

Engineering Thermodynamics
Professor Jayant K Singh
Indian Institute of Technology Kanpur
Department of Chemical Engineering
Module 7
Lecture No 51
Examples on vapour power cycles

Welcome to this tutorial, today we will solve few problems based on vapor power cycles. so let us start with the first question.

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Question-1

An engineer has proposed that a simple ideal Rankine cycle that uses refrigerant-134a be used to produce work with heat from a low-temperature thermal energy reservoir. The boiler operates at 1.6 MPa, the condenser at 0.4 MPa, and the turbine inlet at 80°C. Determine the R-134a mass flow rate needed for this cycle to produce 750 kW of power and the thermal efficiency of the cycle.

$h_1 = h_f @ 0.4 \text{ MPa}$
 $A-1a \quad h_1 = 63.94 \text{ kJ/kg}$
 $v_1 = v_f @ 0.4 \text{ MPa} = 0.0007907 \text{ m}^3/\text{kg}$
 $w_{p,in} = v_1 (P_2 - P_1) = 0.0007907 \frac{\text{m}^3}{\text{kg}} \times (1600 - 400)$
 $= 0.95 \text{ kJ/kg}$
 $h_2 = h_1 + w_{p,in} = (63.94 + 0.95) \text{ kJ/kg}$
 $= 64.89 \text{ kJ/kg}$

An engineer has proposed that a simple ideal Rankine cycle that uses refrigerant 134A be used to produce work with heat from a low temperature thermal energy reservoir. The reboiler operates at 1.6 megapascal, the condenser at 0.4 megapascal and the turbine inlet at 80 degree centigrade. Okay. Determine the refrigerant mass flow rate needed for this cycle to produce 750 kilowatt of power and the thermal efficiency of the cycle. So this is the sketch of the Rankine cycle okay so this pump compresses this liquid from the state 1 to state 1, then heat it being added at a constant pressure in the reboiler and it heated the refrigerant from compressed to superheated state in the state 3.

Then in the turbine this refrigerant gets expanded isentropically to the state 4. So if you see the whole process on TS diagram, so the process 1 will start from the saturated liquid. So it is compressed isentropically from that state 1 to 2 okay. Now heat is being added in the reboiler at

a constant pressure so the state will change and it is added till it is superheated okay. So now from the state 2 to 3, this heat is being added and because of that temperature is increasing.

Now further it is isentropically expanded in turbine so it will come from 3 to the state 4 and finally from the state 4 to state 1 because it loses its heat to the surrounding and comes back into the saturated liquid state. So this is the complete ideal Rankine cycle. So heat is coming in here and it will lose heat here. Okay. Now, we need to find thermal efficiency in the mass flow rate of the refrigerant.

So it is given that initially condenser is working at 0.4 megapascal. So this is the pressure of state 1 and reboiler is working at 1.6 megapascal okay. So this is the pressure at state 3.

Since pressure at state 1 is 0.4 megapascal so enthalpy of that state will be equal enthalpy of the fluid at at this pressure. Why? Because this is saturated liquid so there is no vapor. So from the table A12, we get H_1 equal to 63.94 kilo joule per kg, kilo joule per kg okay. Corresponding V_1 will be volume at 0.4 megapascal okay. So this will be again 0.0007907 meter cube per kg okay. Now, during this compression work, work done by the pump will be equal to change in flow energy of fluid because here, if we calculate work done by the pump, it will be just changing flow energy of the refrigerant which will be V_1 times P_2 minus P_1 .

Okay so V_1 , we have already calculated so it will be 0.0007907 meter cube per kg times pressure P_2 and P_2 is 1.6 megapascal which is 1600 kilopascal minus 400 kilopascal. So this gives total 0.95 kilojoule per kg. So this much of work is being done by the pump on the refrigerant. Okay from the energy balance on the pump, we get H_2 equal to enthalpy of this state 2 will be equal to enthalpy of the saturated liquid plus work done by the pump on the on the refrigerant.

To this will be equal to 63.9 plus 0.95 kilo joule per kg. So after solving it, we get H_2 equal to 64.89 kilo joule per kg.

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Question-1

$$\left. \begin{array}{l} P_3 = 1.6 \text{ MPa} \\ T_3 = 80^\circ\text{C} \end{array} \right\} \begin{array}{l} \text{A-13} \\ h_3 = 305.07 \text{ kJ/kg} \\ s_3 = 0.9875 \text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left. \begin{array}{l} P_4 = 0.4 \text{ MPa} \\ s_4 = s_3 \end{array} \right\} h_4 = 273.21 \text{ kJ/kg}$$

$$q_{\text{in}} = h_3 - h_2 = 240.18 \text{ kJ/kg}$$

$$q_{\text{out}} = h_4 - h_1 = 209.27 \text{ kJ/kg}$$

$$w_{\text{net,out}} = q_{\text{in}} - q_{\text{out}} = 30.91 \text{ kJ/kg}$$

$$m = \frac{\dot{w}_{\text{net}}}{w_{\text{net}}} = \frac{750 \text{ kJ/s}}{30.91 \text{ kJ/kg}} = 24.26 \text{ kg/s}$$

Now, it is given that boiler operates at 1.6 megapascal, that is P3 equal to 1.6 megapascal, T3, the inlet temperature is also given which is 80 degree centigrade. So it is superheated state so from the table, A13 corresponds to these conditions. We get S3, enthalpy at the state 3 equal to 305.07 kilo joule per kg okay. Now further corresponds to this condition H3, entropy will be equal to 0.9875 kilo joule per kg into Kelvin.

Since condenser is operating at 0.4 megapascal okay so P4 is 0.4, S4 equal to S3 because it is an isentropic process okay. So corresponds to this, we get H4 equal to 273.21 kilo joule per kg. Now we can calculate heat coming in the cycle equal to H3 minus H2. So since we have already calculated H3 and H4 so after plugging these, we get 240.18 kilo joule per kg. Similarly Qout will be H4 minus H1. So we know H4 and H1 so after plugging it, we get 209.27 kilo joule per kg right.

So now we can calculate net work coming out equal to Qin minus Qout will be equal to 30.91 kilo joule per kg okay. From here, we can calculate mass flow rate of refrigerant equal to power coming out of the cycle divided by the net work per kg. so it is given that it is producing 750 kilo joule per second divided by 30.91 okay kilo joule per kg. so after solving it, we get 24.26 kg per second so this the mass flow rate of refrigerant.

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Question-1

$$\begin{aligned}\eta_{th} &= 1 - \frac{q_{out}}{q_{in}} \\ &= 1 - \frac{209.27}{240.18} \\ &= \underline{0.129}\end{aligned}$$

Now further we need to calculate thermal efficiency so thermal efficiency is nothing but 1 minus Qout upon Qin. So we have already calculated Qout and Qin. So after plugging it here, we get 1 minus Qout which is 209.27 and the Qin which is 240.18 okay so this gives 0.129 okay. Now moving to the next question.

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Question-2

Consider a steady-flow Carnot cycle with water as the working fluid. The maximum and minimum temperatures in the cycle are 350 and 60°C. The quality of water is 0.891 at the beginning of the heat-rejection process and 0.1 at the end. Show the cycle on a T-s diagram relative to the saturation lines, and determine (a) the thermal efficiency, (b) the pressure at the turbine inlet, and (c) the net work output.

① $\eta_{th,c} = 1 - \frac{T_2}{T_1} = 1 - \frac{(60+273)K}{(350+273)K} = 46.5\%$

② $s_2 = s_3 = s_f + x_3 s_{fg}$
 $= 0.8313 + 0.891 \times 7.0769$
 $= 7.1368 \text{ kJ/kg}\cdot\text{K}$

Consider a steady flow Carnot cycle with water as the working fluid. The maximum and the minimum temperatures in the cycle are 350 and 60 degree centigrade. The quality of water is 0.891 at the beginning of the heat rejection process and 0.1 at the end. Show the cycle on a TS diagram related to saturation lines and determine thermal efficiency, the pressure at the turbine inlet and the net work output. So if you see the process on a TS diagram, since initial conditions are given, it is given that during heat rejection process, initially quality is 0.891.

So heat rejection process will take place at a constant temperature in Carnot cycle and since quality is 0.891 so that point will be somewhere here and it is also given that at the end of this process, the quality is 0.1 so this point corresponds to 0.1 quality because mostly water is here okay. So quality is 0.891 to 0.1. Further it will be compressed isentropically since it is a Carnot cycle so it will look like this, the whole process will be like this.

So 1, 2, 3 and 4 so these 3 and 4 correspond to 0.891 and 0.1 quality okay. It is also given that minimum and maximum temperature of the cycles are 350 and 60 so minimum temperature will correspond to this which is 60 degree centigrade and maximum will correspond to this which is 350 right. So heat will be rejected here and heat is going in here okay. Now it is asked to calculate thermal efficiency so what is thermal efficiency?

Thermal efficiency of the Carnot cycle is $1 - \frac{T_L}{T_H}$ okay. So $1 - \frac{60 + 273}{350 + 273}$ Kelvin divided by 350 plus 273 Kelvin okay. So the thermal efficiency will be 46.5%. Now in the second part the pressure at the turbine inlet okay so we have to calculate P_2 value okay. So S_2 entropy at this state will be equal to the entropy at the state 3 because it is an isentropic process, isentropic expansion.

S_3 equal to entropy of the fluid plus quality at this state 3 and SFG, SFG. So at this temperature from the saturated water table, we get S_F , S_F equal to 0.8313, quality is given 0.891 at the state 3. 0.891 into 7.0769 okay. So after solving it, we get 7.1368 okay kilo joule per kg Kelvin.

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Question-2

$$\left. \begin{aligned} \text{Since } T_2 &= 350^\circ\text{C} \\ S_2 &= 7.1368 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \end{aligned} \right\}$$
$$P_2 = 1.4 \text{ MPa} \quad \text{from A-6}$$

The net work = Area enclosed on the T-S curve

$$\begin{aligned} S_4 &= S_f + x_4 S_{fg} \\ &= 0.8313 + 0.1 \times 0.70769 \\ &= \underline{1.539} \text{ kJ/kg}\cdot\text{K} \end{aligned}$$
$$\begin{aligned} \text{Thus } W_{\text{net}} &= \text{Area} = (T_H - T_L)(S_3 - S_4) \\ w_{\text{net}} &= 1623 \text{ kJ/kg} \end{aligned}$$

Since T_2 equal to 350 degree centigrade okay and S_2 will be equal to 7.1368 because it will be equal to S_3 right so kilo joule per kg into Kelvin. Now we know the state 3 information, state 1 S_2 . So corresponding to this condition, we can calculate P_2 okay which is 1.4 megapascal and this can be taken from the table A6 corresponds to this condition. In the next part of this question, we have to calculate net work okay. So the net work will be equal to area in the TS, area enclosed on the TS curve okay.


So for that, we need to know entropy of the state 4 which will be equal to entropy of fluid plus quality of the state 4 times SFG. Okay so SF is 0.8313, entropy we know which is 0.1 and 0.70769. Now, after solving it, we get 1.539 kilo joule per kg to Kelvin plus W_{net} will be equal to area and this area will be equal to T_H minus T_L times S_3 minus S_4 . So we have to do it here.

So after plugging these values, T_H , T_L , S_3 and S_4 . S_4 , we have already calculated okay so after plugging, we get W_{net} equal to 1623 kilo joule per kg. So now moving to the next question.

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Question-3

A space is kept at -23°C by a vapor-compression refrigeration system in an ambient at 25°C . The space gains heat steadily at a rate of 3500 kJ/h and the rate of heat rejection in the condenser is 6000 kJ/h . Determine the power input, in kW, the COP of the cycle and the second-law efficiency of the system.


$$\begin{aligned} \dot{W}_{in} &= \dot{Q}_H - \dot{Q}_L \\ &= (6000 - 3500)\text{ kJ/h} \\ &= 2500\text{ kJ/h} \\ &= 2500\frac{\text{kJ}}{\text{h}} \times \frac{1\text{ kW}}{3600\text{ kJ/h}} \\ \dot{W}_{in} &= 0.6944\text{ kW} \end{aligned}$$
$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{(3500/3600)\text{ kW}}{0.6944\text{ kW}}$$

A space is kept at minus 23 degree centigrade by a vapor compression refrigeration system in an ambient at 25 degree centigrade. The space gains heat steadily at the rate of 3500 kilo joule per hour and the rate of the heat rejection is 6000 kilo joule per hour. Determine the power input in kilowatt, the COP of the cycle and the second law efficiency okay.

So if we see the process, so it takes heat from the space which is S minus 23 degree centigrade okay and rejects at 25 degree centigrade and it is asked to calculate power input right? So we have to calculate power input, COP of the cycle and second law efficiency of the system. Now the power input would be equal to Q dot H minus W dot L. So it is given in the question Q dot H is 6000 minus Q dot L is 3500 kilo joule per hour ok.

So we we get 2500 kilo joule per hour so we need to calculate in kilowatt so we have to change it further. 2500 kilo joule per hour and since 1 kilowatt is equal to 3600 kilo joule per hour okay so after solving it, we get 0.6944 kilowatt. So this much of power going into the cycle. Next we have to calculate COP of refrigerator. COP of refrigerator is nothing but QL dot upon power going into the cycle. So Q dot L is nothing but 3500 upon this much kilowatt and power input is equal to 0.6944 kilowatt.

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Question-3

$$\begin{aligned} \text{COP}_R &= 1.4 \\ (\text{COP})_{\text{Carnot}} &= \frac{T_L}{T_H - T_L} \\ &= \frac{250 \text{ K}}{(298 - 250) \text{ K}} \\ &= 5.208 \end{aligned}$$

So COP of R will be equal to 1.4 after solving that. Now COP of the Carnot cycle. Now if this cycle operates on Carnot principle okay then the COP will be equal to T_L upon T_H minus T_L okay so T_L is given 250, the minus 23 degree centigrade and the T_H is 25 degree centigrade which is 298 Kelvin. So after solving it, we get 5.208.

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Question-3

$$\begin{aligned} \eta_{II} &= \frac{(\text{COP})_R}{(\text{COP})_{\text{Carnot}}} \\ &= \frac{1.4}{5.208} = 0.2688 \\ &= \underline{\underline{26.9\%}} \end{aligned}$$

So second law efficiency equal to η equal to COP of actual refrigerator to the COP of refrigerator when it follows Carnot cycle okay. So now we know COP of R and COP of Carnot cycles. So after plugging value here, 1.4 upon 5.208, we get 0.26 to 8 which means 26.9%. So this is the second law efficiency of the refrigerator. So we will stop here. We will meet you in the next tutorial, thank you.