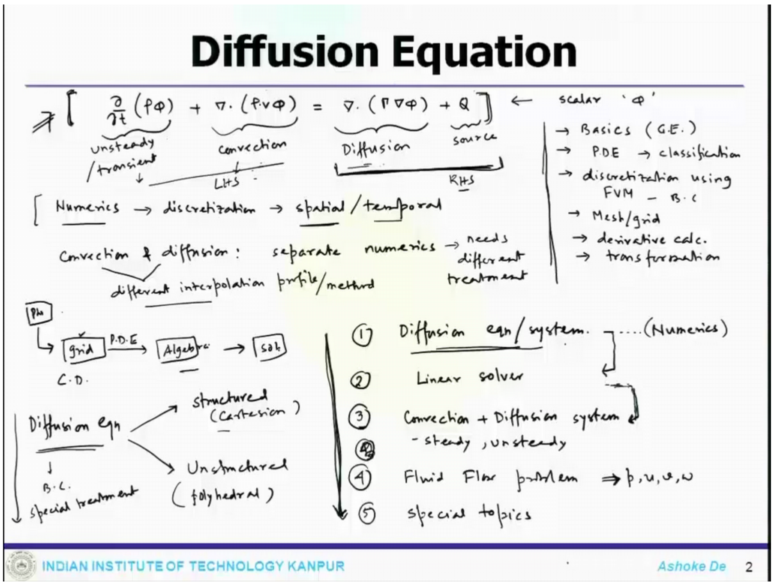
Introduction to Finite Volume Methods-I

Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture – 23
Finite Volume discretization of Diffusion Equation-I

So, welcome to the lecture of this Finite Volume Method. And we will start with a term wise discretization for individual term in the transport equation. So, the first one will begin with is the diffusion equation.

(Refer Slide Time: 00:30)



So, if you recall what is our x equation, the essentially the transport equation for any scalar variable that we have been talking that is in this particular form. So, phi is any scalar variable then you have rho v phi equals to delta dot gamma delta phi plus source term. So, this is the essentially the transport equation for any scalar, scalar phi. Now, this phi once we use so it is generic form for the transport equation. So, from here we can actually get back our momentum equation, we can get back our energy equation, temperature equation, species mass transfer equation any kind of equation.

So, what we have done so far, we have talked about the basic details about this equation system. So, so far what we have covered is the basic of the governing equations. And then the partial differential equation or the classification of the partial differential equation, I mean essentially when you talk about the basics, you talk about all the governing equations or the transport equations. So, the governing equations are essentially or one class of partial differential equation. So, once we talk about the partial differential equation, we have also discussed the classification of partial differential equation.

So, then we have done the discretization using finite volume method. So, once we do that what we have done essentially, we have taken this complete equation, and discretize over a particular cell, and looked at it. So, there we have details some discussions on the issues related to boundary conditions. Then, then we talked about mesh or the grid. And mean while when doing all these also we have discussed the derivative calculations through Taylor series expansions.

So, all this like derivative calculation, then transformation, so all these we have discussed. Now, the important point is that when you look at this complete equation here, this transport equation, there are different terms. So, one is this unsteady term. This term is essentially this is the unsteady or transient term. Then you have this term second term, which is essentially the convection term. Then you have the diffusion term, it is more like and generic diffusion term, so corresponding to every equations, we have for example, if you go to momentum equation, this will lead to the Laplacian form of second derivative of the velocity component. If you go to temperature equation, that will give you the temperature Laplacian. And the last term is the source term. So, these are the different term that there for this transport equation.

Now, what we have done so far is that, we have looked at the complete equation in totality. And then we talked about some basics of discretization method or the rather the numerics. So, numerics when we say numerics, essentially means that, the discretization. So, this discretization could be spatial or it could be temporal. So, when we have the transient terms sitting there or the governing equations which has that unsteady term, then the discretization also needs to be accounted for the temporal discretization that means, with the order of integrations of that transient term. And if there is no transient term, then the discretization needs to handle the spatial calculation only.

So, we have discussed the numerics in some details for the total system. If you look at that, but that time we have not actually discussed. What is going to happen to this individual component? Because of you look at this component for that time being you just drop out the transient term. We will handle it later on. And we drop out the convection term you look at the diffusion term with some source or without source term.

The reason is that convection and diffusion. If you look at this convection and diffusion, these terms are handled separately because the this is you require separate numeric's or discretization technique to handle this individual term. Why, the reason being convection is a different phenomena diffusion is a different phenomena typically, if you have a flow, which is convection dominated that time diffusion become quite less important. So, convection actually becomes predominant phenomena on top of diffusion.

In certain flows like boundary layers or somewhere boundary layer and the flow to close to the wall diffusion becomes dominant. So, when the diffusion becomes the dominant convection, does not play a big role, so that is why the numerics associated with this two terms that essentially the convection and diffusion. They are completely separate, and bottom line is that they needs to be treated separately. So, essentially needs different treatment.

So, the discretization pattern would be completely different and also while dealing g with this two term you need different interpolation profile or method. So, you need that kind of situation you cannot clap them together, and use similar kind of discretization to have these term getting the discretized, and lead to the algebraic system, so that is why you need to teach them treat them separately. And then what we plan to do, now onwards since we have talked about all the basic step, so where for your c f d calculations.

What are the essential components, your essential components are essentially your grid or rather you can start with your physical problem moved to the computational problem or computational grid domain computation domain grid, then governing equation transforms to the, so PDEs these are transforms to the algebraic system and then finally, the solution. So, you got all these different steps.

Now, we are talking about we have talked enough about the grid, we talked about the physical problem including boundary conditions, but anyways the boundary conditions will keep coming back, when we treat them separately. And then finally, we will have looked at the algebraic system, but now what we have planned to do will treat this individual term of this equation.

So, essentially this transport equation will treat them individually the terms and do the discretizations. Once we do that, then only we will be able to see what are the problems associated with each term, and what kind of spatial treatment you need how to handle some individual elements, how to handle the spatial cases. So, all these will be the now point of discussion. When you do that to start with, we will start with the if you will just leave out the left hand side of the equation, we will start with a steady diffusion system with some source or sink term, so that is what essentially the discussion that will follow now on will be primarily on the diffusion system.

And what we are going to do we will look at the plan is that first we will look at the diffusion system diffusion equation or diffusion system once we look at the diffusion system with source term or sinks term then that will actually allow you to lead to a some sort of a linear system. So, we need to know the linear solver. So, we will do that. So, from here we will move to this first, we will do the diffusion discretization. When you talk about diffusion system, essentially we talked about the numerics. And numerics means it will include all sort of natural calculations, sort of spatial calculations, any boundary conditions, any spatial treatment required for a particular kind of discretizations everything that includes all sort of things.

And also what will plan to do will start with, then we will move to the since we are done with the diffusion term. And by that time will have a linear solver also ready will add the convection diffusion system convection plus diffusion system So, that is how we moved to the increasing order of complexity. When you talk about that, there we will talk about both steady and unsteady everything. So, we will do that so that means, by that time we have our for any scalar. Once we reach to the step three for any scalar transport equation, we should have fair amount of idea how to discretize the convection diffusion system whether it is steady or unsteady. And then put it in a linear solver how to get a solution done.

Once that is done then we will move to the fluid flow problem where actually we will solve our Navier-Stokes system. So, this is the way. And then we will move to some speical topics, so that is the plan of action now on I am show that you will see whatever we have discussed so far these are essentially the backbone of our discussion from now on. Thus whatever we have discussed whatever we have learned so far, they are going to be used in each of this step.

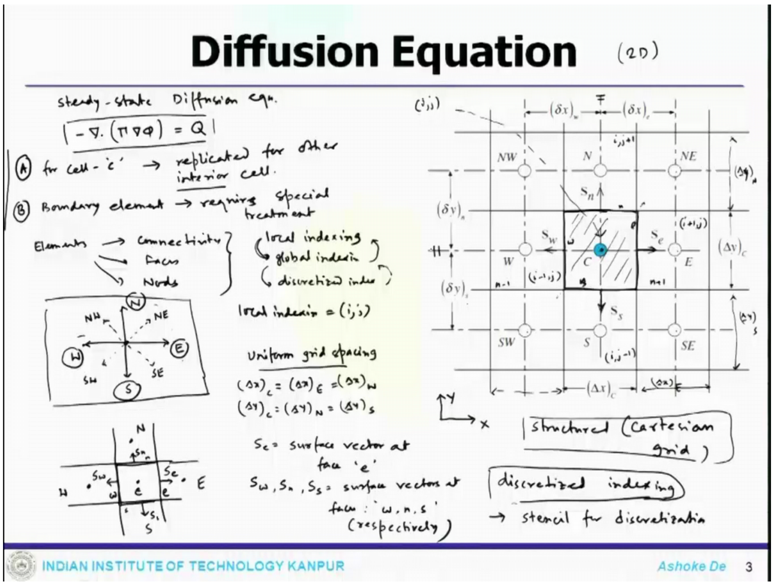
And as we move along this line essentially in the top down approach that means whatever will do in the diffusion system and the linear solver and all these they would be directly applicable or required in the convection diffusion system. And then once we are done with the convection diffusion system, we will move to the fluid flow problem, where you need all the primitive variables like pressure, velocity, all these coupling is required so, that is another challenge where we will see how to handle those kind of variables. And then we will move to some special topic, and do the calculation.

Now, to begin with the diffusion equation, so essentially when we start the diffusion equation, there are two ways one can do. First look at the simplest system that means, we start with the system or the discretization in structured frame work, that means, we have already done enough discussion on the grid. So, structured frame work means we want to remain on the cart structured ordered mesh or Cartesian mesh. So, this is basically your Cartesian mesh.

And then second step would be to look at the discretization on unstructured mesh that means, it would be some sort of a polyhedral system. So, the element would be polyhedral element. And then while doing that each case we will talk about our discretization pattern, how to calculate the equations, how to do the source term calculation, then that will include boundary conditions any special treatment or trivial issues, so that is the way, we will go above to it.

So, as we said we will begin with so the overall framework if you look at it, the overall frame work lies here. We have a transport equation in hand, which is unsteady in nature. And there are different terms, transient term, convection term, diffusion term, source term. And we will now look at the finer details of individual terms through the discretization. So, we will start with the diffusion system that essentially the right hand side of the transport equation. And why the logic behind that is that this system is going to be the simpler to begin with, then we will add the convection to it. So, just to begin with we will start with a structured diffusion system or a Cartesian system. And then move to the unstructured.

(Refer Slide Time: 16:44)



So, have been said that let us have a system of 2D. This is essentially a 2D system 2D structured Cartesian mesh, Cartesian grid. So, it is some generic system it shows. And the equation that will start with the steady state steady state diffusion equation, so that means, our governing equation will be minus delta dot gamma delta phi equals to Q ok, so that is our equation, which will govern the system. So, we have made it simpler. So, drop the unsteady term, and we look at the steady diffusion system with source term.

So, Q could be a source and sink. Now, we have put together these things in a rectangular Cartesian grid ok. And rectangular Cartesian grid if you look at it, this is how you can define your reference frame this direction is x, this direction is y, and the element of interest is some interior cell, which is marked as C. So, essentially we are interested in this particular cell, and one should do that, so the same logic as we have done earlier same logic can be used for all the interior cells. Once we have a algebraic system ready for cell C can be replicated for other interior cell.

While said that one has to note very carefully we are only talking about interior cell, because why when you come down to boundary that requires the boundary cell requires the or boundary element. It requires special treatment so that means, when you have a domain whether it is 2D or 3D, the domain must be divided in two segment; one segment A, which will actually comprise with all the interior cells or elements like this. What we have shown here C w n.

And then you have another segment, which will only represent the boundary cells, that means the element which are which are associated with the boundary that actually provides the boundary conditions at a particular plane or surface depends on the geometry, which is 2D or 3D. If it is 2D, then it would be a phase; of it is 3D, it could be a plane, so that is first task that the physical domain is separated in two segments. One is interior cell; another is boundary cells.

Once we do that you just look at a this picture if you look, we have just actually divided in some cells. And again if you recall from our previous lectures, this particular arrangement can be said that we are in the discretized indexing system. For any finite volume system or finite volume numerical process, you have all these elements their connectivity connecting faces, connecting nodes. These three important information need to be stored in your array or data structure.

And one can have for three different representation of its physical domain. One could be local system or local indexing. Then the connectivity local indexing to global indexing and that can also fit back to local system. And then there would be a discretized system or discretized indexing, which will again lead back to global and global to local. So, here what we see here it is a discretize indexing. So, we have considered one particular element, element C, or the cell C, which is sitting inside the domain somewhere.

Now, once we talk about the discretize indexing or discretize indexing that essentially one can think about this shows the stencil, stencil for discretization that means the particular concern element which is associated or surrounded by some other elements all these information. So, it boils down to that one important things that all connecting elements, connecting faces, connecting nodes and their index. So, all these important information needs to be tacked upon.

So, if you look at this particular C, C is sitting here, C is the centered of this particular cell. Again the discretized system convention will follow the directional convection. So, the directional convention is like this. You have a element sitting here, a head of it, it would be east cell that is why the upstream of it, the cell which is if this cell is in a local indexing, if this is i j, then this guy is essentially i plus 1 j. And discretize indexing it is the east corner of it. Now, behind that particular cell, it is the west. So, this is how we defined our direction also.

So, this could be i minus 1 j, these are local indexing. Then up of it is north that should be i j plus 1 cell, and the down underneath of it is south. So, it is actually how the compass shows the things, so that is the convention it is followed in finite volume algorithm. And eventually that makes like much simpler to understand the discretized indexing or rather it becomes easy to grab the idea. Now, in the down or underneath cell it would be i j minus 1. So, these are the local indexing. So, this i j this is going to be local indexing for this system. And one can transform from local to global, then the cell number would be n, n plus 1 like that, n minus 1. So, there can be always a correlation, which can dictate the indexing pattern.

So, once you do that that means this particular cell ahead there is a cell. And then these corners, all these corners are also going to be the north east corner. This is going to be north-west corner. So, if you see the cell, which is at the sitting at the corner essentially, this is the corner, this is north east cell, this is north west cell, then you come down to this side, it will be south west, and this is south east. So, as I said earlier this stencil or discretize indexing typically follows the directional index or the notation, so that makes life easier to understand that whole ideology.

So, if you see that, this is south west, this is south east ok, so that takes care of the elements. So, theoretically or ideally this c, is surrounded with 1, 2, 3, 4, 5, 6, 7, 8 cells, so that is our directional things. One middle cell or middle element is sort of surrounded with 8 neighboring cells. All these elements or cell they are the neighboring cells to c. Now, the distances this Cartesian system, which is shown in this particular case it has uniform grid spacing so, that means the cell with for element c. So, the delta x C is equal to delta x E equals to delta x W. So, the distance of this particular cell c this is equal to the delta x of E

And similarly they are equal to delta x of W, so that is in the x direction. Y direction also the delta y of c must be equals to delta y of north equals to delta y of south. So, this is the distance of south cell. So, this is delta y south this is the distance of north cell, so that is delta x of delta y of north. So, uniform spacing also makes things bit simpler will start with that way. And as we move along the discretization and all these things we can look at the non uniformity also.

Now, these takes care of the cell size neighboring elements. Now, there will be faces. This particular element C which is essentially look at it, the it has only this four faces which are kind of connecting faces with the east element, north element, west element and south element. So, these are the connecting faces for C. And another representation, which is going to be also used or which will be consistence or rather has being consistence throughout our lecture is that face which is connected with the east node or east cell is marked as small e face.

So, if I look at this particular system here, so this is my south, this is my C, this is my E, this is my north, this is my west. So, this face is E this face is small w, because this face is connected with west node. This face would be small s this face would be small n. So, this would be n; this is s; this is w ok. And the surface vector which are there they are going to be marked accordingly. So, if you look at it the surface vector which will be acting here which will be marked at S e. So, S e is the surface vector at face e. Similarly, this is S n, this is S w, this is S s. So, S w, S n, S s, these are the surface vectors at faces w, n, s respectively, so which is been marked in this particular figure; this is S s, S e like that.

So, now in addition to all this marking, now, we should also have an idea about the distance between the centroid C and E, which is marked as del x e and the c and w del x w. Now, since this is a uniform system these two will be also equal. And now in the y direction the distance between centroid C and S is delta y s. And north and C is delta y n they are going to be again equal, because the whole discretization which is depicted here is on uniform system, so that take care of the all the distance and the notations of this. We will stop here and.

Thank you.