Introduction to Finite Volume Methods - II
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Lecture – 11
Convection term discretisation - III (Private)

Welcome, in this Finite Volume series; right now our discussion is on Convection diffusion system. And where we have stopped so, far we started with a one-dimensional system and we looked at the discretized equation for a convection diffusion system. And, what we have looked at is that the central difference scheme and the upwind scheme. And, also we have compared their accuracy for the against the analytical solution.

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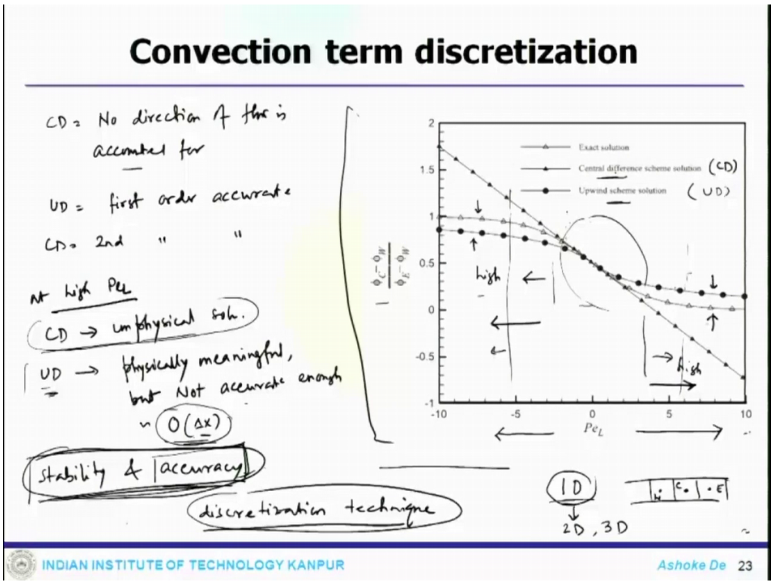

So, if you recall from the last lecture, this is where we stopped and we have seen this particular figure which shows the profile of the scalar against the wearing peclet number. And, we have seen that at the some region which is the for low peclet number region all the numerical scheme like the upwind scheme or the central defense scheme they show or exhibit proper behaviour or exact behaviour while, comparing against the exact solution of the analytical solution.

But both of them start differing as they move out word; that means, this direction and this direction for high peclet number resume. And, CD that the Central Defense scheme shows the complete the different behaviour while the upwind scheme shows the behaviour. It follows proper trend, but still there are certain I mean discrepancy. Now, the point was that there are I mean one has to make some tradeoff between these two. One is that because the central scheme is higher order accurate, upwind scheme is lower order accurate. But, at the same time upwind scheme is much more stable, central scheme is not stable for the kind of system when the convection is dominated.

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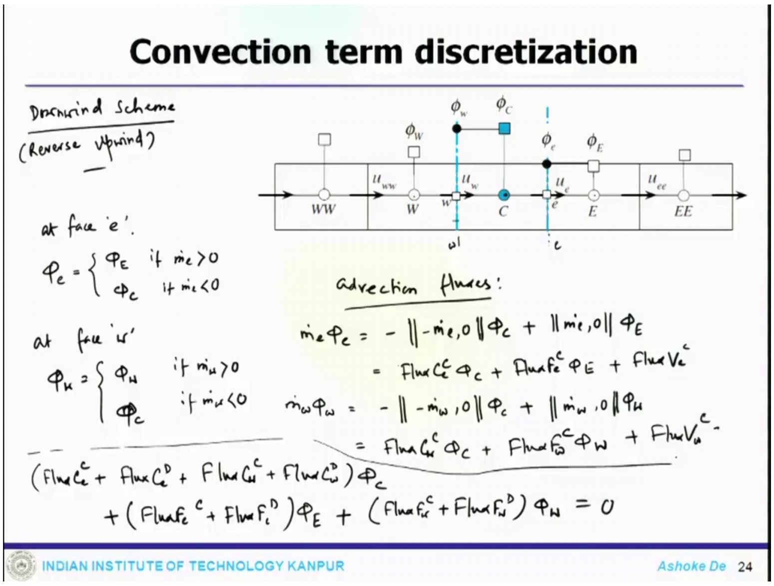

So, with that node now, will start looking at the scheme, a call the reverse upwind or downwind scheme. This is essentially one can think about it is a reverse upwind scheme and we consider this particular stencil as we have done. So, this is our point of interest that particular element, element number C and we have E W and the fluxes at the faces. This is the west face, this is east face of the element. So, primary concern is that how do we or how one evaluates the fluxes at the face s. So for example, at face e; if you look at it then phi e or phi e is going to be phi E if m dot e greater than 0.

So, it is a reverse upwind scheme; while compared with the upwind, if the east face mass flux is greater than 0 or positive then the phi e was supposed to get the contribution from phi C. Now, the other case if m dot e less than 0 this will be phi C. So, in the reverse upwind, it just become flipped. In a conventional upwind scheme, what was there if the mass produced is greater than 0 the face value is assigned to phi C. And, if mass produced is less than 0 face value is assigned to E. Now, similarly at face w phi w equals to if m dot w greater than 0 it is phi W, if m dot w less than 0 it is assigned to phi C.

So, again this is flipped as compared to the conventional upwind scheme. Now, one can write the advection fluxes, if I write the advection fluxes then one has to get m dot e phi e which is nothing, but modulus of minus m dot e 0. So, the maximum of that into phi C plus m dot e 0 phi E. So, that will get me back the flux C e convection phi C plus flux F e c phi E plus flux V e which is again for convection. Similarly, at the west face one can find out m dot w phi W which is nothing, but minus m dot w 0 phi C plus m dot w 0 phi W.

Then we get essentially flux C w convection phi C plus flux F w c phi W plus flux V w convection. So, now if you put everything back in the discretized equation. The equation will look now, flux C e plus flux C diffusion flux C w convection plus flux C w diffusion multiplied with phi C plus flux F e convection plus F e diffusion phi E plus flux F w convection plus diffusion phi W equals to 0. So, that is becomes the equation for the discretized system.

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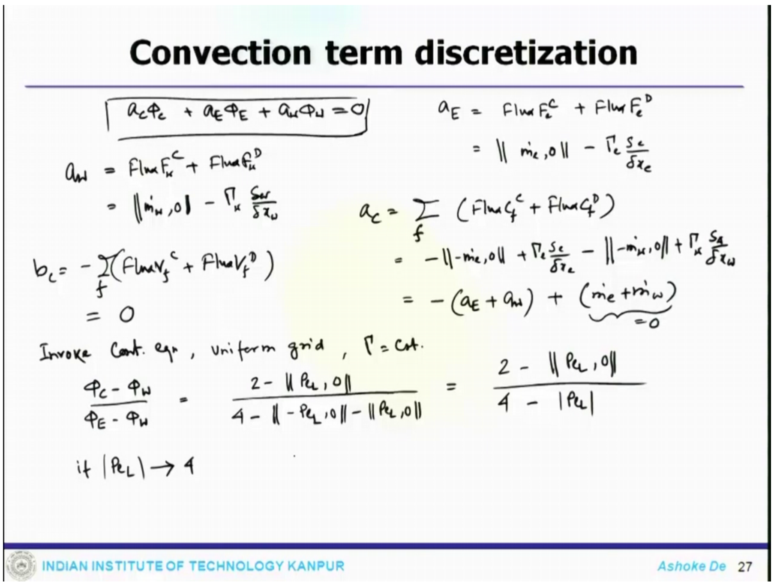

And if you put them in a more compact form; so, this will look like a C phi C plus a E phi E plus a W phi W equals to 0. So, you the discretized equation look similar. Now, once you get that, the coefficients are going to be different compared to conventional fluxes. Here a E is flux F e convection plus F e diffusion which is m dot e 0 minus gamma e S e by del x e. For a W this is flux F w convection plus flux F w diffusion which is m w 0 minus gamma w S w by del x w.

And, a C is summation over faces flux C f convection plus flux C f diffusion which is minus m dot e 0 plus gamma e minus m dot w 0 minus gamma w S w del x w which is once you put them together it is a E plus a W plus m dot e plus m dot w; for continuity this should be 0. So, a C becomes like that and b c is minus f flux V f convection plus V f diffusion which is nothing, but 0 in this case. So, now what one can do you invoke the continuity equation. So, invoke continuity equation and assume uniform grid, diffusion coefficient gamma is constant.

Then the value of phi C minus phi W divided by phi E minus phi W equals to 2 minus peclet number 4 minus peclet number. So, which will essentially boils down to 2 minus 0 divided by 4 minus peclet number. Now, if you plot this equation; it is quite obvious if peclet number goes to 4 this denominator becomes 0 and the whole solution become unbounded. So, the downwind scheme may be beneficial for end blended with other scheme to predict some sharp interfaces. However, its introduction here will be discussed more details in the subsequent analysis.

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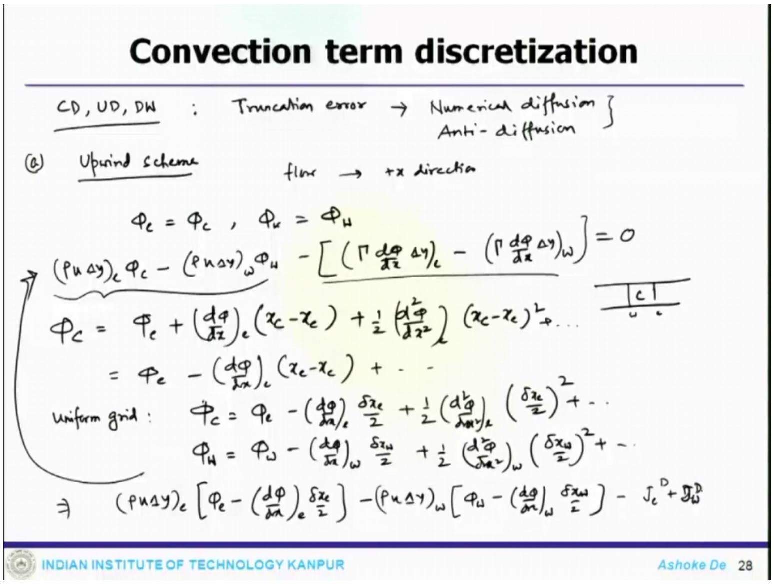

So, what we are looked at is that schemes like central difference scheme, upwind scheme, downwind scheme, downwind different scheme like that. Now, the thing which feature all applicable to all of this scheme are the truncation error. So, this is one thing which is inherently come to this kind of discretization or numerical discretization. So, this truncation error lead to some sort of an numerical diffusion. So, once there is a numerical diffusion, one has to take care in such a way that some anti diffusion also be there to handle this kind of diffusion. So, the numerical diffusion actually takes away the solution from the exact solution.

Now, we will look at again from scheme wise what are the error and what are the issues associated with it. So, to start with we start with the upwind scheme. So, the upwind scheme let us say the flow is in the positive x direction. So, that is the convention that we taking to account. So, which will result that phi e equals to the face phi C and phi w would be phi W. Now, I mean the convection diffusion equation system which was discretized is that rho u delta y e phi C minus rho u delta y w phi W minus gamma d phi by dx delta y at east face minus gamma d phi by dx and delta y w equals to 0.

Now, we go back to our standard Taylor series expansion. So, then what using the value of the flux we calculate. So, we start with the cell and using the value of the faces that we used to calculate that. So, phi e plus d phi by dx at e x c minus x e plus half d 2 phi by dx 2 which is x c minus x e square and so on. What is there is phi e minus d phi by dx e x e minus x c plus so, on. Now, if you have uniform grid then you can phi C equals to phi e minus d phi by dx e del x e by 2 plus half d 2 phi by dx 2 at east del x e by 2 square. Similarly, the phi W can be written as phi w minus d phi by dx w del x w by 2 plus half d 2 phi by del x 2 at w 2 square and so, on. Now, we are removing the higher order terms.

Now, once you use this expression in the equation of this one. So, the resulting equation becomes the left hand side which was there its rho u delta y and e phi e minus d phi by dx e del x by 2 minus rho u delta y w phi W minus d phi by dx w del x w by 2. So, that is the first two term or the convective flux terms can be expanded like that. So, once you do that and; obviously, we can retain the diffusion component as it is. So, these components we can retain J e D and J w D. So, those are the diffusion component. So, they will remain as it is.

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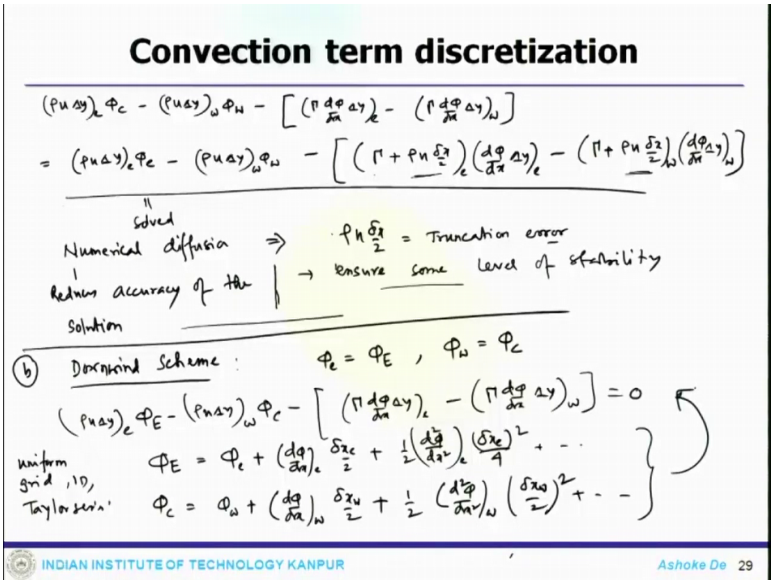

And now, if you rearrange the terms, what it would become is that rho u del y e phi C minus rho u del y w phi W minus gamma d phi by dx del y east minus gamma d phi by dx del y west which will essentially become rho u del y e phi e minus rho u del y west phi W minus gamma plus rho u del x by 2 at east face d phi by dx delta y at east face minus gamma plus rho u del x by 2 at west face d phi by dx delta y w. Now so, it is now very clear that the discretize equation which is solved; so, this is the equation that is now being solved. So, this is now being solved.

Now, in this solved equation we have an added component like this much at the east face and this much of the west face. So, which is essentially act like a numerical diffusion. So, that is there a rho u del x by 2. So, that is the sort of one can think about the truncation error which is knowing there. So, this numerical diffusion actually it reduces the accuracy of the solution. So, this is the one which actually diffuses the solution more. So, the therefore, the convection diffusion equation has an effective modified value of the diffusion effect.

So, not only this gamma d phi by dx that is the term we get an extra term in the context of discretization and which is sort of known as numerical diffusion and that has impact, but at the same time this has some I mean negative impact on the solution because, the accuracy actually degraded. But, at the same time this also some kind of provides or ensure some level of stability. So, that is another issue; it actually make the system quite bit stable because, of this diffusion and make sure that the solution remains bit bounded and physical. Now, another important thing is that theoretically the contribution from this term; the numerical diffusion term should be less.

And, that is exactly what happens when you move down to higher order scheme and you try to reduce the impact or effect of this numerical diffusion. Now, we will look at the second case is the downwind scheme. So, in the downwind scheme since is the reverse upwind scheme your phi e is now phi E and phi w is phi C. So, the discretized equation is rho u del u phi E minus rho u del y w phi C minus gamma d phi by dx delta y east minus gamma d phi by dx delta y west which is 0, that is our discretized equation.

And once we have that discretize equation, again assuming the uniform grid 1D we use the Taylor series. And what does that do? Phi E equals to phi e plus d phi by dx at e del x e plus e square by 4 like that. And, phi C is phi w plus d phi by dx w del x w by 2 plus half d 2 phi by dx 2 w del x w by 2 whole square. And, you can leave out the higher order term and you take everything back and put in the so, you use this information and put it back in the discretized equations.

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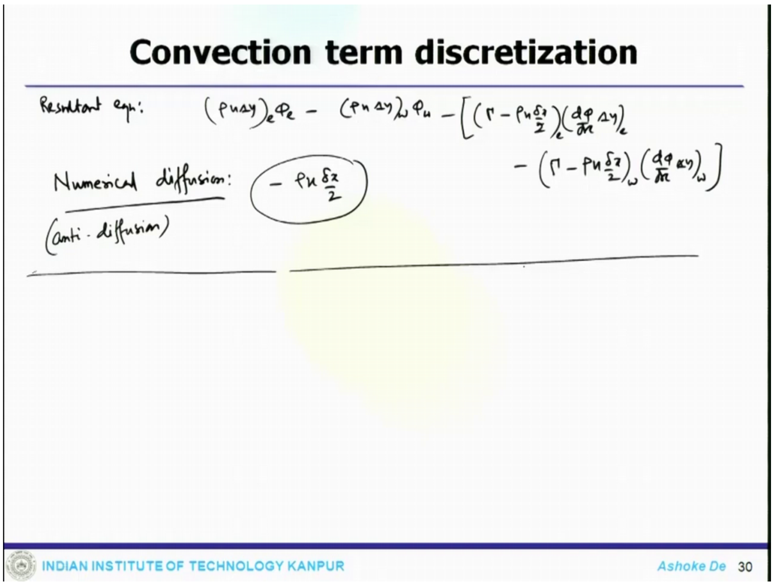

And, once we put it back in the discretized equation the resultant equation; the resultant equation looks like rho u del y e phi e minus rho u del y west phi w minus gamma minus rho u del x by 2 at east face del phi by del x delta y east face minus gamma rho u del x by 2 w d phi by dx delta y w. Again here the numerical diffusion is of the order of minus rho u del x by 2; so, that is the term. Now, here compared to the standard upwind scheme if one see the numerical diffusion comes with the negative sign, which acts as an decreasing the diffusion coefficient. So, and its effect is so, called this will like an anti diffusion.

So, this case downwind scheme this is termed as a anti diffusion because, the error comes with the negative sign. Now, the predictions which are obtained using the downwind scheme are found to cause clipping of advected profiles. In fact, solution to the 1D, convection diffusion equation which are obtained using this particular scheme can be more oscillatory compared to that of its CD scheme. So, we will stop here today and we will take from here in the follow up lectures.

Thank you.