

Engineering Thermodynamics
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Module 07
Lecture No 47

Examples on Gas Power Cycles such as Otto Diesel and Brayton

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Question-1

The compression ratio of an air-standard Otto cycle is 9.5. Prior to the isentropic compression process, the air is at 100 kPa, 35°C, and 600 cm³. The temperature at the end of the isentropic expansion process is 800 K. Using specific heat values at room temperature, determine (a) the highest temperature and pressure in the cycle; (b) the amount of heat transferred in, in kJ; and (c) the thermal efficiency.

Properties The properties of air at room temperature are $c_p = 1.005$ kJ/kg·K, $c_v = 0.718$ kJ/kg·K, $R = 0.287$ kJ/kg·K, and $k = 1.4$ (Table A-2).

Process \rightarrow 1-2

$$T_2 v_2^{k-1} = T_1 v_1$$

Welcome to this tutorial, today we go through few examples based on gas power cycles such as otto, diesel and Brayton, so let us start with the first question. The compression ratio of an air standard otto cycle is 9.5 prior to the isentropic compression process; the air is at 100 kilopascal, 35 degree centigrade and 600 centimeter cube. The temperature at the end of the isentropic expansion is 800 Kelvin, using specific heat values at room temperature, determines the highest temperature and pressure in the cycle, the amount of heat transfer in kilo joule and the thermal efficiency.

So if you see the whole process on a PV diagram, since it is an otto cycle so the first process will be isentropic compression from state 1 to 2 followed by heat addition at constant volume from the state 2 to 3 and then isentropic expansion from state 3 to 4 and heat rejection at constant volume okay and heat will be coming here okay, so this is the complete otto cycle. Now we have to find the highest temperature and pressure in this cycle which will be corresponds to this step which is step 3 okay.

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Question-1

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$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{k-1} = (308 \text{ K}) (9.5)^{0.4}$$
$$T_2 = 757.9 \text{ K}$$
$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1} \Rightarrow P_2 = \left(\frac{V_1}{V_2} \right) \left(\frac{T_2}{T_1} \right) P_1$$
$$P_2 = (9.5) \left(\frac{757.9 \text{ K}}{308 \text{ K}} \right) 100 \text{ kPa} = 2338 \text{ kPa}$$

So starting from the process 1 to 2, since it is an isentropic compression so $T_2 V_2$ to the power $K - 1 = T_1 V_1$ to the power $K - 1$ so from here we get $T_2 = T_1$ times V_1 upon V_2 to the power $K - 1$, okay. So T_1 we know, it is given in the question which is 35 degree centigrade so after putting this value here, 308 Kelvin, V_1 by V_2 which is compression ratio is given to us which is 9.5 so 9.5 and $K - 1$ since K is 1.4 so it will be 0.4 so from here, we get $T_2 = 757.9$ Kelvin, okay. Now here we assume air as an ideal gas. So from ideal gas equation of state, $P_2 V_2$ upon T_2 will be equal $P_1 V_1$ upon T_1 okay so from here we get $P_2 = V_1$ by V_2 times T_2 upon T_1 times P_1 okay, so after plugging these values, we get $P_2 = 9.5$ times 757.9 divided by 308 Kelvin into initial pressure which is 100 kilo Pascal, so this gives us pressure at the state 2 which is 2338 kilo Pascal.

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Question-1

From the process 3-4

$$T_3 = T_4 \left(\frac{V_4}{V_3} \right)^{k-1} = (800 \text{ K}) (9.5)^{0.4}$$

$$T_3 = 1969 \text{ K}$$

From the process 2-3

$$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2} \quad V_3 = V_2$$

$$P_3 = \left(\frac{T_3}{T_2} \right) P_2$$

$$P_3 = \left(\frac{1969 \text{ K}}{757.9 \text{ K}} \right) (2338 \text{ kPa})$$

$$P_3 = 6072 \text{ kPa}$$

From the process 3 to 4 which is an isentropic expansion so T_3 will be T_4 times V_4 upon V_3 to the power $K - 1$. We know T_4 which is 800 Kelvin even in the question, compression ratio is 9.5 to the power 0.4, so from here we get temperature of step 3 is 1969 Kelvin, okay. Now from the process 2 to 3 which is constant volume heat addition, so from the ideal gas equation $P_3 V_3$ upon T_3 will be = $P_2 V_2$ upon T_2 . Okay. Since volume is same so from here, P_3 will be T_3 upon T_2 times P_2 okay, so P_3 will 1969 upon 757.9 Kelvin times 2338 kilo Pascal, so P_3 will be = 6072 kilo Pascal.

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Question-1

$$Q_{in} = m (u_3 - u_2) \quad \Delta V = 0$$

$$= m c_v (T_3 - T_2)$$

$$m = \frac{P_1 V_1}{R T_1} = \frac{(100 \text{ kPa}) (6 \times 10^{-4} \text{ m}^3)}{0.287 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K} \times 308 \text{ K}}$$

$$m = 6.788 \times 10^{-4} \text{ kg}$$

$$Q_{in} = 6.788 \times 10^{-4} \times 0.718 \times (1969 - 757.9) \text{ kJ}$$

$$= 0.59 \text{ kJ}$$

$$Q_{out} = m c_v (T_4 - T_1) = 0.24 \text{ kJ}$$

$$w_{net, out} = Q_{in} - Q_{out} = 0.35 \text{ kJ}$$

$$\eta_{th} = \frac{w_{net, out}}{Q_{in}} = \frac{0.35}{0.59} = 59.4 \%$$

Now we are asked to calculate the heat coming into this cycle okay, so heat is given during the isochoric process from the state 2 to 3, so Q in will be = mass times change in internal

energy okay because delta V is 0 since it is air is behaving as an ideal gas so MCV T3 - T2, it will be reduced in this form. Now we need to know MCV we can get it from the air properties table. T3 and T2, we know already.

Now from the ideal gas equation, we can get $M = P_1 V_1$ upon our T1 okay. So after plugging values of these variables 100 kilo Pascal P1, volume is given in the equation so it is 6 into 10 to power - 4 meter cube divided by 0.287 kilo Pascal meter cube upon kg into Kelvin and temperature T1 which is 308 Kelvin. So after solving it, we get $M = 6.788$ into 10 to the power - 4 kg okay. So now after plugging these values, we will get $Q_{in} = 6.788$ into 10 to the power - 4 into 0.718 into change in temperature 1969 - 757.9 kilo joule so after solving it, we get 0.59 kilo joule heat is coming into the cycle.

Now we need to know thermal efficiency, for that, we need to know heat coming out of the cycle okay, so Q_{out} will be = MCV T4 - T1 okay, we know M, we know CV and we know T4 and T1, so after plugging these values here, we get 0.24 kilo joule okay and W_{net} will be W_{net} out will be = $Q_{in} - Q_{out}$ so this gives 0.35 kilo joule. So now thermal efficiency would be W_{net} out divided by total heat going into the cycle so from here we get 0.35 upon 0.59 so this gives 59.4% so this is the thermal efficiency of the otto cycle so moving to the next question.

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Question-2

An ideal diesel engine has a compression ratio of 20 and uses air as the working fluid. The state of air at the beginning of the compression process is 95 kPa and 20°C. If the maximum temperature in the cycle is not to exceed 2200 K, determine (a) the thermal efficiency and (b) the mean effective pressure. Assume constant specific heats for air at room temperature.

Properties The properties of air at room temperature are $c_p = 1.005$ kJ/kg·K, $c_v = 0.718$ kJ/kg·K, $R = 0.287$ kJ/kg·K, and $k = 1.4$ (Table A-2).

$$\eta_{th} = \frac{W_{net, out}}{Q_{in}}$$

From the process 1-2

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{k-1} = 293 \text{ K} (20)^{0.4} = 971.1 \text{ K}$$

2 → 3 $\frac{V_3}{V_2} = \left(\frac{T_3}{T_2} \right)$ since $P_2 = P_3$

$$\frac{V_3}{V_2} = \frac{2200}{971.1} = 2.265$$

$$V_3 = 2.265 V_2$$

An ideal diesel engine has compression ratio of 20 and uses air as the working fluid. The state of air at the beginning of the compression process is 95 kilo Pascal and 20 degree centigrade. If the maximum temperature in the cycle is not exceeding 2200 Kelvin, determine the thermal

efficiency and the mean effective pressure. Assume constant specific heats at for air at room temperature. So this is an ideal diesel engine, so if we see the cycle on a PV diagram, the first process will be isentropic compression from the state 1 to state 2.

Then heat is added in this cycle at constant pressure okay so heat will be going in at constant pressure. Now from the state 3, it will expand isentropically to the state 4 and finally it will reject heat at constant volume okay, so this is the complete diesel cycle. Now we are asked to do calculate thermal efficiency and mean effective pressure so what is thermal efficiency? Thermal efficiency is W total work coming out of the cycle divided by total heat going into the cycle from the process 1 to 2 which is an isentropic compression, so temperature at the state 2 can be obtained using temperature and volume data even at state 1 so V1 upon V2 to the power K - 1 so T1 is given 293 Kelvin.

Compression ratio is given to us which is 20 to the power 0.4 okay since K is 1.4. So this gives T2 = 971.1 Kelvin okay. Now from the process 2 to 3 which is an isobaric process so from that V3 upon V2 = T3 upon T2 okay since P2 = P3 and we have assumed that air is behaving as an ideal gas. So this ratio V3 upon V2 will be = 971.1 Kelvin so this gives 2.265 okay. So now here from here we can get V3 in terms of V2 which is 265 into V2.

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Question-2

$$q_{in} = h_3 - h_2 = c_p (T_3 - T_2)$$

$$= \left(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right) (2200 - 971.1) \text{ K}$$

$$q_{in} = 1235 \text{ kJ/kg}$$

$$q_{out} = u_4 - u_1 = c_v (T_4 - T_1)$$

$$= 0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (920.6 - 293) \text{ K}$$

$$q_{out} = 450.6 \text{ kJ/kg}$$

$$w_{net, out} = q_{in} - q_{out} = 784.4 \text{ kJ/kg}$$

$$\eta_{th} = \frac{784.4}{1235} = 63.5\%$$

We will make use of this in the next process so now at constant pressure heat is going into the cycle which is Qin = change in enthalpy right, S3 - S2 which will be = since this is constant pressure process so this will be = CP times T3 - T2 okay and we know CP from the air properties table, kilo joule upon kg into Kelvin times temperature difference T3 - T1, it is

971.1 Kelvin. So from here we get Q in per kg which is 21235 kilo joule per kg okay. Now heat is rejected at constant volume so Q out = U4 since it is heat is being rejected from the process 4 to 1 okay, so Q out will be U4 - U1 okay which will be = CV T4 - T1 okay, so from the properties table, we can get the CV value which is 0.718 kilo joule per kg Kelvin change in temperature which 920.6 - 293 Kelvin okay.

So after solving it, we get Q out = 450.6 kilo joule per kg okay. So since we know Q in, Q out, we can calculate net work coming out of the cycle = Q in - Q out which is = 784.4 kilo joule per kg okay. So since we know Wnet out, now we can calculate thermal efficiency of this cycle which is 784.4 upon 1235 okay, so now okay from here we get 63.5%. Now moving to the second part of the question where we we are asked to calculate mean effective pressure, so mean effective pressure will be = total net work coming out of the cycle divided by the volume 1 - 1 by R.

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Question-2

$$MEP = \frac{W_{net,out}}{V_1 \left(1 - \frac{1}{r}\right)}$$

$$V_1 = \frac{RT_1}{P_1} = \frac{0.287 \frac{\text{kPa} \cdot \text{m}^3}{\text{kg} \cdot \text{K}} \times 293 \text{ K}}{95 \text{ kPa}}$$

$$V_1 = 0.885 \text{ m}^3/\text{kg}$$

So MEP = $\frac{784.4 \text{ kJ/kg}}{0.885 \frac{\text{m}^3}{\text{kg}} \left(1 - \frac{1}{20}\right)}$

$$MEP = \underline{\underline{933 \text{ kPa}}}$$

So since we do not know initial volume but using ideal gas equation of state, we can get P1 = RT1/P1 which will be = 0.287 kilo Pascal meter cube upon kg into Kelvin times temperature, initial temperature and P1 is 95 Kelvin kilo Pascal. So from here get V1 = 0.885 meter cube per kg so after plugging this MEP will be 784.4 kilo joule per kg upon 0.885 meter cube per times 1 - 1 by 20, so this gives Mean Effective Pressure = 933 kilo Pascal okay, okay so now moving to the next question.

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Question-3

A simple Brayton cycle using air as the working fluid has a pressure ratio of 10. The minimum and maximum temperatures in the cycle are 295 and 1240 K. Assuming an isentropic efficiency of 83 percent for the compressor and 87 percent for the turbine, determine (a) the air temperature at the turbine exit, (b) the net work output, and (c) the thermal efficiency.

$T_1 = 295 \text{ K}$

$h_1 = 295.17 \text{ kJ/kg}$

$P_{r1} = 1.3068$

$P_{r2} = \left(\frac{P_2}{P_1}\right) \times P_{r1} = 10 \times 1.3068 = 13.07$

$h_{2s} = 570.26 \text{ kJ/kg}$

$T_{2s} = 564.9 \text{ K}$

$\eta_c = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_2 - h_1}$

A simple Brayton cycle using air as the working fluid has a pressure ratio of 10, the minimum and maximum temperature in this cycle are 295 and 1240 Kelvin. Assuming an isentropic efficiency of 83% for the compressor and 87% for the turbine determine the air temperature at the turbine exit, net work output and thermal efficiency. Now since this is a Brayton cycle, so if you see this process on a TS diagram okay, the first process will be an isentropic compression from the state 1 to 2. Then heat addition will take place at a constant pressure and isentropic expansion will take place in the turbine and heat rejection will also take place at constant pressure, so 1, 2, 3 and this process will be a 4S since this is an ideal process so 1 to 2S, but actual process will follow the path 1 to 2 and same here in the turbine as well, it will follow the path 3 to 4 because of irreversibility of the system.

So it is given that minimum temperature is 295 which is corresponds to this state 1, so $T_1 = 295$ Kelvin okay. It has already given that maximum temperature is 1240 Kelvin which will be corresponds to this state 3. Since we are asked to calculate the thermal efficiency and turbine exit temperature which is T_4 , we need to know temperature at all these states, so starting from the first state, $T_1 = 295$ Kelvin so from the table A17 corresponds to this temperature we can get enthalpy = 295.17 Kilo Joule per kg, corresponding reduced pressure will be 1.3068 okay.

Now to get enthalpy at 2S, P reduced at this second state will be P_2 upon P_1 times PR_1 . Now since the compression ratio is given = 10 so 10 times 1.3068 which gives 13.07 okay. Now corresponding to this reduced pressure okay, we can get corresponding enthalpy = 570.26 kilo joule per kg from the A17 and T_{2s} as well which is 564.9 Kelvin okay. Now from the

compressor efficiency which is = isentropic work to the actual work ratio of isentropic work to the actual work will be = $h_{2s} - h_1$ upon $h_2 - h_1$ okay.

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Question-3

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c}$$

$$h_2 = 295.17 + \frac{(570.26 - 295.17)}{0.83} = 626.1 \frac{\text{kJ}}{\text{kg}}$$

$$T_3 = 1240 \text{ K} \Rightarrow h_3 = 1324.93 \text{ kJ/kg}$$

$$Pr_3 = 272.3$$

using this $Pr_4 = \left(\frac{P_4}{P_3}\right) \times Pr_3 = \frac{1}{10} \times 272.3 = 27.23$

$$h_{4s} = 702.07 \text{ kJ/kg}$$

$$T_{4s} = 689.6 \text{ K}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_T (h_3 - h_{4s})$$

So from here, after solving it, we get $h_2 = h_1 + h_{2s} - h_1$ divided by compressor efficiency so compressor efficiency is given in the question, so from here we get $h_2 = h_1$ we have already calculated which is $295.17 + 570.26 - 295.17$ upon 0.83 okay, so this gives 626.6 kilo joule per kg. It is given in the question that $T_3 = 1240$ Kelvin okay, so from the properties of air, we get $h_3 = 1324.93$ kilo joule per kg and $Pr_3 = 272.3$ okay so using this, we can get $Pr_4 = P_4$ upon P_3 times Pr_3 , which is $= 1$ upon 10 times 272.3 which gives 27.23 okay.

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Question-3

$$h_4 = 1324.93 - 0.87 (1324.93 - 702.07)$$

$$h_4 = 783.04 \text{ kJ/kg}$$

A-17 $T_4 = 764.4 \text{ K}$

ⓑ $q_{in} = h_3 - h_2 = 698.8 \text{ kJ/kg}$

$$q_{out} = h_4 - h_1 = 487.9 \text{ kJ/kg}$$

$$w_{net} = q_{in} - q_{out} = 210.4 \text{ kJ/kg}$$

Ⓒ $\eta_{th} = \frac{w_{net}}{q_{in}} = 0.3013 = 30.1\%$

So corresponds to this reduced pressure, h_{4s} can be taken from the table which is 702.07 kilo joule per kg and $T_{4s} = 689.6$ Kelvin, so from the thermal efficiency of turbine we can get $s_3 - h_4 = s_3 - h_{4s}$ so from here after rearranging, we get $h_4 = h_3 - \text{thermal efficiency of turbine} \times (s_3 - h_{4s})$ okay, so after plugging the values of these variables, we get $h_4 = 1324.93 - 0.87 \times (1324.93 - 702.07)$ okay, so this gives $h_4 = 783.04$ kilo joule per kg and corresponds to this h_4 and p_4 , T_4 from the table A17 can be calculated as 764.4 Kelvin okay.

So in the second part of the question it is asked that we have to calculate total work so Q_{in} will be $h_3 - h_2$ right so this will be 689.3 kilo joule per kg and Q_{out} will be $h_4 - h_1 = 487.9$ kilo joule per kg, so W_{net} will be $Q_{in} - Q_{out}$, so this will be = 210.4 kilo joule per kg. Now thermal efficiency of the whole cycle will be W_{net} out divided by heat going into the cycle, so this E will be = 0.3013 that means 30.1% so we will stop here, we will meet you in the next tutorial, thank you.