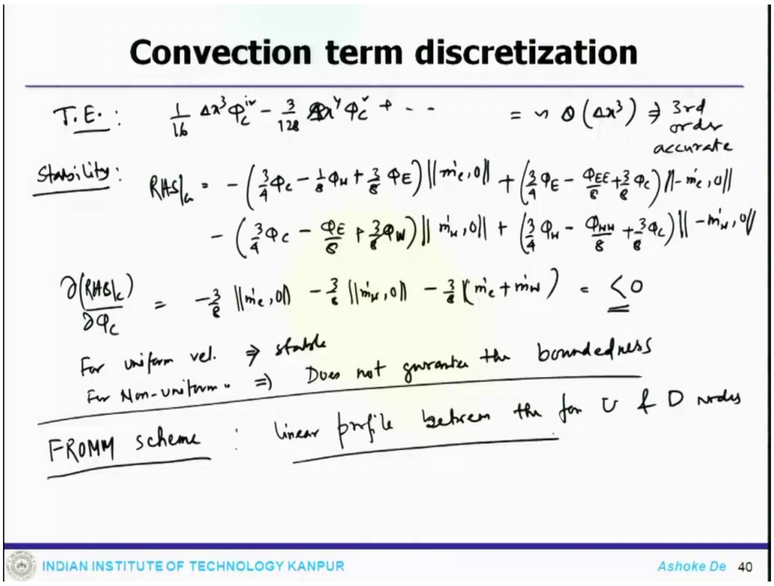
Introduction to Finite Volume Methods - II
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 14
Convection term discretization – VI (Private)

So, welcome back to the lecture series of Finite Volume.

(Refer Slide Time: 00:21)



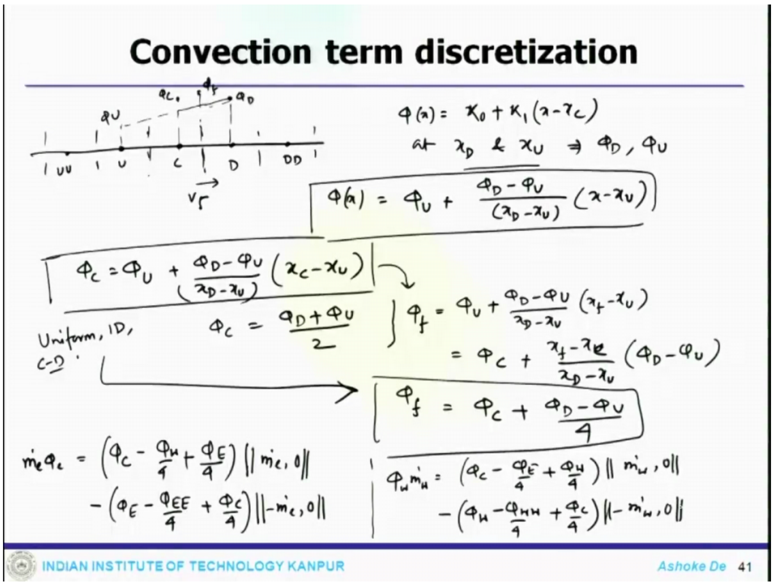
And, this quick scheme if you analyze there are truncation error which would turn out to be 1 by 6 del x cube minus 3 by 128 del x to the power 4 phi C phi and so on. So, which essentially give you some sort of an 3rd order accurate, this is 3rd order accurate scheme. So, as we move along the hierarchical direction 2nd order upwind was 2nd order accurate quick becomes 3rd order accurate. Now, the third point one has to check its stability. So, the stability if you look at the right hand side convection term it should be minus 3 by 4 phi C phi W 3 by 8 phi E m dot e 0 plus 3 by 4 phi E minus phi EE by 8 3 by 8 phi C m dot e 0 minus 3 by 4 phi C minus phi E by 8 3 by 8 phi W then 3 by 4 phi W minus WW by 8 phi C.

So, to get the stability limit check you get this right hand side term with respect to del phi C which will turn out 3 by 8 m dot e minus 3 by 8 m dot w minus 3 by 8 m dot e plus m dot w. So, this is again less than 0. So, this shows that it is a stability I mean stable scheme. So, if you have some sort of a uniform velocity this will be always stable;

however, this does not guarantee the solution boundedness specially in case of non uniform velocity.

So, one has to for uniform velocity always stable, for non-uniform velocity does not guarantee the boundedness. So, that is the point one has to note that how it goes. Now moving ahead that is another scheme which is called FROMM scheme. So, which is uses the linear profile between the for upwind and downwind nodes connecting the faces.

(Refer Slide Time: 04:35)



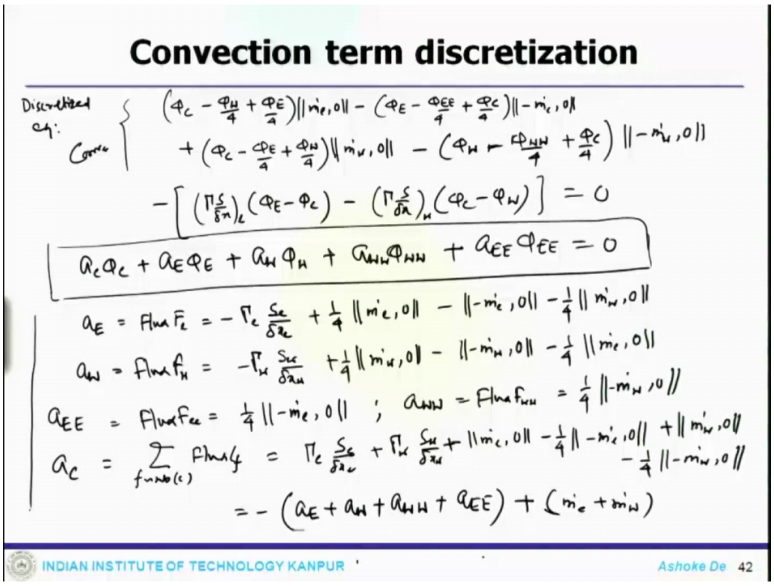
So, how would that look like, if one has to put the stencil again this is our C this is the phase, these are the downstream; this is downstream and this is upstream U U across this cell phases assuming velocity in this direction. So, what it uses it uses some sort of an profile calculation between these points this is phi D phi U and in between you have phi C and phi f. So, this is how it takes into account and the function definition is K naught plus K 1 x minus x C and using the value at x D and x U which is corresponding to phi D and phi U what you get the phi x equals to phi U plus phi D minus phi U x D minus x U x minus x U that is the function that you get.

Now again one can carry out the similar exercise for this particular scheme and what you end up getting is that. Now, if you have this upwind note phi C which you can calculate as phi U plus phi D minus phi U divided by x D minus x U into x C minus x U. So, that is how you get. Now, again the assumption remains uniform grid 1 D convection diffusion system then phi C becomes phi D plus phi U by 2 which is the centroid value becomes the average between this and this.

Now, with that phi f would be phi U plus phi D minus phi U x D minus x U x f minus x U which one can write phi C plus x f minus x C divided by x D minus x U multiplied with phi D minus phi U. So, you use this information here and you get this and for the uniform grid it reduces to phi f equals to for uniform system it reduces to phi C plus phi D minus phi U divided by 4. So, that is what you get for uniform system.

So, to put this thing information back in the discretized equation you calculate the mass fluxes I mean m dot e fluxes at the phases which will be phi C minus phi W plus phi E by 4 multiplied with m dot e 0 minus phi E phi E E by 4 plus phi C by 4 multiplied with m dot e 0. Similarly you get phi W mw which is phi C minus phi E by 4 phi W by 4 multiplied with m dot w 0 minus phi W phi WW by 4 phi C by 4 minus m w 0. So, this we are doing it for the uniform system.

(Refer Slide Time: 09:29)



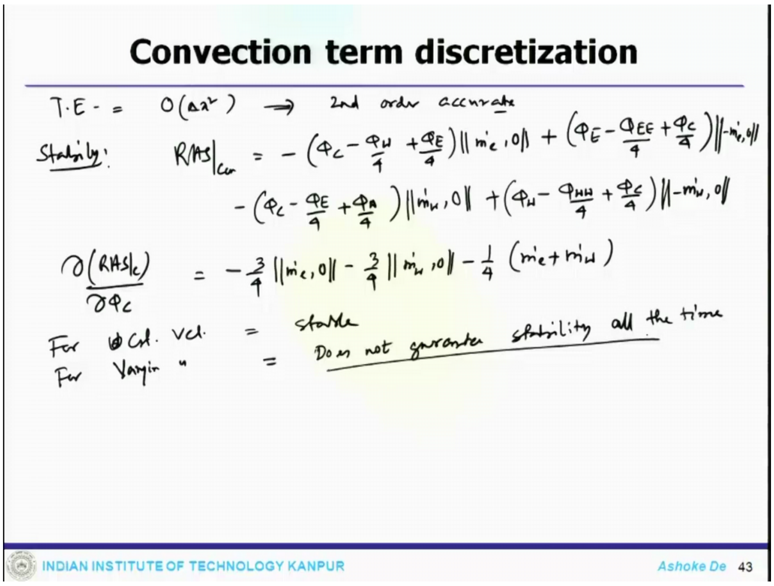
And then the converted discretized equation would be looking like phi C minus phi W by 4 plus phi E by 4 multiplied with m dot e 0 minus phi E phi EE by 4 plus phi C by 4 multiplied with m dot e 0. And then you get phi C phi E by 4 phi W by 4 multiplied with m dot w 0 minus phi W minus phi WW by 4 plus phi C by 4 multiplied with minus m dot w and then we have the gamma term for the diffusion terms. So, this portion contributes the convection and then this is the diffusion term by del x at w which is phi C minus phi W equals to 0.

So, eventually the equation will look like the compact equation will look like a C phi C a E phi E plus a W phi W a WW phi WW plus a EE phi EE which is 0. So, this I mean the fact which remains same in the final volume once you get a discretized equation they look exactly similar for different different scheme. So, if you have a one discretization in place one can actually just using different coefficient you have a different scheme in space minus 1 by 4 m dot e 0 minus m dot e 0 1 by 4 m dot w 0.

Then this flux F w minus gamma w S w by del x w m dot w 0 minus, minus 1 by 4 minus m dot 0. Now a E which will become flux Fe 1 by 4 minus m dot e 0 a WW which will be flux f small ww 1 by 4 m dot w 0. And finally, a C which is over the phases flux C f which will get you gamma e Se plus gamma w plus m dot e 0 minus 1 by 4 m dot e 0 m dot w 0 minus 1 by 4 m dot w 0 which will again remain as the summation of a E a W a WW EE plus m dot e m dot w.

So, these are the only places the calculation of the coefficients, were you see the differences and if you look at the truncation error for this Fromm scheme.

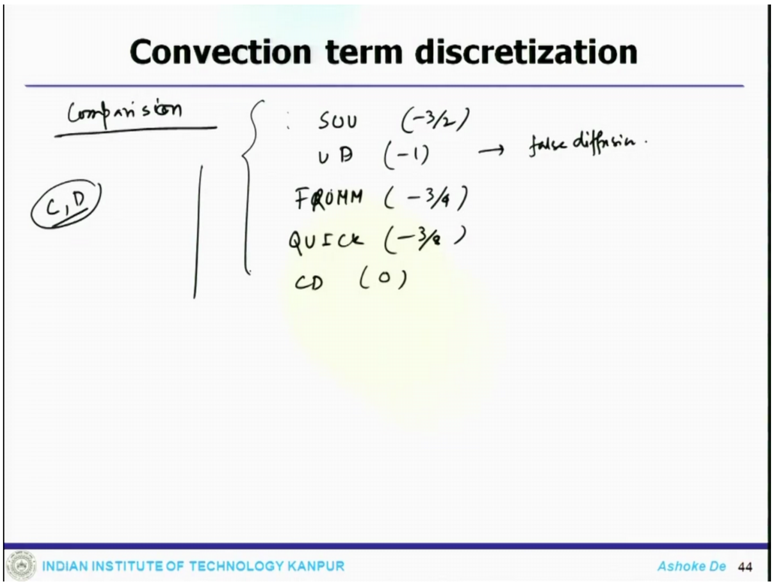
(Refer Slide Time: 14:11)



The truncation error would be order of delta x square which also provides you 2nd order accurate system and the stability criteria the stability if you look at the convection term the right hand side. So, that will have these components phi C minus phi W by 4 plus phi E by 4 m dot e plus phi E minus phi EE by 4 phi C by 4 multiplied with minus m dot e 0 minus phi C phi E by 4 phi W by 4 m dot w 0 plus phi W minus. So, if you take the derivative of this term with respect to phi C which will become minus 3 by 4 m dot e 0 minus 3 by 4 m dot w 0 minus 1 by 4 m dot e plus m dot w.

So, like the quick scheme for constant velocity this guy is always stable and for varying velocity this is I mean it does not guarantee stability all the time it may not. So, but if you have constant on uniform velocity this will always provide you back the stability. So, if you look at the comparison of all this profiles.

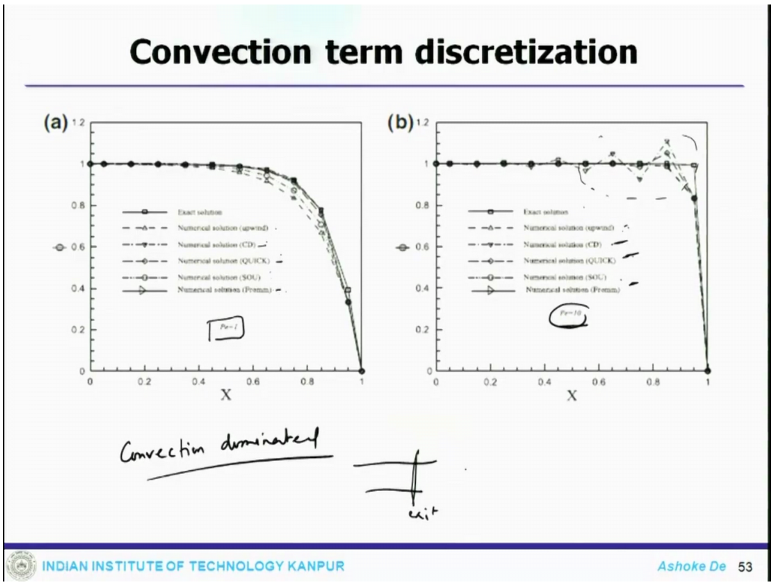
(Refer Slide Time: 17:01)



So, basically some sort of an comparison if you look at various schemes what you see is that with the most negative coefficient is associated with 2nd order upwind scheme that is minus 3 by. And then you have upwind scheme which is minus 1 then FROMM scheme F R O M M FROMM scheme with minus 3 by 4 and then finally, the quick which is minus 3 by 8 and C D is coefficient 0, which is sort of a neutral stability. So, the self corrective action is the sum of both convection and diffusion contribution.

So, the self corrective measures comes from both the convection and diffusion contribution and the false diffusion essentially produced by the upwind scheme which adds to. So, upwind scheme actually the false diffusion or numerical diffusion which adds to the stability even though the coefficients is minus 1 and it is the most stable scheme among this all that we have discussed.

(Refer Slide Time: 18:33)



So, if you put them together and plot then you see this curve for varying Peclet number this is Peclet number 1 and this is Peclet number 10 and if you plot all the scheme against the exact solution or the analytical solution. So, you get upwind C D quick 2nd order form similarly exact numerical upwind scheme C D scheme quick scheme 2nd order upwind and Fromm scheme.

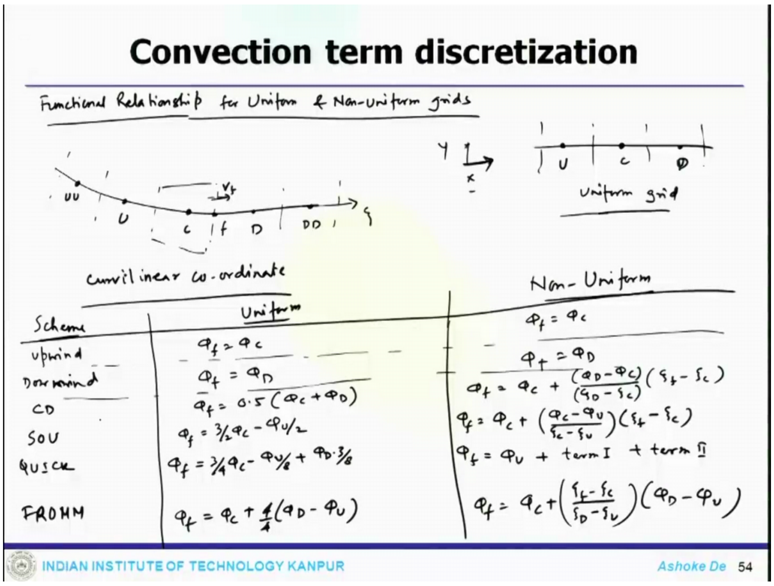
So, once you look at all the cases for Peclet number 1 which is at the low Peclet number. So, you see that C D Fromm and quick this guy this guy and this guy they I mean some, I mean basically the Peclet number 1 case they show some sort of an regals here I mean the high Peclet number case, but in the low Peclet number case they do not show anything.

But, the schemes at the high Peclet number case show some regals and amongst these is the least accurate is the solution by the 1st order upwind scheme. Now and the solution of this guy is slightly more accurate than 2nd order upwind and; obviously, it is going to be accurate than the 1st order upwind also. Now when you look at this guy this guy and this guys C D quick and Fromm they show some regals in this locations of the solution.

So, here the solution which is obtained by the 1st order upwind and the 2nd order upwind their profile being the most accurate close to the exact solution. And now coming back to the Fromm and quick scheme on the other hand they show some regals, but with some small amplitude and the reason for this regals is that they imposed boundary recondition at the exit of the domain. So, since the problem is convection dominated it is affected by the. So, these are for convection dominated flow the solutions at the downstream is partially affected by the upstream calculations or upstream information.

So, solution which are actually at the exit of the domain which are specified this is the exit of the domain that have some impact on this regals and all these things, but. However, the 2nd order upwind scheme is expected to give rise to some oscillation in the presence of high gradient in the domain if you have a shock wave or like that. So, that give some idea on some sort of an comparison of this.

(Refer Slide Time: 22:03)



Now, moving ahead you can actually find some functional relationship for uniform and non uniform grids. So, far our discussion was restricted to uniform grid. So now, one can find out some correlation or the mapping between uniform and non uniform grid and how you can do that essentially let us see if you have some sort of an element C here which will be connected with a phase this is the phase f and the velocity here will be in this direction then there is a cell D then there would be cell D D downstream the upstream there is U upstream U U. So, this goes in a xi direction. So, sort of an curvy linear coordinate system.

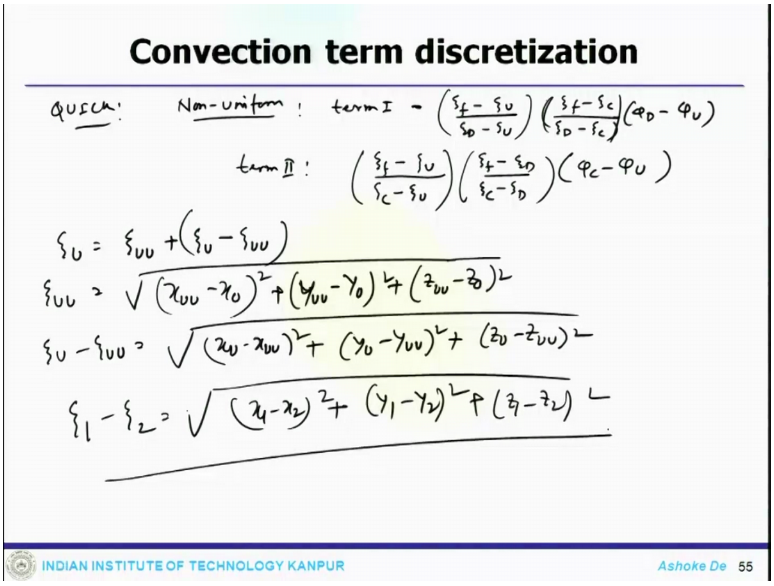
So, typically my x y system or the uniform x y system the cells would be sitting like that this is the cell down of that up of that. So, this is where your uniform grid system or orthogonal grid system and this is in curvilinear grid system. So, one important point which is here the functional relationship for uniform grid remain exactly same and for the non uniform also independent of whether the Cartesian of the curvilinear grid system is used. For non uniform grid the independent variable x which is appearing in the functional form should be replaced by xi.

So, what essentially it says the functional form would not get change it only the notation like inform grid used get a functional best of x; in this case it should be function of xi which will actually represent the system with the discussion. Now, if you get all this functional relationship for different scheme then it should be put together let us say this is for uniform system this is non uniform system, this is non uniform system then this are the scheme that we have so far have discussed.

So, when you talk about the upwind scheme this will be uniform case phi f is phi C this case also phi f is phi C when you talk about downwind scheme then phi f is phi D this case phi f is also phi D. When you talk about C D this is phi f is 0.5 into phi C plus phi D and this case phi f equals to phi C plus phi D minus phi C by xi D minus xi C multiplied with xi f minus xi C. When you go to 2nd order upwind get phi f equals 3 by phi C minus phi U by this case it would be phi f equals to phi C plus phi C minus phi U divided by xi C minus xi U multiplied with xi f minus xi C.

Now, quick you get phi f equals to 3 by 4 phi C minus phi U divided by 8 plus phi D into 3 by 8 and this case this is phi U plus you have term I plus term II we will see how they look like. And, if you put Fromm scheme this would be should be phi f equals to phi C plus 1 by 4 phi D minus phi U and this case it is phi C plus xi f minus xi C divided by xi D minus xi U multiplied with phi D minus phi U. So, you can see this difference in the functional form of this expression.

(Refer Slide Time: 28:07)



Now, once for the quick the non-uniform system the term I is xi f minus xi U divided by xi D minus xi U multiplied with xi f minus xi C divided by xi D minus xi C with phi D minus phi U. And, term II they would look like xi f minus xi U xi C minus xi U if minus xi D xi C minus xi D phi C minus phi U.

So, where xi U is xi U U plus xi U minus xi U U and xi U U is nothing, but x U U minus x O square. So, in this particular this somewhere centre is sitting. So, plus y U U minus y O square plus z U U minus z O square and xi U minus xi U U equals to x U minus x U U square y U minus y U U square z U minus z U U square. Now so, if you put them together xi 1 minus xi is essentially x 1 minus x 2 square y 1 minus y 2 square z 1 minus z 2 square.

So, this is how you can actually get the functionality between uniform and non-uniform system and the functional form look similar only thing the calculation of the non-uniform system the coordinate system becomes xi system. So, we will stop here and will continue the discussion in the next lecture.

Thank you.