Introduction to Finite Volume Methods - II
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Lecture - 13
Convection term discretization – V (Private)

So, welcome back to the lecture series of Finite Volume and what we are in the middle of the discussion of the Discretization of Convection diffusion system. And so far what we have discussed is that the central difference scheme and the upward scheme and the downward scheme. And the important message that has come out, it is a trade of between your stability and the accuracy what we have looked at is that central difference scheme is accurate. I mean its second order accurate, but it is not that way stable or rather it does not provide you the accurate solution.

I mean the other way once you look the upwind scheme that is stable, but the accuracy is an issue. So, now, today that actually provides the platform for discussion of the higher order scheme. So, we will now look at the higher order scheme in the system.

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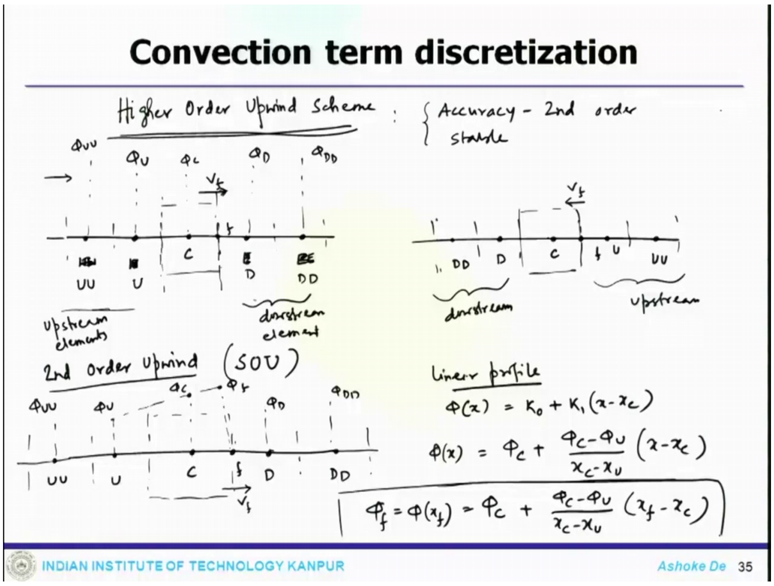

So, what higher order upwind scheme? So, essentially uses the information of your upwind scheme. So, it is some sort of an upwind interpolation scheme. So, now, and what is the genesis of that because of the genesis is that the accuracy and stability of the other scheme like CD upwind and all these scheme. Now, what this scheme is aiming at it? Producing at least a second order accurate solution so, it would be accuracy vice it would be second order and also it stable. So, it uses both the advantages of upwind and CD scheme and then we formulate these kind of scheme.

Now while doing that this also uses if you look at that one-dimensional stencil; this is your C then that would be your cell interfaces. So, these are the cell interfaces it uses the similar kind of stencils this is I mean E west WW this is EE. So, it uses similar kind of stencil in a one dimensional system. But now once the important property which associated with the flux at the interface is that the element C and what it follows.

Now to get an upwind kind of system, one can say that ahead of this one instead of E; you can say it is a downstream of that if the profiled is in this direction and the flux at this particular element faces this is the f, then the V f is in this direction. So, it is a then this should be the upwind information of that and instead of this you can say this is upwind and instead of this you can say its a downstream why? The reason is that you are interested in this particular cell and at this phase where the flow field of the flow velocity direction is in this direction.

So, the weight of the upstream cell would be important or pre dominant compared to the downstream cell. So, that is the convection because this is the upstream cell, this is the downstream cell and the values which corresponds to these values is phi C this value would be phi U, this value would be phi UU. So, that follows our standard notation system phi DD. So, this is what one can use. So, similarly, one can see if the velocity is in this direction rather. So, this is our C and if you have an element like that and for example, let us say for this element phases at these phase the flow field goes in this direction the V f is in this direction.

Then this is the cell phase and it goes in the, this direction this is D this is upstream of that this would be the upstream upstream and this will be the downstream downstream. So, in this case the cell velocity was in this direction. So, these are counted as upstream nodes. So, these are all upstream elements and these are downstream elements while the cell speed is this way and these are considered as an upstream and these are considered as downstream. So, depending on the direction of the flow field, one has to decide which will be the upstream elements and which will be the downstream elements and accordingly you can define your I mean the variables in that fashion.

So, now, we will start with a 2nd order upwind what we have seen the standard that is a 1st order upwind now we will start with 2nd order upwind. So, one can say it would be 2nd order upwind SOU scheme. So, how you go about it, you can define in one dimensional stencil like that lets say like this and in between that you have C so, you define your cell interfaces like that. So, this will be the cell elements associated with that now at this particular element this is the phase and the value which is upstream of it lets say this is U, this is UU, this is D and this is DD.

So, if you consider the value between all this so, they will go like that. So, this will have phi D, this is phi DD and this is where phi U and there would we some sort of an phi C, phi f and this is phi UU. So, all these information's are now stored and why we are saying this is upstream because at the phase we are assuming the flow field is in this direction. So, the mass flux is positive at that particular phase. So, you start with a linear profile, if you start with a linear profile, then the system you can define as phi x equals to K not plus K 1 x minus x e this whole idea is to find out the phase value at this f phases.

And fitting it to the lower values x c and x u add all these one can write that phi x equals to phi C plus phi C minus phi U divided by x C minus x U into x minus x C. So, that is how you define this system. Now what happens then? At phi f which is essentially phi at x f equals to phi C plus phi C minus phi U divided by x C minus x U multiplied by x f minus x c.

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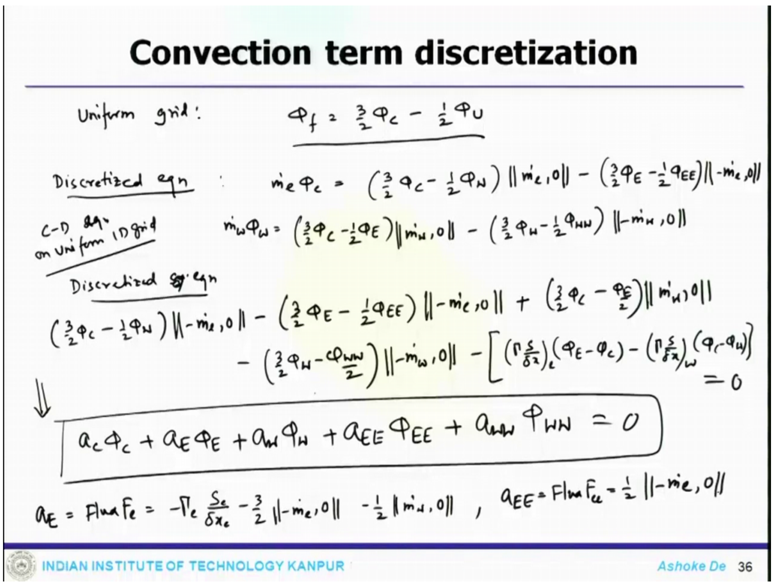

So, that is how you get the phase value and if you have a uniform grid then this guy will (Refer Time: 09:11) down to for uniform grid this phi f will (Refer Time: 09:19) down to 3 by 2 phi C minus half phi U. So, that is the special case if you have uniform grid to calculate this information.

Now, once has to look at the discretized system and how. So, if you look at the discretized equation then, one needs to calculate the like m dot e phi e which is going to be 3 by 2 phi C and minus half phi W. So, this we are looking at the system where one has assuming the equation which is convection diffusion equation which is convection diffusion equation on uniform 1D grid. So, that is the that is why you write these things and multiply it with m dot e 0 minus 3 by 2 phi E minus half phi EE multiplied with minus m dot e 0. And then you get the similarly m dot W phi W equals to 3 by 2 phi C minus phi E by 2 multiplied with m dot W 0 minus 3 by 2 phi W minus phi WW now multiplied with m dot w 0.

Now, these if you substitute in the discretized equation that we have obtained earlier, now discretized system or equation. If you add those or put those back there this will get you back phi W minus m dot e minus 3 by 2 phi E minus phi EE, then minus m dot e plus 3 by 2 phi C minus phi E by 2 m dot w minus 3 by 2 minus phi W minus phi WW by 2 minus m dot w 0 minus gamma s by del x e phi E minus phi C minus gamma s by del x w phi C minus phi W equals to 0.

So, which will lead to the discretized form like a C phi C plus a E phi E plus a W phi W plus a EE phi EE plus a WW phi WW equals to 0 know there is no source term. So, the discretized equation will look like that, since it is a so when we looked at the it was a 1st order system we had only this much, now because of the second order you get two more pointsead and downstream. In these a E is flux F e which is minus gamma e S by del x e minus 3 by 2 m dot e 0 minus m dot w 0 a EE is flux F EE which is half of minus m dot e 0.

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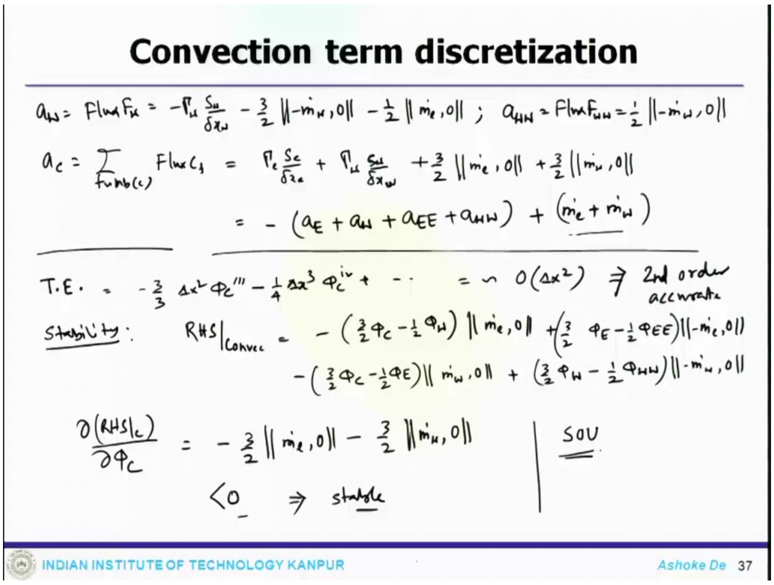

Now, a W equals to flux F W which is gamma W S W by del x W minus 3 by 2 m dot w 0 minus half of m dot e 0 and similarly you get a W W equals to flux of F W W half of minus m dot w 0. And a C which is the summation of all the fluxes which is flux C f that will get you gamma S e by del x e plus gamma W s W by del x W plus 3 by 2 m dot e 0 plus 3 by 2 m dot w 0 which one can write with negative sign a E plus a W plus a EE plus a W W plus m dot e plus m dot w.

So, this for a continuity equation to be conserved this will (Refer Time: 15:38). Now the truncation error; so, the truncation error one can estimate equals to. So, for this scheme as we have used for the upwind scheme or central scheme one can follow the similar procedure on what one can obtain this will be 3 by delta x square phi C triple prime minus 1 by 4 delta x cube phi C forth derivative and so on which indicates that the higher the term is of delta x square which is essentially 2nd order accurate in special discretization.

Similarly, if you look at the stability of this particular scheme what you can obtain that right hand side of the convection term equals to minus 3 by 2 phi C minus half of phi W multiplied with m dot e 0 plus 3 by 2 phi E minus phi EE multiplied with minus m dot e 0 minus 3 by 2 phi C minus phi E by 2 multiplied with m dot w plus 3 by 2 phi W minus phi W W multiplied with m w 0.

So, the rate of change of right hand side right hand side with respect to phi C would get you minus 3 by 2 m dot e 0 minus 3 by 2 m dot w 0. So, which is always negative and indicating that the scheme is also stable. So, with the 2nd order upwind scheme you can somehow come out of those problems with both the central difference and the upwind scheme and this particular scheme essentially the SOU scheme is not only stable it also gives you 2nd order accuracy.

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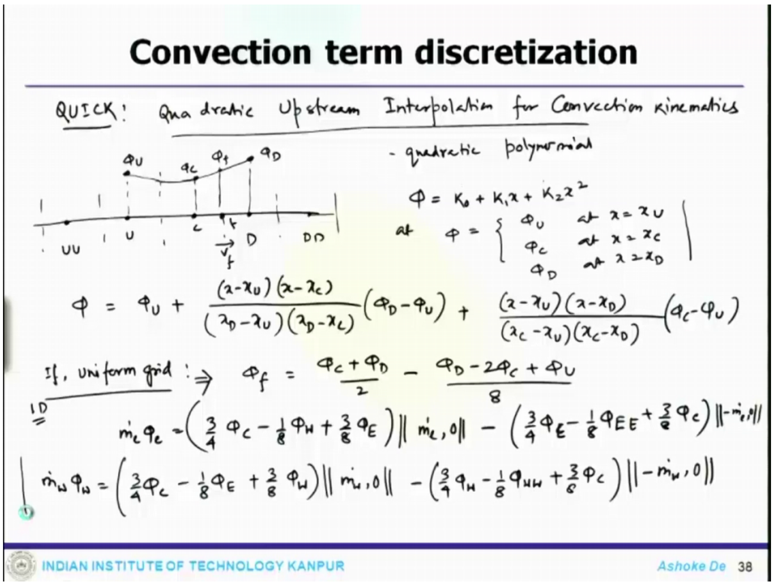

Now in that similar direction one can device another scheme which will call the quick; quick is quadratic upstream interpolation for convection kinematics. So, that is what the quick transfer. So, again if you look at a stencil like that where this is the cell then you have it is the downstream of that it is upstream of that and then this is further upstream this is DD and at the cell phase you can say the v f in this direction and this is the phase value.

So, using these and this value one can calculate the profile. So, this is phi C phi f phi D phi U. So, all the information of the neighbouring elements are taken into consideration. So, what as the name suggest the polynomial would be quadratic polynomial? So, that is where the interpolation is going to be obtained and what profile one can obtain from this you can assume the phi would be or phi x would be some sort of a K not plus K 1 x plus K 2 x square.Now which will say that at you have this phi values at different at phi E at x equals to x U it will be phi C at x equal to x C it is phi D at x equal to x D.

So, now you need three points to find out the coefficients the 1 2 3 K 0 K 1 K 2. So, to in order to find out the coefficient you need 3 set of information and if you put this back and solve it what one get phi equals to phi U plus x minus x U x minus x C x D minus x U multiplied with x D minus x C phi D minus phi U plus x minus x U x minus x D x C minus x U x C minus x D phi C minus phi U. Now, if you have uniform grid then this (Refer Time: 22:34) down to at phi f it becomes phi C plus phi D by 2 minus phi D 2 phi C plus phi U divided by 8. So, for uniform grid the face value becomes like that.

Now, using these for the same uniform case on 1 D the convection diffusion system if you put it you calculate that m dot e phi e which is going to be 3 by 4 phi C minus 1 by 8 phi W plus 3 by 8 phi E which is now m dot e 0 minus 3 by 4 phi C phi E 1 by 8 phi double E plus 3 by 8 phi C multiplied with minus m dot e 0. Similarly, you get m dot W which is 3 by 4 phi C minus 1 by 8 phi E plus 3 by 8 phi W multiplied with m dot w 0 minus 3 by 4 phi W 1 by 8 phi W W plus 3 by 8 phi C multiplied with m dot w 0.

So, this is what you get. Now again after getting this information you put everything back in the discretized equation.

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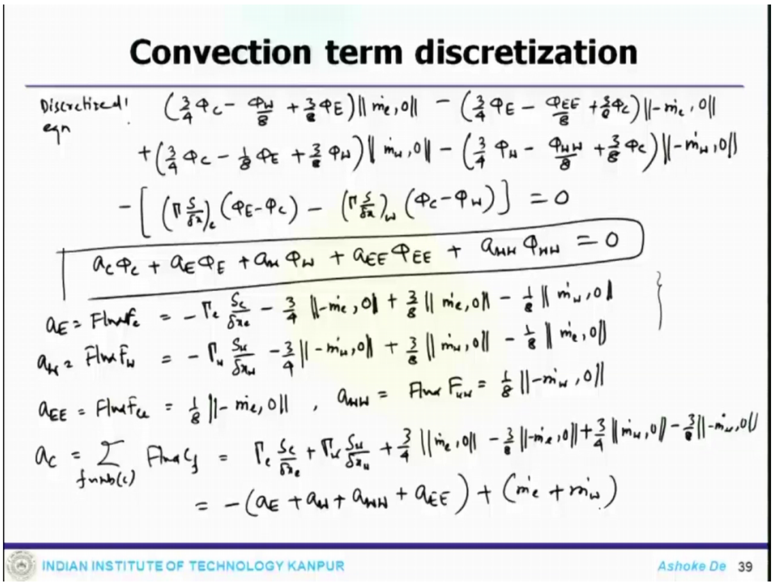

Now, once you do that the discretized terms would look like 3 by 4 phi C minus phi W by 8 plus 3 by 8 phi E that is the first term multiplied with m dot e 0 minus 3 by 4 phi E minus phi EE by 8 plus 3 by 8 phi C multiplied with minus m dot e 0 plus 3 by 4 phi C minus 1 by 8 phi E plus 3 by 8 phi W multiplies with m dot w 0 minus phi W W by 8 plus 3 by 8 phi C. So, is all the term what we have calculated now putting back in the discretized system and then left with the diffusion term which is gamma S by del x e phi E minus phi C gamma S by del x W phi C minus phi W which is 0.

So, then the discretize form will become again looks similar, but the final discretized form will have the different coefficient. So, it will be a E phi E plus a W phi W plus a EE phi EE plus a W W phi W W and the term a E equals to flux F e which will be gamma e S e by del x e minus 3 by 4 m dot e 0 plus 3 by 8 m dot e 0 minus 1 by 8 m dot w 0. A W would be similarly is flux F W which is gamma W S W by del x W 3 by 4 minus m w by 0 plus 3 by 8 m w 0 minus 1 by 8 m dot e 0.

So, the pattern for this coefficients are pretty much similar if you can notice, but the thing is that these are looking similar as we have assumed or what we are deriving here for the uniform gird system. If that gird is not uniform this terms would look completely messy or rather complicated, but the formulation will not change. Only thing you will get to see the differences in the coefficient calculation. Now a W W equals to flux F W W equals to 1 by 8 minus m w 0. So, a C is f n b C flux C f equals to gamma e S e by del x e plus gamma W S W by del S W plus 3 by 4 m dot e 0 minus 3 by 8 m dot minus e 0 plus 3 by 4 m dot w 0 minus 3 by 8 m dot w 0 which is again minus a E plus a W a W W a EE plus m dot e plus m dot w. So, we will stop here today and will take from here in the follow up lectures.

Thank you.