Introduction to Finite Volume Methods-I
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Lecture-15

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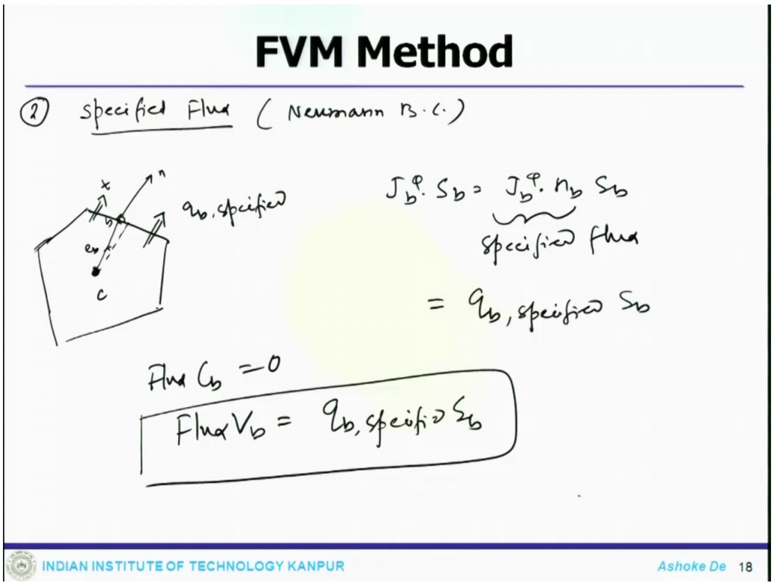

So, the last lecture we have discussed about this boundary condition. Now, we will move from here. So, 2 types of boundary condition that we have discussed one is the dirichletand, the Neumann. Now, we will see how you find out the accuracy level?

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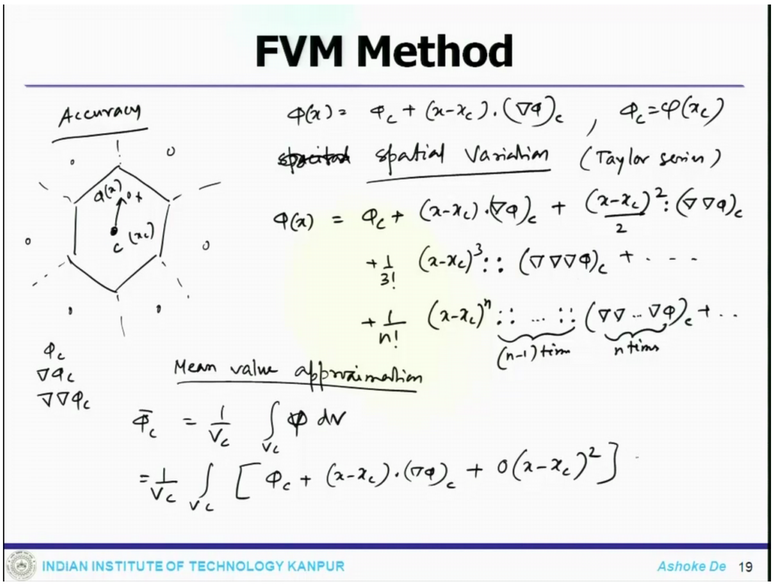

Because, that is one of the important criteria so, what you are trying to get is that, if you have a let us say element like this which is centre c, and the coordinator is x c and you are moving to a point which is called x.

So, function would be phi x and you have all these connecting phases sitting there ok. So, you have all the terms since it is an element c you have phi c delta phi c you have delta delta phi c all these are there. So, what you can see phi x which could be phi c plus x minus x c dot delta phi c where your phi c is phi x c. How do you get the spatial variation? So, first you look at the spatial variation; that means, it is again it would be similar kind of or rather same Taylor series.

But, we will now write it explicitly phi x equals to phi c plus x minus x c dot delta phi c plus x minus x c square 2 double dot del del phi c plus 1 by 3 factorial x minus x c cube del del del phi c plus. So, on and you can have x minus x c to the power n. So, this would be multiplied of n minus 1 times del del del phi and this should be n times.

So, x minus x c is the essentially the distance vector between these 2 points and if you look at this del del c del del triple del c this will return the second derivative third derivative nth derivative like that ok. Now, this is how you express your spatial variation getting or given a value at centroid c. Now, mean value if you do mean value approximation what is that. So, I am calculating phi c bar which will be nothing but my

Vc integration of Vc phi dv. So, that I can write 1 by Vc Vc phi c plus x minus x c dot delta phi c plus order of x minus x c square d v.

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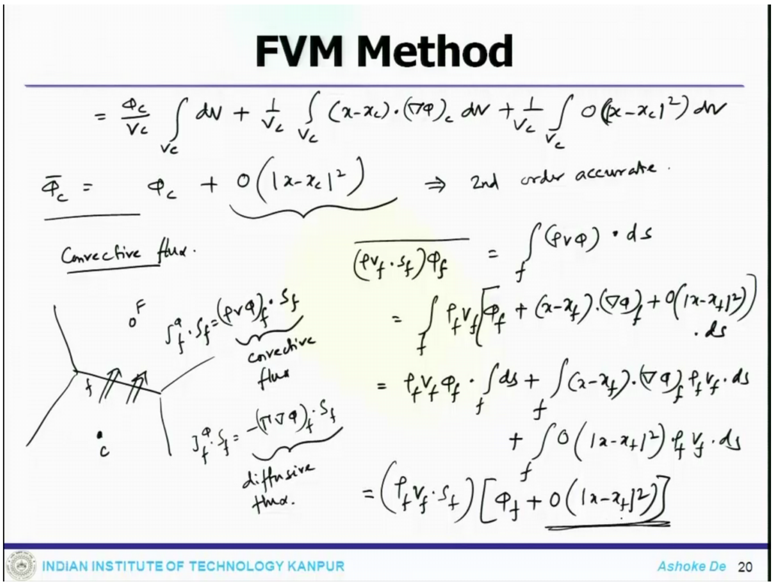

So, if you write phi c by Vc Vc dv plus 1 by Vc Vc x minus x c dot delta phi c d V plus 1 by Vc Vc order of x minus x c square d v. So, essentially if you look at it, it turns out to be the mean value approximation is phi c plus order of x minus x c square so; that means, the mean value approximation what you get this is a very important term which is your second order accurate.

Now, this should be a very important calculation to have because when you calculate the other derivatives and other terms, in the as explicitly, in finite volume formulation. So, you can estimate the order of accuracy and all this now similarly you can have the fluxes. Let us say convective fluxes approximated and see the order of convective flux. So, you just take this side of the element and this is the c this is the face f. So, this is f so j f dot s f is rho v phi dot s f so that my convective flux ok.

And my diffusive flux would be minus gamma del phi f dot s f. So, that is my diffusive flux. So these are 2 components. Now, if you look at the convective flux so the convective flux you have rho V f dot s f phi f. So, essentially this is integration of f rho V phi dot ds rather you write the rho f Vf phi f plus x minus x f dot delta phi f plus order of x minus x f square dot ds. And then, if you segregate this out you right rho f Vf phi f dot f ds plus integration x minus x f dot delta phi f rho f Vf dot d s plus f order of x minus.

So essentially in a Taylor series expansion, you just written 1 or 2 term initially and then, truncated the higher order term. So, this will actually get you back rho f b f dot s f phi f plus order of x minus x f square. So, again if you look at the higher order term sitting here, this is second order that accurate. So, what you got once you calculate the mean value at the cell centre. So, that also second order accurate once you calculate the convective flux that become second order accurate.

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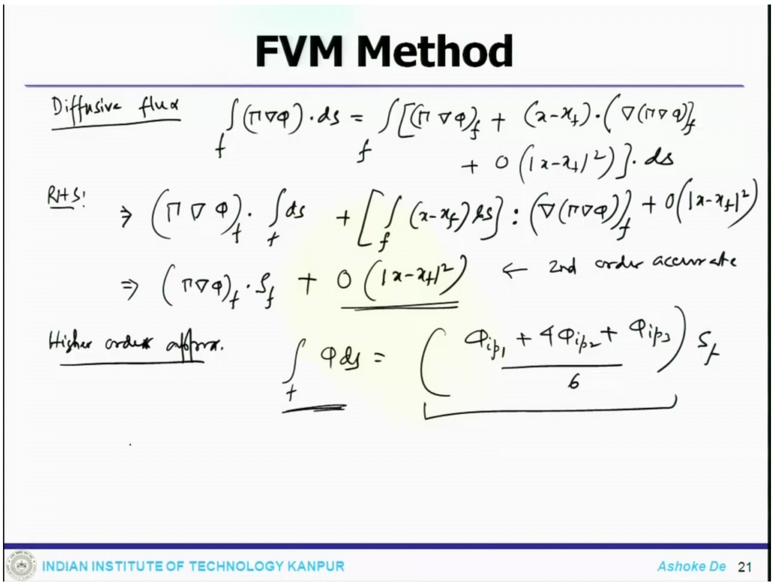

Now, you can look at the diffusive flux. So, what accuracy it possesses. So, the diffusive flux you have f gamma delta phi dot ds. So, you can write over the surface gamma delta phi f plus x minus x f dot delta gamma delta phi plus x minus x f square dot ds, if collect the term in the right hand side.

So, the first term will get you back the gamma dell phi f dot f ds plus f x minus x f ds dot product of the tensor delta gamma delta phi plus order of x minus x f square. So, if you collate, so this will be gamma delta phi f dot s f plus order of square ok. Again, the higher order term sitting here is the second order accurate so; that means, if you use this particular approach and discretize the system essentially. The generic finite volume discretization gives you back all the convective diffusive terms are of the second order accurate one can always achieve higher order accuracy. So, that case higher order approximation you have to use.

So, if you want to do that so you can get the integration of phi ds with some sort of a weighting function like this. So, you can achieve higher order system, but in general all finite volume discretization return you back the second order accurate system which is handy to work with the generic finite volume system, but one can always achieve the higher order system with this kind of a approximation. These get you an idea about the steady state system if you move to the transient system.

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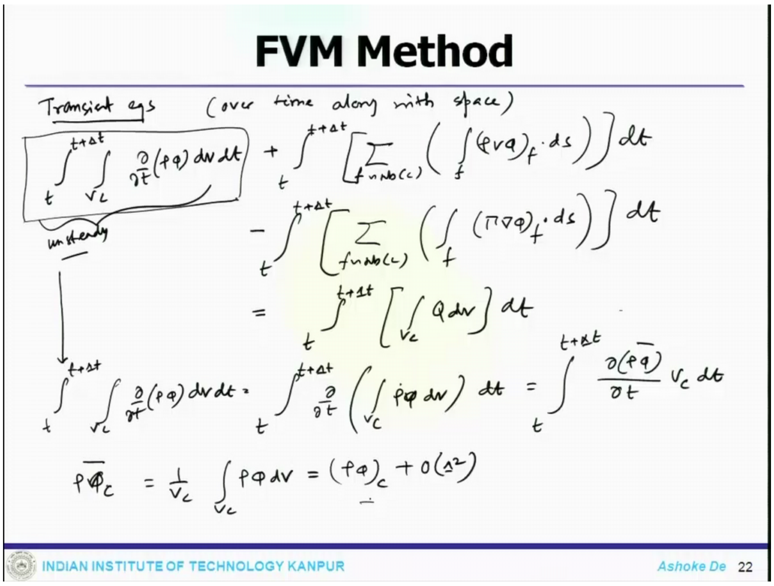

So, what happens to the transient equation ok? So, the transient equation you integrate over now time along with space. So, your transient term will be integrated over a small interval of delta t and this term now will be returned the (Refer Slide Time:14:22) term and the other term will be also integrated over the time. So, these are my convective fluxes minus t plus delta I will get diffusive fluxes gamma delta phi f dot ds. So, you get now along with the spatial variation you get the integration d1 over temporal variation also.

Now, this term previously we have dropped out, now this is the unsteady term or the term which will be written and this should be equals to your generic source term. So, if you compare between the steady state and the transient system except this particular term and the integration over time. If you drop out everything else has been integrated and you obtained a linear system for the steady state system. So now, when you move to the transient system, now you have integration over time. So now, you have to simplify each term so first you look at the unsteady term what happens to that ok.

So, once you do that integration of the unsteady term. So, that get you back the integration of so, that is essentially the integration over the time interval t plus del t rho phi by del t Vc dt. Now, we have already seen the mean value approximation for this. So, this is nothing but rho phi dv which we can write second order accurate system. So, that is the mean value approximation that we have seen so, if I put this back what I will get.

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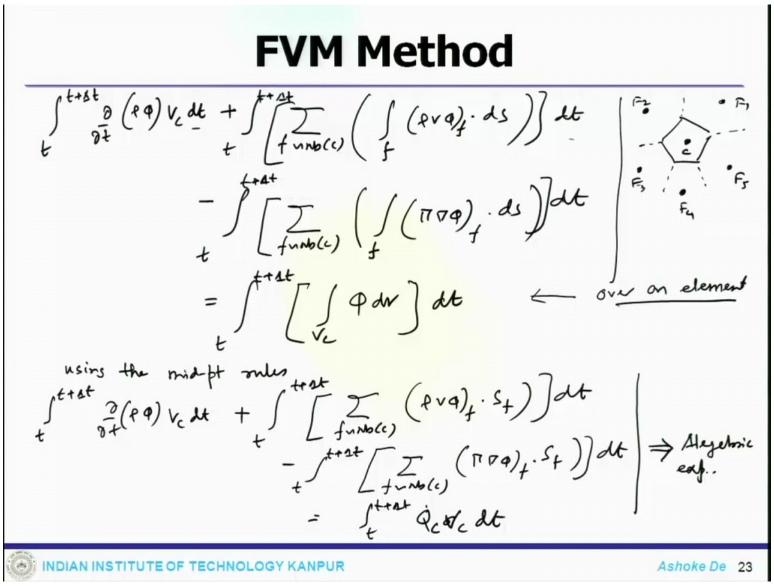

I will get that integration over time with the first term sitting there Vc dt plus integration over time summation of the convective fluxes f rho V phi dt minus the diffusive fluxes, f gamma del phi dt equals to the time integration of the source term ok.

So, each term now retains the integration of dt and you get the integrated system like this. So, the whole objective here is that from this system so if you recall this is a kind of element we are dealing with I mean it could be something else and it has some surroundings cells. So, these are F1 F2 F3 F4 F5. So, it could be 4 faces 6 faces does not matter now essentially all these integration is d1 over an element. And then from here you get back the linear system and from the linear system you finally, solve the problem of the linear equation.

Now, using some midpoint rule you put it back. So, using the midpoint rule you get this equation. Now, t plus delta t del del t rho phi Vc dt plus t plus delta t. So, if gets summation of faces rho V phi minus t plus delta t equals to. So, you get a simplified system now from here you can always get the algebraic expression which will lead to the system of linearized equation. Now, there are certain things for your numerical method should have one of the or rather some important properties of discretized equation ok.

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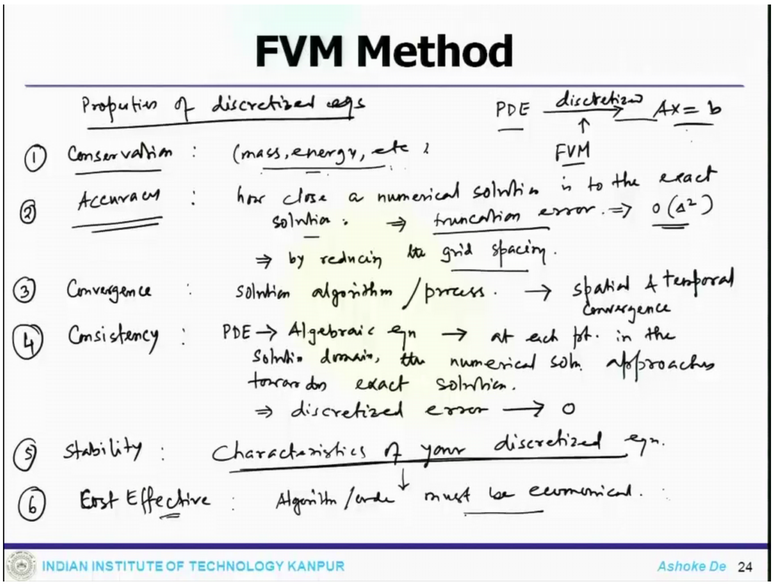

So, what we are doing here we have PDEs now PDEs are discretized to get linear system ok. Now, this descritization process you must have and this is where we have applied our numerical technique of finite volume method. Now, one of the important property that we have also discussed about the conservation ok; that means, all the conservative quantities like mass energy etcetera. They must be conserved even after discretization. So, what is important here is that we start with our mass equation energy equation.

I mean basically momentum equation energy equation continuity equation all this conservative properties like mass, energy all this things. They must be conserved in the discretized equation. You cannot afford to lose the conservation property this is one of the very, very important of fundamental property of this second is obviously. So, first thing I have one has to make sure that the physical properties or conservative properties are properly conserved when you move from the differential equation to the discretized equation. So, they cannot be afford to lose any conservative property second important property that comes out to be the accuracy ok.

So, this is again it essentially refers how close a numerical solution is to the exact solution. So, this is where actually you come down to the accuracy of the system and in 1 or 2 lectures before we have talked about the accuracy and how do you achieve and just right now, you have also seen how one can actually get the and this essentially, means that order of truncation error ok. And we have seen that, when you do final volume discretization all your mean value convective flags diffusive flag everything is of second order.

So, the higher order term in the truncation expression or truncated expression will lead to the accuracy of the system ok. So, more and more order is I mean the higher order term is retained in the expression; you get higher order expression for your numerical scheme. So at the other time, also one can increase the accuracy by reducing the grid spacing. So, I mean since these are leading order term would be of some delta square delta cube of something if you have a small delta or the grid spacing is less. So, this will reduce I mean reduce the error and you can increase the accuracy.

So, third is that so this is where the accuracy is essentially has to do with, how you have discretized your partial differential equations. And what kind of approximation you have made. So, the approximation made and the leading order term that are kind of truncated of from your Taylor series expansion, will lead to the third important property is the convergence. So, that is again has to do with the solution algorithm or process so one may say my solution has converged so; that means, when you do some physical iterations and the error between the process of the values. They are not changing one can argue that solution has converged.

So, sometimes it is also used for the transient calculation when people say the solution it not converts for the time discretization. So, it has to do with both spatial convergence and spatial and temporal convergence ok. So, this is how one can talk about the convergence. Now, 4th property which is again going to be very important is the consistency of the system consistency of the system; so, that essentially when you approximate your PDEs to an algebraic equation.

So, this is a approximation when you do that how consistent it is, is essentially at each point in the solution domain. The numerical solution approaches towards exact solution. So, that is essentially means that whatever you have the discretized error that approaches to 0 and the decretization error comes from essentially. The primarily the truncation error and also the some sort of a numerical errors I mean round off errors.

Now, 5th property which is also very important is the stability ok. So, the stability is that something, when you talk it is the characteristics of your discretized equation ok. So, this has to do with the stability that means, it is a very, very inherent property of your discretized system. How you can say when because the discretized systems only will finally, take you to the solution. So, the part of the solution is be important and that can dictate without the solution is stable or not.

So, it has to do with both the discretized systems and the solution process. Now, and their properties that your scheme and all this process should be also cost effective; that means, I should have some algorithm or code that must be economical; that means, here economical does not mean in terms of money I mean it directly not in terms of money.

It essentially in terms of the computational overhead; that means, how much computational cost is associated with this particular algorithm or the code that you have obtained and how do you have obtained that you have the PDS. You discretize the PDEs get a linear system you have a linear solver and when everything is actually put together through a programming language. And you get a either a sequential or paralyzed version of your code that, is when you use that particular code to solve a problem you can actually estimate whether, it is cost effective or not. Because, it depends how much time it takes to do the calculation.

Thank you.