Introduction to Finite Volume Methods-I
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture – 26
Discretization of Diffusion Equation for Cartesian orthogonal systems-I

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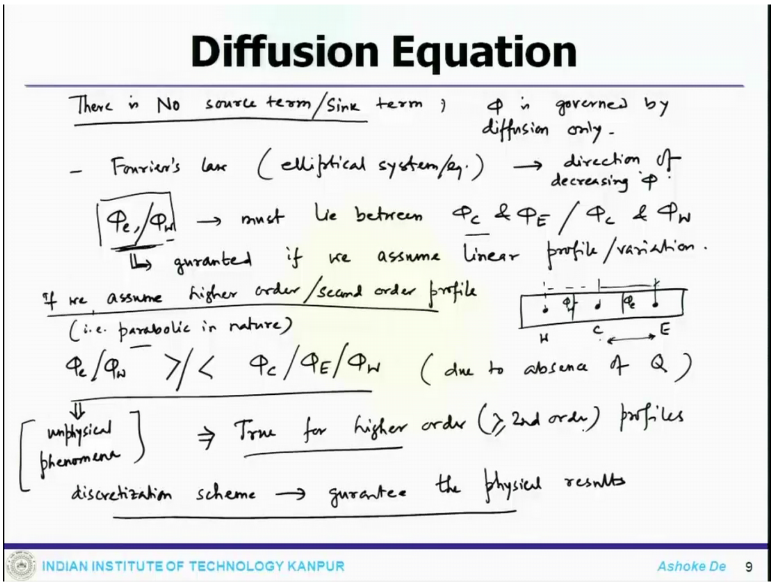

So, welcome to the lecture of this Finite Volume Method.

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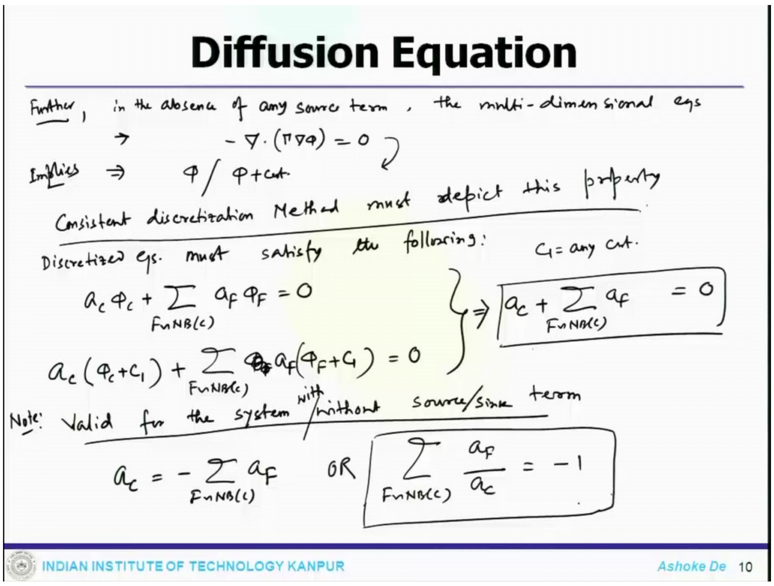

Now further the in the absence of any source term, in the absence of any source term, the multidimensional heat conduction equation, the multidimensional equation reduce to the equation reduce to this equation delta dot gamma phi equals to 0.

So, this implies that phi and that implies straight away one important property phi or phi plus some constant are the solution of this particular governing equations. Since this is a an elliptical system, the solution of this equation would be same whether it is a phi or it is a phi plus any constant. A constant discretization method should reflect this property or rather the important property of a consistent discretization method must depict this property which is one of the characteristics of which is one of the characteristics of solution to a elliptical PDEs.

So, this is very important and what this happens, once the discretization method has to reflect this property, this discretized equation must satisfy the following criteria. What is that? It should have a c phi c plus summation of F NB C capital F phi F equals to 0 or a c phi c plus some C 1 constant plus summation of F NB C phi F a F phi F plus C 1 must be 0. So that means, where C 1 is any arbitrary constant which will lead to essentially a c plus F N B C a F phi F a F equals to 0.

So, this has to be satisfied by the consistent discretized equation. So, this is valid, one has to note that this is only valid; please note this is valid for the system without source or sink term. This is very important that without that this is valid. Valid for system without or with or without source and sink term. So, this if I rewrite, this can be rewritten as a c equals to minus summation F N B C equals to a F or one can write the this things as F NB C a F divided by a c equals to minus 1.

So, which means, one can see this particular equation tells an very important criteria that phi c is the weighted sum of its neighbour.

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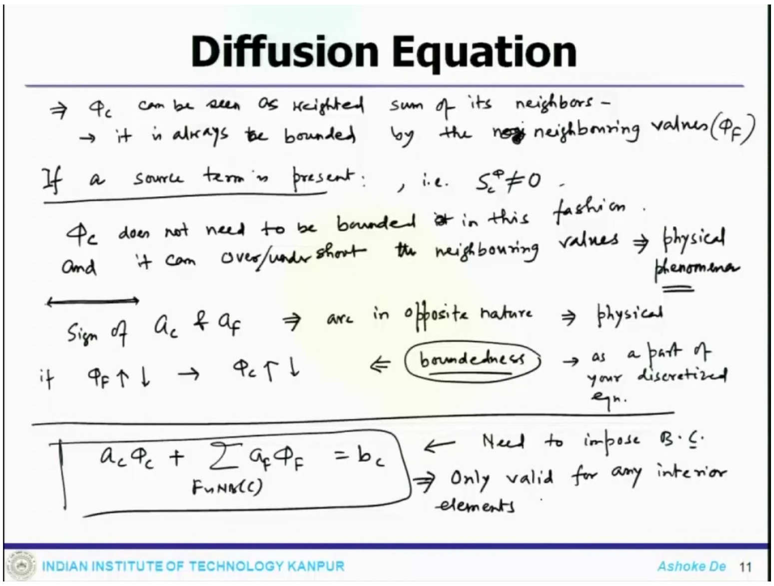

Essentially which tells that phi c can be seen as weighted sum of its neighbours ok. And in absence of any source term, it is always be bounded by the neighbouring values ok. So, this important equation which clearly one can infer that phi c must be looking at as a weighted average or sum of its neighbouring elements and it remains always bounded by the neighbouring values like phi F.

So, whether its east or west so, they actually, now if a source term is present. So, we got an very nice criteria for in the absence of source or sink term but if there is a source term which is present there in the system; that means, where S c phi not equals to 0. So, then phi c does not need to be bounded in this fashion and it can over or undershoot the neighbouring values.

So, now one can immediately ask that when I have a source term, so phi c does not need to be bounded in a particular fashion. Fashion is that in this particular fashion. So, repeatedly this is mentioned that or I am reiterating the fact that this is only true when there is no source or sink term. Now, phi c if there is a source term, then this need not to be bounded through that particular equation.

At the same time the phi c could overshoot or undershoot from its neighbouring values. Now one may immediately think that is this physical, but when there is a source or sink term, this is absolutely physical phenomena. So, this is possible ok.

So, which gets you an important message that when you have this assumption of the linear profiles, it comes with certain restriction. But if you keep on refining your greed, this does not have significant impact on the results. Now at the same time if someone uses higher order profile, then one has to be careful whether the system as any source or sink term.

If the system does not have any source or sink term, then the higher order assumption can overshoot the values at the faces which is unphysical as we have seen and the values should be satisfied with this kind of criteria. Now this criteria is also valid for the situation or the equation system where you have the or one has source or sink term, but in that case things can overshoot and which is perfectly fine.

Now, have been said that one more important thing to note here is that coefficients the sign of a c and a F. If you look at they are in opposite nature. So, this is also physical. Why? Because this has a complete physical meaning because the value of phi F, the value of phi F can increase or decrease or the rather if phi F is increasing, then which will lead to phi c to increase.

If phi F is decreasing, then this also do in the other way around ok. So, this has a physical meaning that the sign opposite signs are of physical nature. This also other way one can think about this has to do it with the boundedness of the nature. So, the boundedness is actually doing that thing. If one goes down, other goes up; in one goes up, other one goes down and that has a complete physical significance.

So, one can think that this particular property comes inherently as a part of your discretized equation, one discretized equation. One need not to forcefully enforce this particular properties ok. So, these are some important systems that one can.

Now once you get that discretized system. If you just go back this particular system your a c and a F. So, this is my system; a c phi c summation of a F N BC phi F a F equals to b c. That is my discretized equations system. Now, once you get that discretized equation system, you can always find out the coefficients, but you need now need to impose boundary condition and these equation is only valid for any interior elements for any interior elements, it is not true for any boundary elements.

So, the boundary elements needs to be treated separately. Now, boundary conditions could be of different type and we can see how the boundary elements are imposed now on the system.

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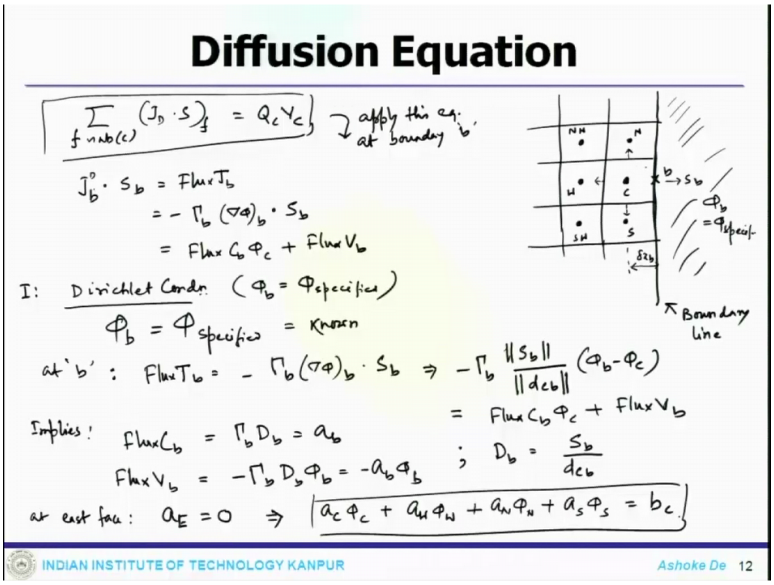

So, you can think about that you have these particular let say, system. You get this, this, this you get this elements ok. And this is your C, this is your S, this is your south west, west, north west, north and this line is my boundary line ok. So, you have this flags going this way and then here is a point boundary point where this would be S b ok. So, you can think about this side is your boundary side. And now if you had that so, this distance would be a distance of delta x small b and the rest remains same as for our notation.

Now, for the boundary also what we need to do? We need to put the same system of equation. This is our equation system equals to Q c V c. So, that is still this is my governing equation, this must be valid. But the one which we got here, this is only valid. So, there is a difference what we are starting that this is valid with any interior element, why? Because this requires some surrounding elements and that is what the validity comes from. But when we starts from here, then we will get the equation system from at that particular boundary where you say that my J b, if we apply this equation at boundary b. Then this would be J b dot S b equals to flux T b which will be minus gamma b delta phi b dot S b which is nothing, but flux c b flux c plus flux V b.

Now, the specification of boundary condition would be different. So, number 1, it could be Dirichlet condition. So, which means you have specified phi equals to phi b is phi specified. So, at this face, I am saying that phi b equals to specified; that means, this is known. So, this is my Dirichlet condition or constant boundary condition.

So, once I say that at that face phi b is known, then I can use that condition and write my total flux at b my flux T b is minus gamma b del phi b dot S b which is going to be my minus gamma b S b divided by d c b phi b minus phi c. So, that is equivalent to flux c b phi c plus flux V b which will which will get you which implies that flux C b equals to gamma b D b. Let us say small a b and flux V b equals to minus gamma b D b and phi b which is minus a b phi b where your D b is defined as S b by d c b ok.

So, if you look at this particular elements shown here in this particular figure, the coefficients at the at east face that is a E which is essentially 0 and the discretized equation would yield like a c phi c plus a W phi W plus a N phi N plus a S phi S equals to b c. So, if you compare with your these equation, the complete equation which you got. This is my complete equation for any interior element where you have a c phi c a E phi E a W phi E a W a N phi E and a S so; that means, in one interior cell has all these east south west north element. So, that is the governing equation.

Now, in this particular case Dirichlet boundary condition, it boils down to a different equation, why? This particular cell and this particular face this is essentially east theoretically east face and the east face is exposed to the given boundary condition and other elements like north, west or south, they are there as it is.

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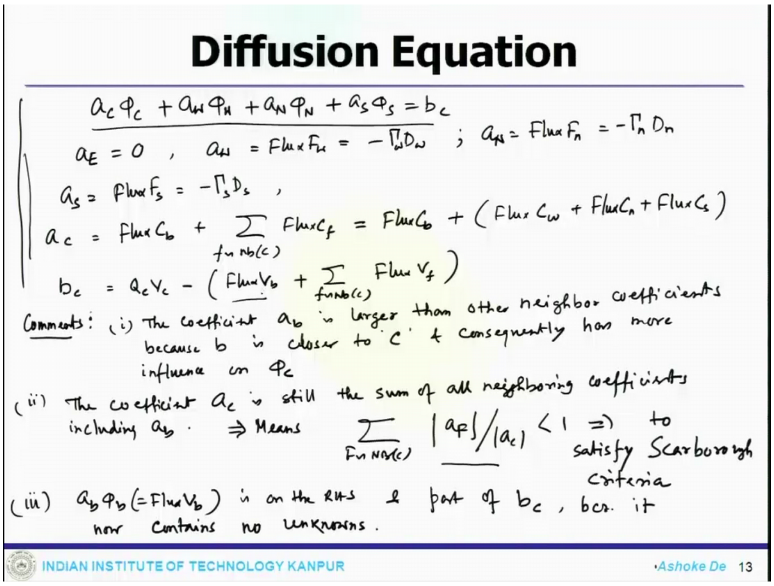

So, your east coefficient goes off and you get back this system where essentially if you write it down a w phi w plus a N phi N plus a S phi S equals to b c, where your a E is 0, a w equals to flux F w equals to minus gamma w D w.

Similarly, a N equals to flux F n which is minus gamma n D n; a S is flux F S which is gamma S D S and a c equals to flux C b plus small f goes from all the faces flux c F which is nothing but flux C b plus flux C w plus flux C n plus flux C s.

So, that is what you get on the b c is the source term which is Q c V c minus flux V b plus summation of N b C and flux V f. So, this is what you get for the given Dirichlet condition. So, one can note few comments. So, some important comments or observation; number 1, it is the coefficient a b is larger than other neighbour coefficients because b is closer to C and consequently has more influence more influence on phi c.

So, second is the coefficient a c is still the sum of all neighbouring elements or neighbouring coefficients including a b. This means important criteria means for the boundary element, this F which is the criteria mod of a F divided by mod of a c less than 1 gives the second necessary condition to satisfy Scarborough criteria.

So, this is second observation and the third one would be so, this criteria the Scarborough criteria would be achieved through the iteration. And the third one is that that the product a b phi b which is the product essentially equivalent to your flux V b is now on the right hand side of the equation is on the right hand side of the equation and part of b c. And the reason is that because it now contains no unknowns ok.

So, there are three very important observations one can make out of these Dirichlet boundary condition implementation. So, this is my system that I obtain and these are my coefficients. So, one of the important observation is that a b is larger than the other neighbouring coefficients because b is closer to c. So, it will have more impact on the element c then the Scarborough criteria is also satisfied through iteration. And the third one is that now the product a b phi b which is flux V b is sitting on the right hand side of the equation because it is no more unknown all the terms has known values. So, this is where how one can treat the specified boundary condition. So, we will look at the other boundary conditions in the subsequent lectures.

Thank you.