**Introduction to Finite Volume Methods – II**
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**Lecture – 10**
**Convection term discretisation – II**

Welcome, to the lecture of this Finite Volume Method.

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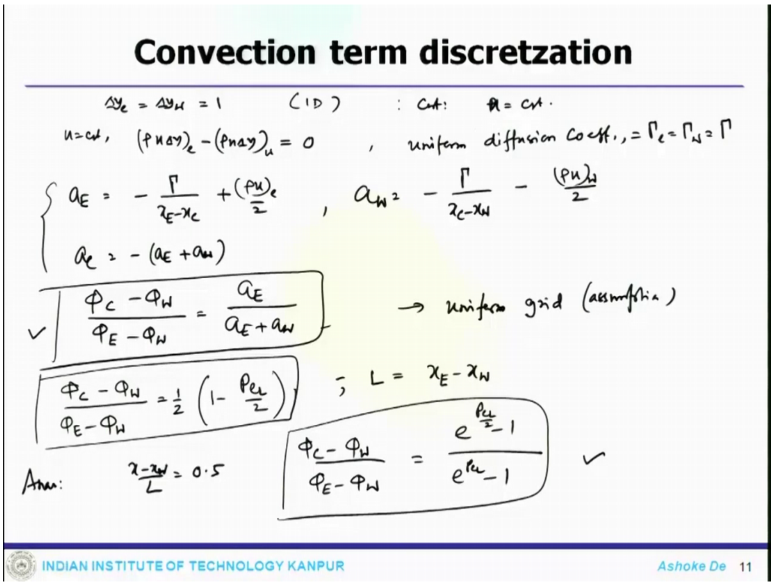


Now, these two solutions one is the discritized solution and one is the analytical solution. So, the analytical solution provides you the exact one and the discritized solution will give you the numerical solution and one can see for the variable Peclet number because this is also a function of Peclet number, this guy is also the analytical solution is also a function of Peclet number. So, one can plot them for varying Peclet number and one can see how the solution actually varies.

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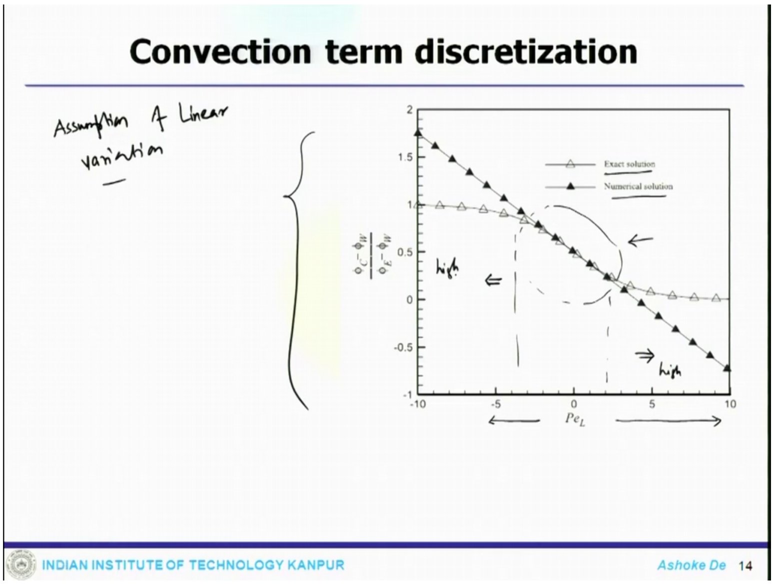


So, the plot is. So, this is where you compare your exact solution with your numerical solution and the numerical solution is obtained based on C-D scheme and the Peclet number is varying between minus 10 to plus 10. So, what you can immediately see from here? This is my exact solution the pattern of the exact solution; this is where my numerical solution provides, which is more like an linear profile which we get from the numerical solution. So, what is happening here is that at the low Peclet number if you see; that means, this is the region where the low Peclet number region one can see in this region solution are quite exact.

But, as you move towards this side or; so this is the region one can see the solutions are quite good or if you go to either of the side with the increasing Peclet number the solutions are quite different. In the sense the numerical solution differs significantly from your analytical solution, ok. So, what it is doing it is essentially the solution indicate clearly that the assumption used. So, the assumption of the linear variation; so, the assumption of linear variation which was used in the discritization appears to be unrealistic or unphysical because in this particular case it does not get you back the realistic solution.

So, what is the reason for this? I mean why is this happening? So, what we can see right now there is a difference between the numerical solution and the exact solution that is happening as we move to the high Peclet number region. These are high Peclet number region and why is that happening so, that can be kind of shown in a schematic.

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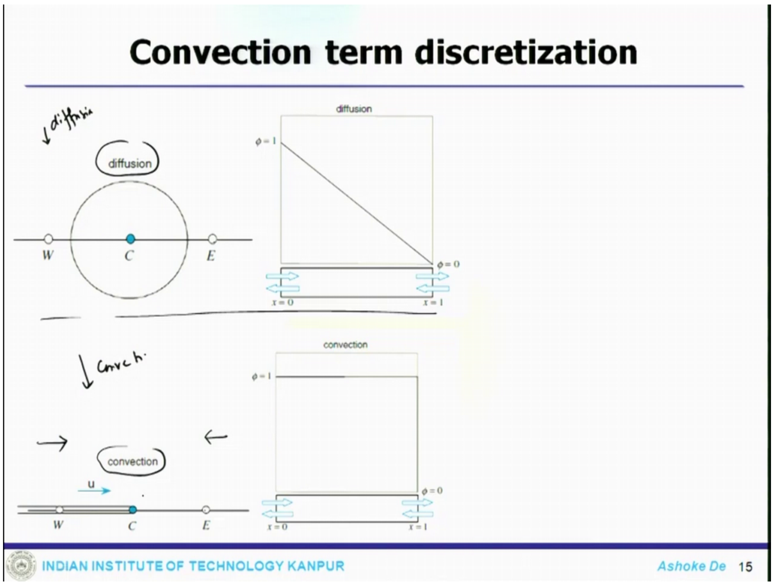


So, first thing that you look at this particular picture where at the point C, the diffusion only here we are talking about diffusion. So, at this particular point C so, the diffusion is affected by the upstream and downstream element. So, now the advection process, which is shown here or the convection process; convection process is a highly directional process of transporting the properties because it takes care of the underlined flow field.

So, the assumption of linear profile which assigns the weight to both upstream and downstream nodes can be a good approximation for diffusion system, but cannot be a good approximation for the convection diffusion system, because it does not take care the directional preference. If u is moving in this direction then the information or the scalar is getting transported from left to right.

And, so, information should propagate from left to right with an weightage on the points which are there and when the information will propagate from this side to the side then the weightage must be provided to the node sitting that side. But, when you assume linear profile or the central difference scheme this does not take the effect of this directional properties.

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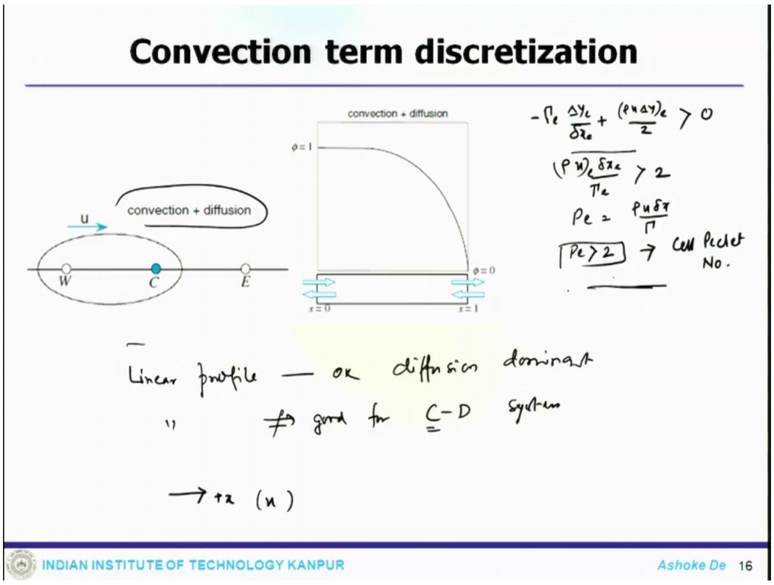


So, if you look at the combined convection diffusion influence, so, one can see what is happening to the profile. The zone of influence actually approaches the diffusion region which is displayed earlier, and the advection region which is also depicted earlier.

Now, here at low and high Peclet number values; now, as long as your diffusion is dominant the transfer mechanism the use of linear profile is so, the linear profile assumption is for diffusion dominant system, but that is not the case, once convection overwhelms over the diffusion you get this is not good for C-D system or convection diffusion system because now the convection takes over this.

So, assuming the flow to be positive x direction; so, we assume the flow to be positive x direction, this is my flow direction. So, one can write the possibility of a E coefficient to become positive thus leading to some sort of an unphysical results. Now, if the flow is in the negative direction then a W may become positive. So, what one can write is that write an system that minus gamma e delta y e by del x c plus rho u delta y e by 2 greater than equals to 0. This is the coefficients which corresponds to a E that is the coefficient corresponds to a E.

So, now, a E to be positive which means the rho u v delta x c by gamma e to be 2. So, if you define the cell Peclet number like P e equals to rho u del x by gamma and for a uniform grid the Peclet number has to be 2. So, that is the restriction. So this is called the cell Peclet number or cell Peclet number. So, for cell Peclet number greater than 2, the discritization process actually becomes inconsistent or else now an increasing an neighbouring values will lead to the decrease in the value of C. So, that is why you are getting the significant error when the convection is. So, these situation one can avoid or can be avoided by decreasing the grid size.

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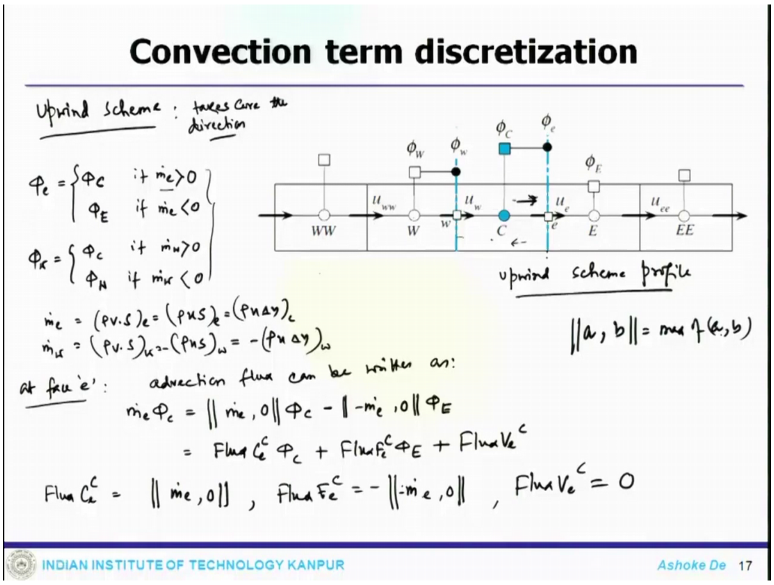


So, the cell Peclet number is smaller than 2 or you can use some sort of an other scheme which is called the impact of the or upwind scheme called the up wind scheme which takes into account. So, one thing is clear by this time that C-D system has or the central difference scheme has the equal weightage which has a problem. So, the upwind scheme actually takes care the direction. So, that is where it can. So, now, this is what the shows the schematic for this one d stencil and this is how the upwind scheme profile look like.

So, these takes into account the direction of the flow field and how it does now you what it does is that when the flow is in this direction; that means, the fluxes at their faces, if the flow is in going from these direction then it should take the advantage. So, at face phi e it should be phi C, if the mass flux is positive; that means, flow is going from this side to that side. So, that is the information passing or transferred being transferred from left to right or if the flow is in the reverse direction then the phi e should be phi E.

So, when the mass flow rate at this face is negative; that means, at this place face e you look at the mass flux or intern you try to find out the direction of the flow field whether the flow is going from left to right. If it is going to left and right then the weightage of C has been assigned. So, the flux value should be assigned to the central value of that cell. If at that face the flow is coming from the other side then the dominance of the upstream cell like e should be assigned.

Similarly, at w face it should be phi C if m dot w greater than 0 or it will be phi W if m dot w less than 0. So, at this face if the flow is positive; that means, m dot W is going out then this should be the information if it is negative; that means, it is going in this direction. So, it will only tell you the direction of the mass flux or the mass flow rate at the faces and. So, m dot e which is nothing, but my rho v dot s e which is rho u s e is rho u delta y e and m dot w is rho v dot s w which is rho u s w minus minus rho u delta y w.

Now, the advection flux at face e. So, at face e the advection flux can be written. One can write that m dot e phi e equals to magnitude of m dot e 0. So, it takes care of the magnitude and then phi c minus magnitude minus m dot e 0 phi E. So, that is how it is done. So, m dot e phi e is written like that. So, which can be also written as, flux C e convection phi c plus flux F e convection phi E plus flux V e convection. So, it provides you all these coefficients. So, my flux C e convection coefficient would be the magnitude between these then flux F e coefficient for the convection would be my m dot e 0 and flux V e convection is 0. Now, what this guy denotes is the maximum of a and b between that it takes care.

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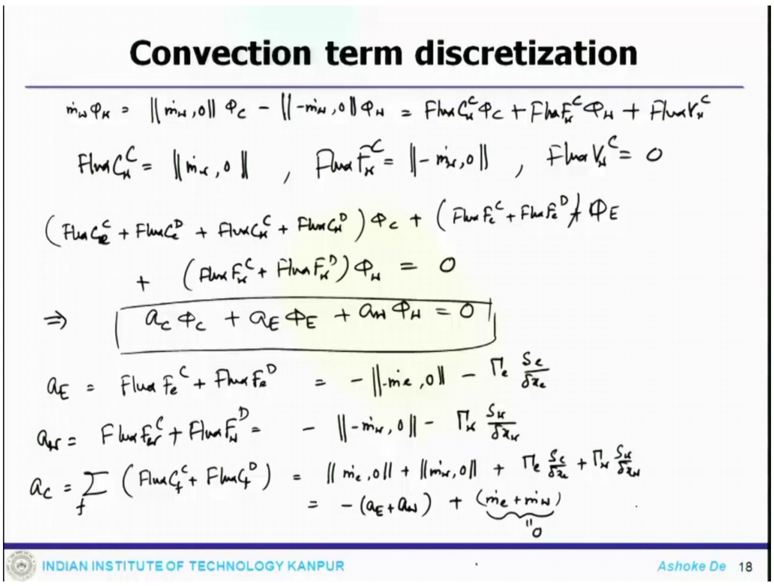

Similar calculation one can get for the west face which is m w phi w equals to m w 0 phi c minus m w 0 phi w which is also flux C w convection phi c plus flux F w convection phi w plus flux V w convection. And, what we get flux C w convection is m w 0, flux F w convection is minus m w 0 and flux V w convection is 0. So, you get for these different systems the solution coefficients.

Now, what you put back everything in the discritized system then we get that flux C w or C e convection flux C e diffusion plus flux C w convection plus flux C w diffusion. This with phi c plus flux F e convection flux F e diffusion phi E plus flux F w convection plus F w diffusion phi w equals to 0.

So, here it will lead to the same system of equation a C phi C plus a E phi E plus a W phi W. So, the discritized system will look similar and the coefficients are here a E is flux F e convection plus flux F e diffusion which is minus m dot e minus gamma e S e by del x e. Similarly, a W is flux F e F w convection plus F w diffusion which is m w 0 minus gamma w S w by del x w. And, what you get at a C which is quite interesting this is the integration over F. So, you get flux C f convection plus flux C f diffusion. So, that will be m dot e 0 plus m dot w 0 plus gamma e S e by del x c plus gamma w S w by del x w. So, that leads to a very nice property like a E plus a W plus m dot e plus m dot w which actually lead to 0, due to mass conservation.

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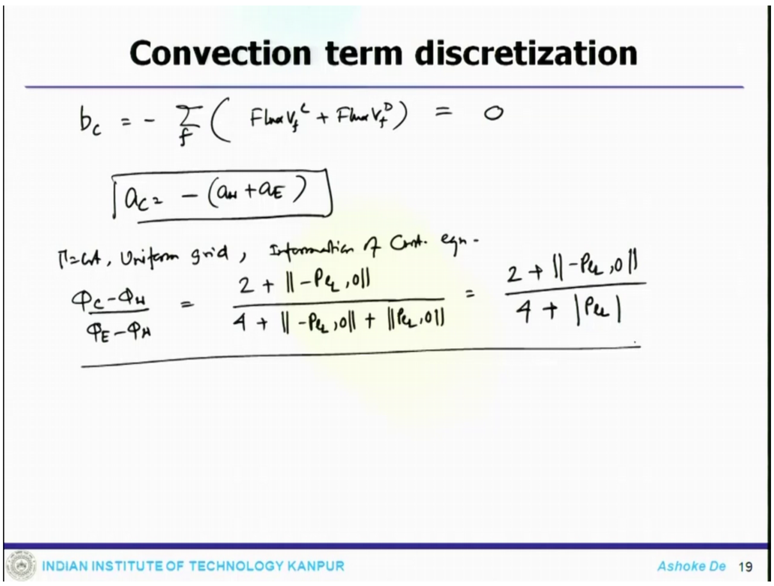

And, the source term or the right hand vector is summation over F flux V f convection flux V f diffusion which is nothing, but 0.

So, one can see that upwind scheme yields essentially the negative neighbor coefficients as the continuity satisfied it reads that a C equals to minus a W plus a E which guarantees the boundedness of the property. Now, once you use this continuity information and use the information of continuity and the uniform grid for, so gamma is constant uniform grid and information of continuity equation is you can write the phi C minus phi W by phi E minus phi W equals to 2 plus minus P e L 0 divided by 4 plus minus P e L 0 plus P e L 0 which will lead to 2 plus minus P e L 0 divided by 4 plus P e L magnitude. So, one get the solution in this pattern.

So, the thing is that now using the upwind scheme what we have obtained is again a solution which involves the Peclet number. So, it would be good idea to see because previously we have seen that the analytical solution differs quiet largely with the C-D scheme.

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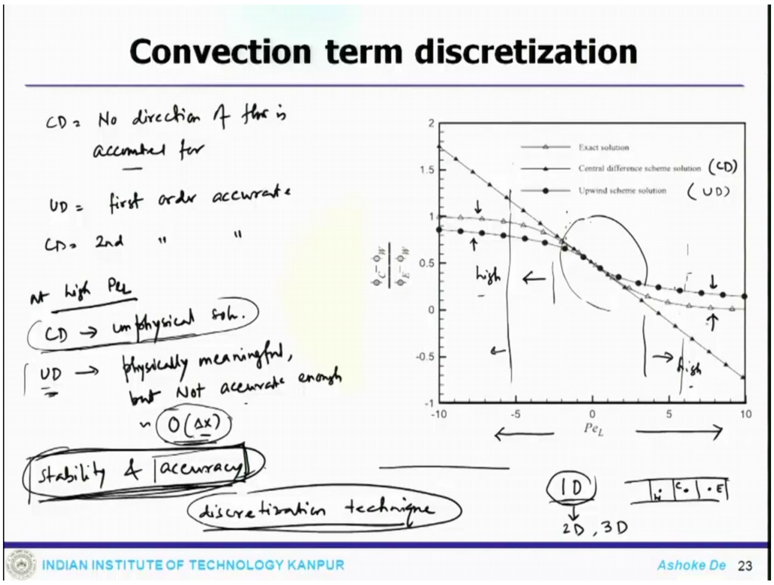


So, the idea is to see how they vary. Now, here this is how the Peclet number is varying and we got three different solution, this is CD which we have seen earlier exact solution follow this curve. So, this is the curve of the exact solution.

CD or the central difference scheme this is upwind scheme. So, the CD scheme shows some sort of a linear profile and if you look at the upwind scheme it follows that trained. So, now, there is a interesting feature. Now, but still this region of the low Peclet number all most all the solution they collapse together; that means, the analytical solution is close enough to the numerical solution, but as you move along from this side at the high Peclet number side, the solution are getting deviated from the exact solution. In this case the CD deviates the most, because the as we have said the CD does not take into account the direction of the flow velocity. And we have assume for CD no direction of flow is accounted for and it provides the equal weightage to the upstream and downstream node.

But, in the upwind scheme due to that accounting it provides a proper trained, but at same time upwind scheme is also not accurate. There are some differences at high Peclet number zone and this is expected. Again this particular feature what we are observing here in this high Peclet number region this is quite expected and why it is expected? Because upwind profile here whatever the upwind we have used this is first order accurate scheme. So, CD is second order accurate.

Now, for the first order accurate scheme at high Peclet number values the CD scheme is unstable and its solution is unbounded and that is why the solution is for the CD is at high Peclet number CD provides unphysical solution. Now, on the other hand upwind scheme provide some its actually provides the proper trained. So, the solution is physically meaningful, but not accurate enough and why is that happening? That is primarily because this guy the upwind scheme is first order accurate.

So, you have a leading order term which is of delta x. So, this can be avoided if you do some sort of a grid refinement. So, that also applies for the CD, if we do grid refinement the local cell Peclet number actually reduces then this band actually can be expanded to this much. So, then you have been slightly flexibility of using higher cell Peclet number to get an solution or the stable solution. But, otherwise also since being an first order accurate; that means, the leading order truncation term is of this order it becomes slightly diffusive.

So, the diffusive nature actually creates some sort of an smoothening of the solution so, you start getting not accurate results. So, what appears from this one end you have a second order accurate CD which provides you unphysical results at the high Peclet number other end you have a upwind scheme which provides you physically meaningful result; that means, it follows the proper trained but, there is accuracy is not enough.

So, the tradeoff is between stability and accuracy. So, one hand CD is unstable at the high Peclet number because the solution becomes unboundedness and that is why you get a wrong result. But the accuracy wise that is second order accurate or higher order accurate and other hand UD or upwind scheme is stable, but accuracy is not there. So, the thing is that these case using the upwind it is better behave results you can obtain at the high Peclet number range, but the accuracy is low. And the CD provides the higher order accuracy, but unstable beyond certain level of Peclet number.

So, the both the scheme seem to be affected by some sort of error or infected by some sort of an errors and one affecting accuracy while the other affecting the stability. And what are these error? So, these are the errors which arise due to the discritization technique. So, this is due to the discritization technique or approach that is adopted.

So, one can kind of. So, one has to keep in mind. So, as we started of that that is what we are doing all our discussion in 1D system. Why? The 1D state still if you look at it, it will be easy to understand what is happening and then once we understand the 1D stencil then we can carry forward this things to or extend this concept to the because the problem which arise at in or arises in 1D cases this will be existing or this will present in 2D and 3D cases.

So, one case the always the when you define. So, these are the problems if you see you never encountered when you started talking about diffusion system or when we started talking discussing the diffusion system. Diffusion system does not have this kind of problem. These starts appearing once you have the convection system because you have a underline flow field. So, the underlined flow field is going to affect. So, we will see how we can avoid all this and discuss about the other problem in the subsequent lecture.

Thank you.