine with regenerator.

Okay now we are going to consider a Brayton cycle with regeneration. This is merely to increase the efficiency of gas turbine engines and just we will take a look at specific conditions. Now many systems or the temperature of the exhaust gas often is considerably higher than the temperature of the air leaving the compressor okay. So this is the case where we can make use of regeneration, when you have the temperature of the exhaust gas higher than the air leaving the compressor which is going to be 2 okay and in that case what we can do is, we can heat this gas which comes out from the compressor using the heat of the exhaust gases okay by making use of counter flow heat exchanger.

And this particular way of heating the gas from the compressor by the exhaust gas in a counter flow heat is called regenerator okay, so let this is the schematic diagram of it. You can see here, this is the inlet compressor and this is the outlet okay which goes to the combustion and after the combustion, high temperature and pressure get expanded and 4 is the outlet of the turbine this turbine is essentially used to transfer the heat to the air leaving the compressor okay and this is the presentation in a steady flow condition.

So by making use of this, we try to increase thermal efficiency because less amount of fuel is now needed in order to heat the air from the compressor and this is something which we can understand from here so what is being said here, if you look at to this 1, 2, 5, these are the representatives of the stream conditions, so 1 to 2 is of course the compressor so 1 to 2 is basically the isentropic compression and from 2 to 5, there is a heat transfer from the regenerator which is essentially from 4 to 6.

Okay in other word, you can see that this 2 to 5 is the point where the energy is being transferred from this particular and where typically we reject the heat okay. Instead of rejecting this part of the heat from 4 to 6, you transfer this energy to heat up this particular part of the air or this particular process and this is the reason that this part 4 to 6 is the one which essentially is nothing but your Q regeneration and this is the part which is taken internally, not from external thus you are saving this much energy okay.

And the effective Q in is nothing but from 5 to 6, so 5 is an actual point where you can take, but theoretically you can raise the temperature till 5 dash which is equivalent to 4 okay where this 4 nothing but your exit of turbine okay. So the maximum heat generation would be your so regeneration maximum is going to be H5 dash - H2 which is nothing but H4 - H2 okay so this would be the case for ideal ideal case where there is no losses and so forth okay and thus your T5 is going to be T4 okay.

And but in in a for the actual case your Q regeneration is nothing but your H5 - H2 and the ratio of this Q regeneration to the Q regeneration maximum is we call effectiveness okay so this is something which is called effectiveness okay. So note that this particular regeneration is useful only when exhaust temperature okay is higher than the compressor exit temperature okay. So as we can clearly see there is there is many ways to enhance the efficiency of a gas turbine and essentially you can think of many other ways. We have already noticed earlier that we have already looked into compressor work can be minimized by inter cooling and similarly we can also increase the efficiency of your turbine by reheating okay.

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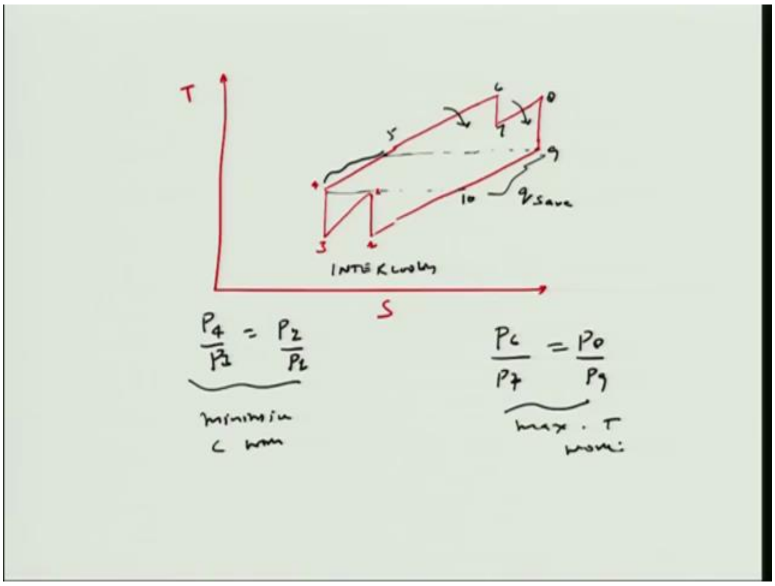

So this is something which we can try to come up with Brayton cycle with inter cooling and reheating, so this is an example okay where I I can come with this kind of a diagram. T of S okay so let us say we have compressor okay at 1 to 2 and in order to reduce the compressor work so we try to cool this okay and then we further apply another we pass through this working fluid through another compressor so that is going to be 1, 2, 3 and 4 okay.

And then we undergo again the same Q heating process at constant pressure. Here we try to reheat okay and then this we have this connect here and then of course we can make use of the fact that these temperatures are higher than the exit temperature of the compressor and thus we can use also regeneration. So this is an example of inter cooling so this is inter cooling okay and this is an example of reheat okay and then of course we can and this is particular amount, it will be Q safe so this is nothing but Q regeneration okay and this is going to be reheat, this is your primary heat and this is your energy saved, so thus you can clearly see that this would lead to a better or more efficient gas turbine.

Now as we have discussed earlier that in order to have minimum work for compressor so in other word, if we have a 2 stage compressor and to minimize the work, the pressure ratio should be maintain at each stage that means P4 by P3 should be = P2 by P1 okay. Similarly, one can show that for the case of reheat, in order to maximize the work, your P6 which is so if we can say this is 5, this is 6, 7, 8, 9 and 10, P6 by P7 should be same as P8 by P9 okay, so this is to minimize compressor work okay and this is to maximize turbine work okay.

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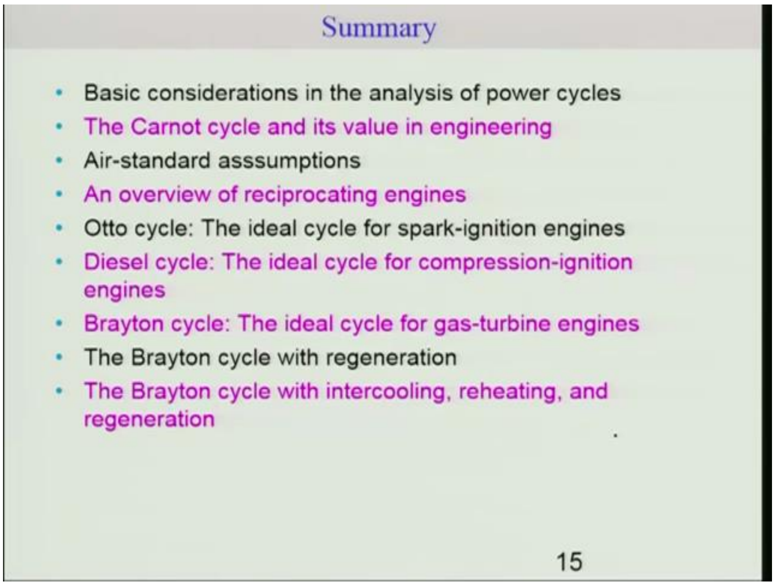

Summary

- Basic considerations in the analysis of power cycles
- The Carnot cycle and its value in engineering
- Air-standard asssumptions
- An overview of reciprocating engines
- Otto cycle: The ideal cycle for spark-ignition engines
- Diesel cycle: The ideal cycle for compression-ignition engines
- Brayton cycle: The ideal cycle for gas-turbine engines
- The Brayton cycle with regeneration
- The Brayton cycle with intercooling, reheating, and regeneration

Okay so this is what I wanted to cover or the gas turbine with inter cooling, reheat and regeneration and this is would the end of the lecture so let me just summarize what we have covered in this particular topic on gas power cycles. We have looked into the Carnot cycle in this value engineering which acts like a ideal cycle. We have considered the air standard assumption in our analysis and a overview of a reciprocating engine, we have considered 3 different cycles, otto cycle for spark ignition engine and diesel cycle for compression ignition engine and today we have looked into a Brayton cycle which is an ideal cycle for gas turbine engines.

We also considered the regeneration inter cooling, reheating and regeneration as a part of increasing the efficiency of Brayton cycle, so that will be the end of today’s lecture. I will start new topic in the next lecture, I will see you in the next lecture.