Introduction to Finite Volume Methods - II
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Lecture - 12
Convection term discretisation – IV (Private)

So, welcome to the lecture of this Finite Volume Method.

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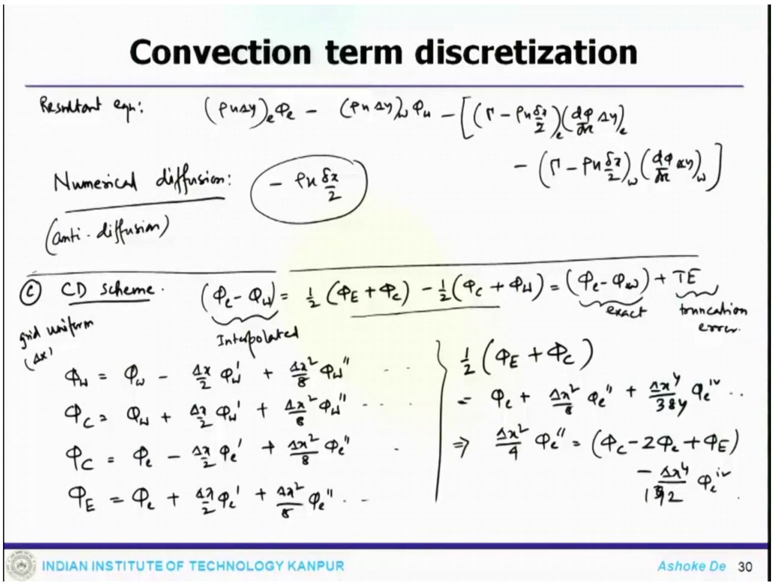

Now, the third one we will look at the central different scheme. Now, the central different scheme, the truncation error to obtain that it requires some sort of a little bit involved calculation because, it uses the stencil from the both the sides. So, what one can introduce in the calculation, let us say we have phi e minus phi w which is nothing, but half of phi E plus phi C minus half of phi C plus phi W which is phi e minus phi w plus the truncation error. So, this is the exact term, this is what interpolated ok.

So, assuming the velocity is known everywhere and again the grid is uniform which size delta x. So, we can write these things in this fashion. So, that TE here is nothing, but the truncation error. So now, we use Taylor series expansion to find out all this terminology one by one. So, the first one we get phi W is phi at small w or face w minus del x by 2 phi w prime plus del x square by 8 phi w double prime and so on. Now, phi C equals to phi w plus del x by 2 phi w prime plus del x square by 8 phi w double prime and so on.

Phi C again can be represented phi e minus del x by 2 phi e prime plus del x square by 2 phi e double prime and phi E is so, nothing. Now, what one can do that if you get the average of this term half of phi E plus phi C which could be phi e plus delta x square by 8 phi e double prime plus delta x 4 by 384 phi e like that which is delta x square by 4 phi e double prime equals to phi C minus 2 phi e plus phi E minus delta x 4 by 192 phi e 4. So, that is what we get.

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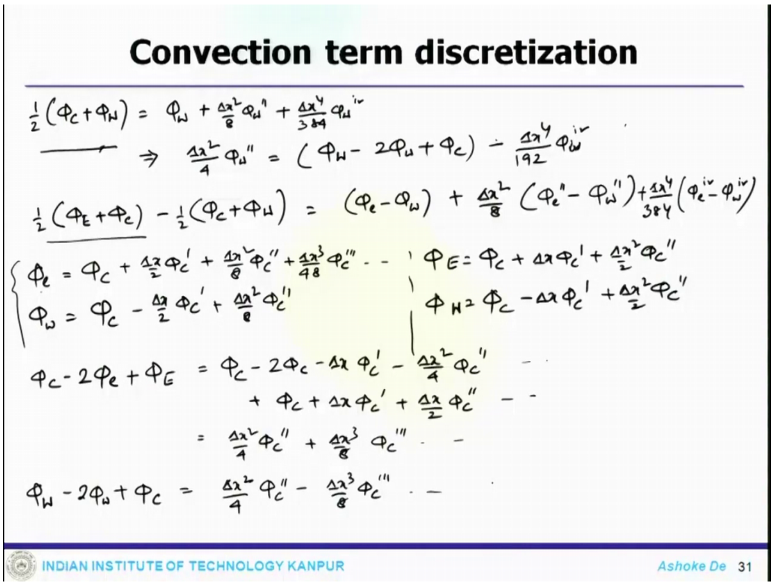

Now, similarly you get this term half of phi C plus phi W, which will be phi W plus delta x square by 8 phi W double prime plus delta x 4 by 384 phi W prime. So, which one can write delta x square by 4 phi W double prime equals to phi W minus 2 phi W plus phi C minus del x 4 by 192 5 4 phi W. Now, you substitute this term and what you have got here in the discretized equation. So, what we get phi E plus phi C minus half of phi C plus phi W which is going to be phi small e minus small w plus delta x square by 8 phi e double prime minus phi w double prime plus del x 4 by 384 phi e fourth derivative minus phi w fourth derivative.

Now, one can expand the small phi e in terms of phi C which is also like this, phi C double prime cube by 48 phi C triple prime and so on. So, one can represent that phi C minus del x by 2 phi C prime plus del x square by 8 phi C double prime and so on. In addition to that similarly, one can write phi E equals to phi C plus delta x phi C prime delta x square by phi C double prime so on and phi W equals to phi C minus delta x prime delta x square by 2 phi C double prime so on. So, in terms of cells centre value phi C you can expand or rather you can actually obtain all this expression. Now, we got one this set of information another is this. Now, in this set of information if you put this things there, like phi C minus 2 phi e plus phi E.

So, this term this term and this term; if you do that then what we get phi C minus 2 phi C minus delta x phi C prime minus delta x square by 4 phi C double prime and so on plus phi C plus delta x phi C prime plus delta x square phi C double prime and so on. So, which will simplify to delta x square by 4 phi C double prime plus delta x cube by 8 phi C triple prime and so on. Similarly, one can write phi W minus 2 phi small w plus phi C which will get you del x square by 4 phi C double prime minus del x cube by 8 phi C triple prime and so on.

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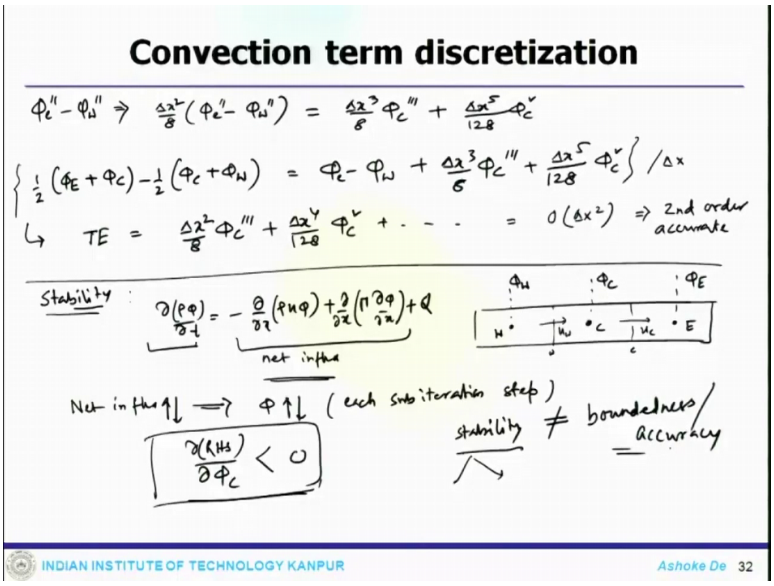

Then one can compute phi e double prime minus phi small w double prime as del x square by 8 into phi e double prime minus phi w double prime equals to del x cube by 8 phi C triple prime plus del x to the power 5 128 phi C fifth term. Now, you substitute back everything in that equation of half of phi E plus phi C minus half of phi C plus phi W which will be phi e minus phi small w plus del x cube by 8 phi C triple prime plus delta x 5 by 128 phi c fifth term. Now, you divide by delta x then the truncation error term will be order of delta x square by 8 phi C triple prime plus delta x 4 by 128 phi C 5 and so on.

So, this we divide by delta x you get this truncation error. So, which essentially shows that the higher order term is order of delta x. So, CD is second order accurate. So, that is what it turns out. So, CD becomes second order accurate screen. Now so, this takes care of the accuracy. Now, the other point which comes to that thing is the stability. Now, the confusion related to truncation error first order for the physical solution and second order for the un physical one has led to extrapolate this term; so, accurately that the diffusion gets reduced. Now, at the same time while doing that one has to also keep in mind the stability of the system. Because, while doing this things stability can be also affected. For example, if one takes this kind of a pencil like this C east west. So, this is w face. So, this goes in this direction this is u e u w.

Now, the quantity here is phi W scalar here is phi C, here is phi E ok. Now, if I look at the change of this term is nothing, but del del x rho u phi plus del del x gamma del phi by del x plus source term. Now, this left hand side is the rate of change of phi C and the right hand side shows the net influx across the element surface. So, if the numerical error exist then the right hand side may increase or decrease depending on the situation. Now, if in a unstable scheme a small deviation from the correct value of the phi C gives a corresponding increase or decrease in the net influx at the right hand side. So, when an iterative procedure is used as a part of the solution mechanism; an increase or decrease in the net influx will further increase or decrease the value.

So, either increase or decrease in the net influx will have the similar impact on the solution of the phi C at each sub iteration step. So, in a stable scheme this change in phi C is due to the error in the right hand side and that should feedback negatively in to the right as a self collective device. So, far this kind of numerical stability what one has to do the right hand side with respect to phi C should be less than 0. So, that is indicating the sensitivity of phi C in the combined system. Now, if there is increase or decrease in phi C which will correspond to increase or decrease in the influx which will also intern pushes phi C download or upward to correct the value.

So, it is a self correcting situation; however, the stability should not be confused. So, stability has nothing to do with; they are not same the boundedness or accuracy. So, stability is something whether the stable numerical scheme could actually be unbounded or giving rise to some over suit or understood or oscillations riddles or can be very diffusive ok; also can provide the solution with lower order of accuracy. So, stability here refers to the controlling the numerical error to remain bounded in order to not to increase infinitely; so, that the system or the numerical code actually diverges. So, this has nothing to do with boundedness or accuracy.

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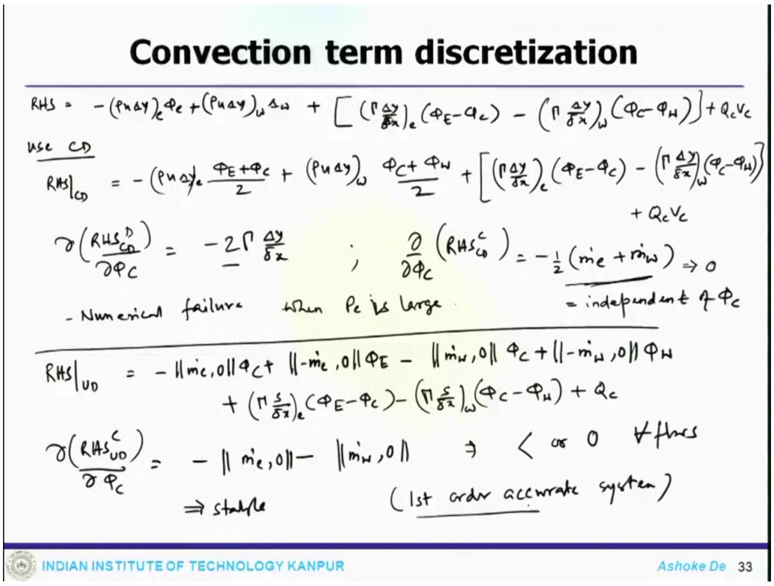

So, stability is something that one has to now, keeping that in mind if you look at that discretize right hand side of the previous equation; the right hand side was minus rho u del u at east phase phi e plus rho u at the plus gamma del y by del x minus phi C minus gamma del y by del x w phi C minus phi W plus Q c V c. Now, one can use CD scheme, then the right hand side for CD scheme it will become rho u del y e phi E plus phi C by 2 plus rho u del y w phi C plus phi W by 2 plus gamma del y by del x e phi E minus phi C minus gamma del y by del x w phi C minus phi W plus Q c V c..

Now, once if you analyse the scheme the criteria which is found for the diffusion term that is sensitive to that is the right hand side CD diffusion term by del phi C is 2 gamma del y by del x. And so, there is a negative sign into that. And similarly, if someone look at it the right inside for CD the convection term it gives minus half of m dot e plus m dot w. So, for steady flows which is essentially 0 for steady flows, but not necessarily it will happen for the unsteady case. For unsteady situation its value will be positive for disleading flow and general flow such regions will act as an regal sources and can easily lead to some sort of a numerical failure ok.

When the peclet number when peclet number is large so, but even for steady flows as its value is 0 here; for steady flow this value is going to be sort of 0. It cannot fit to the self correcting system. So, what it suggest that steady flow the net convective flux computed using the CD scheme is some sort of independent of phi C. So therefore, this will result in same net convective flux over centroid C. Now, similar things one can find out for the upwind scheme. So, the right hand side for upwind scheme will be minus m dot e 0 phi C plus minus m dot e 0 phi E minus m dot w 0 phi C plus minus m dot w 0 phi W plus gamma S by del x e phi E minus phi C minus gamma is by del x w phi C minus phi W plus Q c.

So, what one can get the right hand side for convection upwind scheme by del phi C is m dot e 0 minus m dot w 0 which will be either negative or 0 for all flows. So, it will be 0 when the both the mass flow rates are negative and a situation that does not arise in one-dimensional situation, of constant area cross section. But, when added to a false diffusion into the induced by the first order approximation it indicates that the scheme is very stable. So, that suggest that there is a stability. However, this can be achieved at the expense of some sort of an accuracy. So, this is always first order accurate system. So, though it is stable, but it can come with a price that so; now the other one which is there is the downwind wind scheme.

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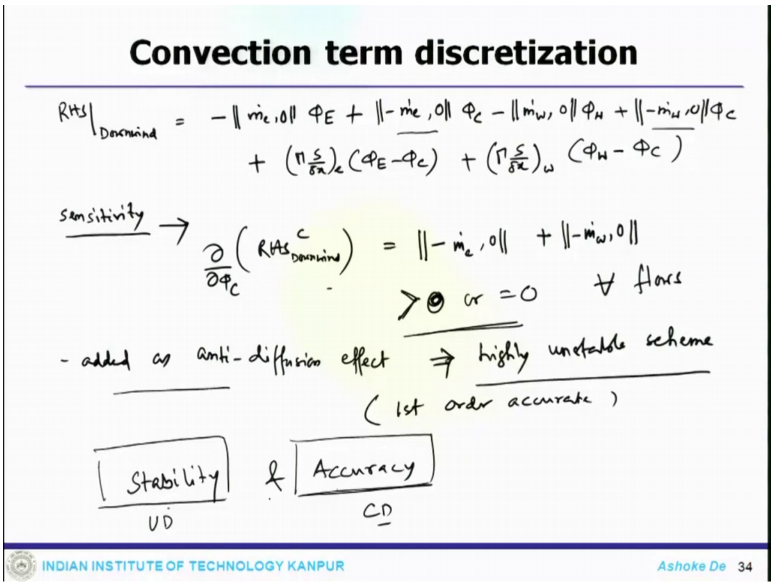

So, the downwind scheme the right hand side one can write the downwind scheme is equals to minus m dot e 0 phi E plus minus m dot e 0 phi C minus m dot w 0 phi W minus m dot w 0 phi C plus gamma S by del x e phi E minus phi C plus gamma S by del x w phi W minus phi C. Now, the sensitivity analysis leads to that, the right hand side due to phi right hand side of downwind scheme for the convection part which will lead to minus m dot e 0 plus minus m dot e 0; this should be minus w. So, that is m dot w. So, the contribution from this guy and the contribution from this guy will come because, again if you look at this term this is either always positive. So, this is always positive or equals to 0 so; that means, for all sort of flows.

So, when this is added to anti diffusion effects this means this whole term is as long as it is 0 there is no problem. But, when is positive and it is added as anti diffusion effect this things becomes or rather it leads to the highly unstable scheme. So, one hand you get the upwind scheme which is stable, but lower order accuracy. CD scheme has some sort of a problem and this guy it is unstable. And, then so the issue is that, this is also by the way first order accurate. So, that is where since, as we said the trade-off between 2 component: one is stability another one is accuracy.

So, highly stable scheme can be completely providing or can completely give inaccurate results which happens with the CD scheme; as we have seen that for one-dimensional case. But, accurate scheme cannot be not necessarily always be stable or at the other way around also the stable scheme like the first order upwind cannot be accurate. Or so, the upwind scheme is stable, but does not provide accurate solution CD scheme is higherly accurate; but it does not provide a I mean great level of stability. So, these are two important things where one has to make a trade-off between stability and accuracy. And the scheme that we have seen so far that is what and carry forward the discussion.

Thank you.