Introduction to Finite Volume Methods-II
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Lecture – 17
Convection term discretisation-IX

So, welcome back to the lecture series of Finite Volume series. And we are in the middle of our Discretisations of Convection diffusion system. And so for we have looked at different kind of discretisation scheme; like upwind, central difference, second order upwind, downwind, quick scheme, and Fromm scheme. And we have analysed their stability accuracy and everything.

So, now we can look at that there are certain issues or rather I would like to say there are couple of errors which may arise due to discretisation of the convective term. One is the diffusion error another is the dispersion error. Now there are always an trade off between your higher order scheme and the less stable scheme. So, one case it provide stability, but it is lower in accuracy other case it is more in the error prone for the dispersion kind of error prone, but it is higher ordered system.

So, the idea is that which will actually give us in platform to find out some scheme which is of higher order accuracy, but at the same time it is stable, I mean in the sense it should not give you any sort of small riggles or oscillations where the sub gradient occurs. So, we look at the higher order schemes now and how to actually formulate those schemes from the basic information that is what we will discuss today.

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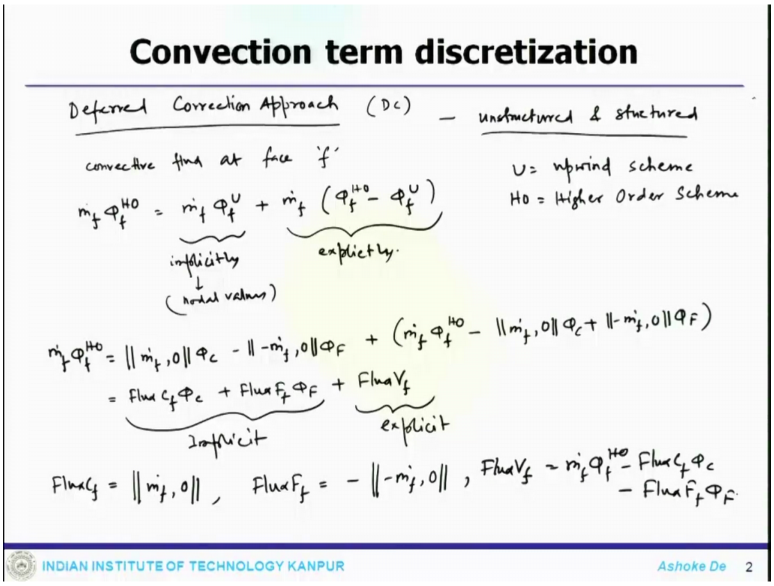

Now, before we move to that one important information or the topic that we started discussing, but we could not finish in the last lecture is the deferred corrections approach. So, we will first talk about that and then we will move to the higher resolution scheme or higher order scheme with better stability. So, the deferred correction approach it is accept essentially what it does that it enables to use the higher order scheme and easily inside these formulation.

So, this is also applicable for unstructured and structured grid. So, there is no restriction on the grid. Also this method is based on the calculation of the convective flux. So, the convective flux at face f which is calculated using some sort of blending of the higher order resolution and the upwind resolution. So, the way it is formulated is that m dot f phi f at higher order is m dot f phi f from upwind plus m dot f phi f higher order minus phi f upwind.

So, here U stands for upwind scheme and Ho stands for higher order scheme. So, it is essentially a blending function of upwind scheme and higher order scheme. So, what it does that when you get these things the first term of this equation this can be calculated implicitly and the second term which can be calculated explicitly ok. The reason is that why can be can calculated can be implicitly it because it can be expressed in terms of nodal values.

So, in terms of nodal values these guy can be expressed. The other one has to be evaluated explicitly the second term on the right hand side because at the latest available phi values from the previous iteration that is going to be used in this iterative process. Now in terms of nodal values this equation can be written as m dot f phi f higher order system m dot f 0 phi C minus m dot f 0 phi f plus m dot f phi f higher order minus m dot f 0 phi C plus m dot f 0 phi F which will get you flux C f phi C plus flux F f phi F plus flux V f.

So, these term again is a implicit term this is the term which is explicit term where one can see the flux C f is m dot f 0 your flux F f is minus m dot f 0 and flux V f is m dot f phi f higher order minus flux C f phi C minus flux F f phi F. Now substituting this all this information in the convection flux and then what we will get the algebraic or the compact form of the discretized system.

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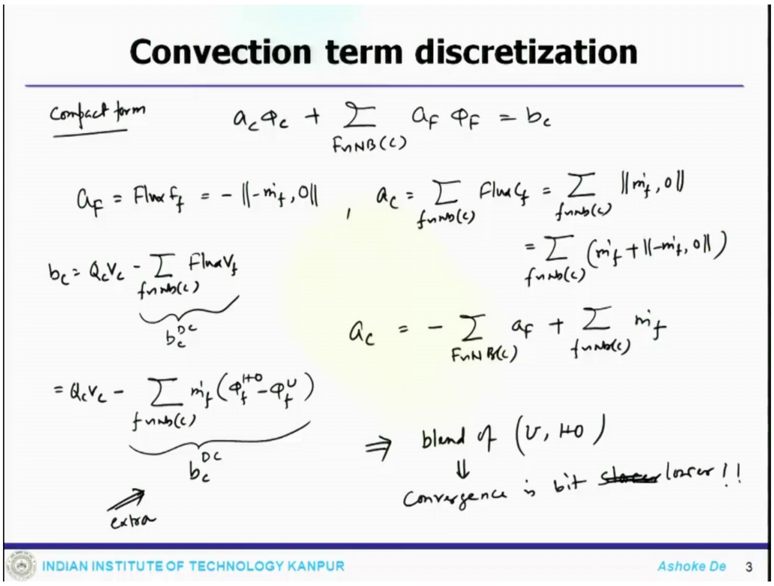

Like a c phi C plus summation of capital F goes over NB C, a f phi F equals to b c where coefficient a F is flux F f which is minus m dot f 0. And a c is summation of small f goes over all the faces. Flux C f which is goes over all the faces m dot f 0 that will get you it is the integration over all the faces m dot f plus minus m dot f 0. It is nothing, but minus summation of capital F goes over all the neighbouring elements.

So, a F plus f NB C m dot f that is what a c is. And the source term which will look like Q c V c minus f goes over all the faces flux V f. So, that is essentially will have the correction term due to deferred correction. So, this is Q c V c minus f NB c m dot f phi f phi f U. This is the term which actually is represented as source term due to deferred correction. So, these term one can see is an extra term which arising due to the deferred correction procedure. In top of that these deferred connection technique results in an equation for which the coefficient matrix is always diagonally dominant since it is formed using the some sort of an upwind scheme.

So, the compacting procedure is simple to implement; however, as the difference between the cell faces values calculated with the upwind scheme and the calculated with higher order scheme becomes larger the convexer convergence rate diminishes. So, it essentially is a blend of upwind and higher order scheme. So, that is why there is a convergence rate is bit slower or bit lower one can think about that way.

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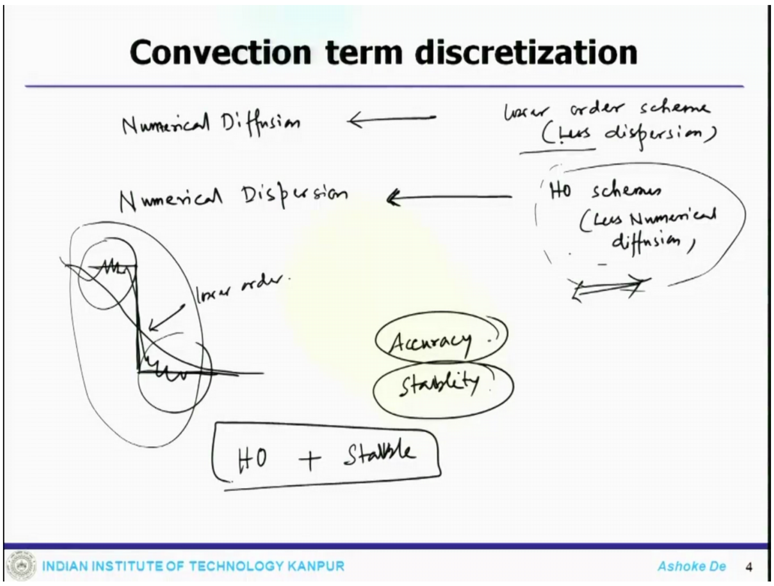

So, have been said that when we talk about all these things the two issues which are very pertinent is that when you discretize the convection term these are one is that; numerical diffusion other is the numerical dispersion. Now this is more prone for higher order schemes because the diffusion would be less so less numerical diffusion. But due to dispersion wherever the solution like we have seen the when there is a sub gradient the solution become rig some sort of an oscillatory nature and then it goes like that.

So, this is a problem of the dispersion error which will arise in primarily second order upwind, quick, Fromm scheme all the CD scheme because they are at the spatial accuracy is higher. So, they are not stable when there is sub gradient. So, it is essentially due to the stability problem of these different schemes. So, one hand these are highly accurate because spatial accuracy is higher like second order or third order, but at the same time they lead to some sort of a riggles or oscillations when there is a sub gradient in the flow field.

So, that is one problem other end dissipation will come from the lower order scheme or the first order scheme so, but that case less dispersion which does not mean then the dispersion error would be completely absent in the lower order scheme. We have already seen while doing the analysis for the upwind scheme we have seen that the value of the k where the both real and imaginary part actually exists. So, which immediately tells this lower order schemes are prone to have both kind of errors both diffusion and dispersion.

But diffusion is more dominant because their spatial accuracy is less and that is what come from your truncation error. So, when you use lower order scheme they might not give you this kind of oscillations or riggles along the sub gradient, but the solution will smear out like that. So, this will happen with the lower order scheme because of the diffusion things become much smoother and then it goes away from these capturing of these particular gradient.

So, these are the problems. One hand you want higher accuracy, but at the same time you want stability. So, these are the two partnering questions that one has to answer that one hand accuracy one hand stability. So, higher order scheme are accurate, but not very stable lower order scheme they are not accurate, but more stable.

So, we need to device or rather that actually provides the platform for finding a different kind of scheme or class of scheme which will be of higher order plus stable. So, that is what one needs to find out. And how we can device those kind of high resolution scheme that we will see. Now when you talk about those high resolution scheme the thing which actually comes is or they are formulated based on normalized variable formulation.

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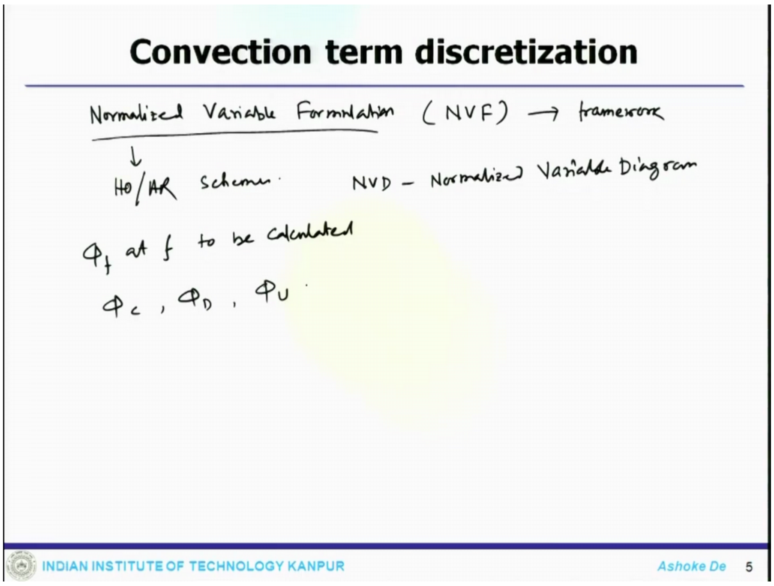

So, which is known as NVF. So, this higher order resolution scheme these are sort of formed or can be coined using this concept of normalized variable formulation. So, this is the rather the framework. So, one can think about this is the framework which can be used to for description as well as analysis of higher order, high resolutions schemes. Now it was introduced long time back. So, what happens in the normalized variable diagram is useful tool. So, there will be NVD which is Normalized Variable Diagram.

So, this is an useful tool for the development and analysis of these higher order high resolution scheme. Now, NVF is a face formulation based on locally normalizing the dependent variable for which the value phi f at face to be calculated or constructed. So, the approach relies on the upwind scheme phi C the downwind point phi D or upwind phi U and this kind of nodes.

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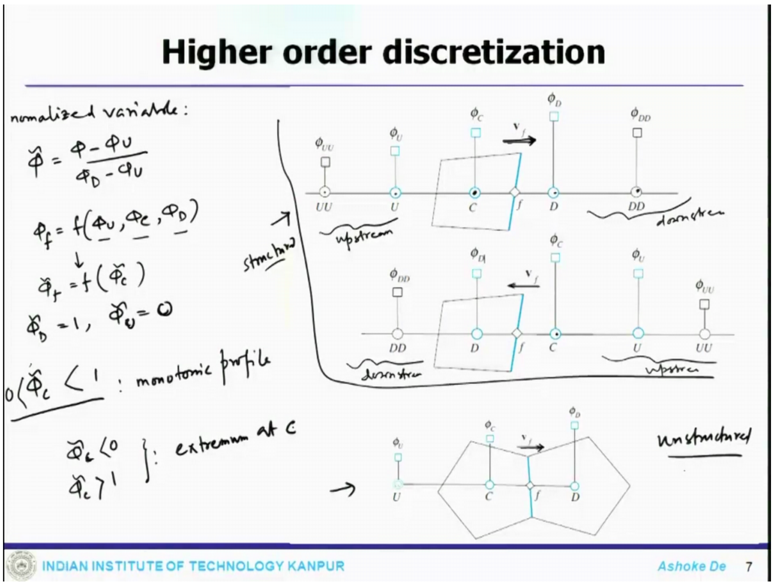

So, if you see how one can do that this is your particularly this is a the schematic the schematic shows that there are cell which we are interested upstream of the cell is U, downstream of the cell is D, further downstream DD because our velocity direction at that face in that direction and for upstream is UU. So, these are used in the structured system for deriving or describing this kind of system. The second picture shows that only C and D and the extrapolated this picture where C and D that are used to extrapolate the variable on unstructured grid.

So, these picture shows for structured system and this one for unstructured system where you can use this cell centre values and there would be slightly upstream value U which is in another cell and the at the face the velocity vector actually moves in this direction. So, one case is structured and other case is unstructured. And the structured case when the face velocity goes in these direction this should be considered as upstream condition and the subsequently this side would be downstream condition.

And when the face velocity goes in these direction these guys would be upstream and this is downstream. So, it just changes the direction depending of the flow velocity direction. So, this is a very useful information and we have come across this information once we started deriving the discretized equation for the convective term. Because this problem was not present when you looked at the diffusion term because diffusion term there is no convection which is associated with that.

So, now, if you define your normalized variable; your normalized variable such that phi tilde equals to phi minus phi U divided by phi D minus phi U. So, the normalization relation stands for function of U, function of C, function of D which means it takes into account upstream point the cell which is concerned of that the downstream point. And it could be either way round if flow goes this side this information will be taken into account, this information also will be taken into account, if it goes in this direction these are in the information which will be.

So, this can be transformed to phi f equals to f of phi C. And the normalized value of phi D tilde equals to 1 and phi C tilde phi U tilde is 0 with the normalized value of phi C phi C tilde becomes an indicator of smoothness. So, now if the value of phi C tilde lies between 0 and 1. So, what happens? So, it represents a monotonic; monotonic profile while if the phi C tilde C is less than 0 or phi tilde t greater that 1 it indicate the extreme extremum at C. Now top of that so this you can see how the normalized variable of phi C tilde which provides this kind of expression.

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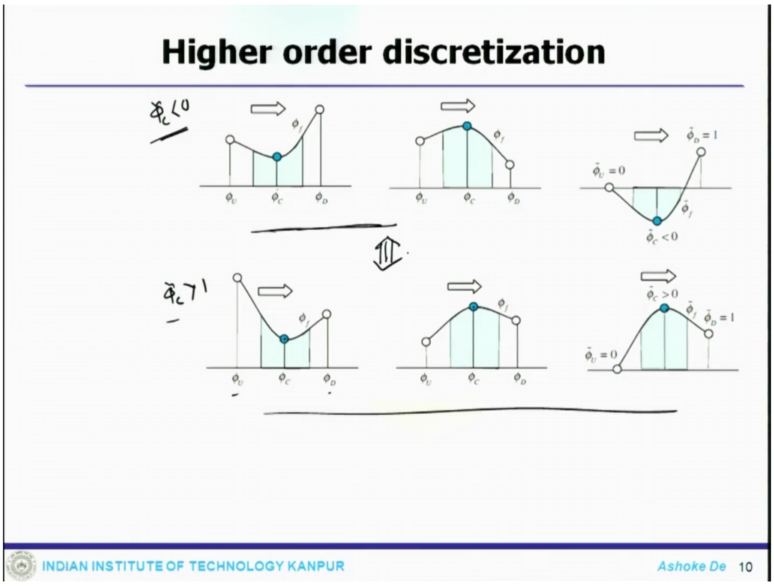

This is shown in this particular graph where you can see that this particular case which is shown here this case is the case where phi C tilde less than 0. So, phi C tilde less than 0 at indicate that there is a extremum at point C this is my cell which is concerned this, is the downstream, this is the upstream and this is the flow direction. So, either it could be extreme maximum or could be minimum this is what the normalized conditions.

Now the second case, it actually the case where phi tilde C is greater than 1. So, if it is greater than 1 then also you will see there could be extremum at point C. It could be minimum it could be the maximum value when you consider phi U phi C. Now the other case when it lies between 0 to 1 and then this case where it is phi C tilde equals to 1, this case is phi C tilde equals to 0 and this is the case where phi C tilde lies between 0 to 1.

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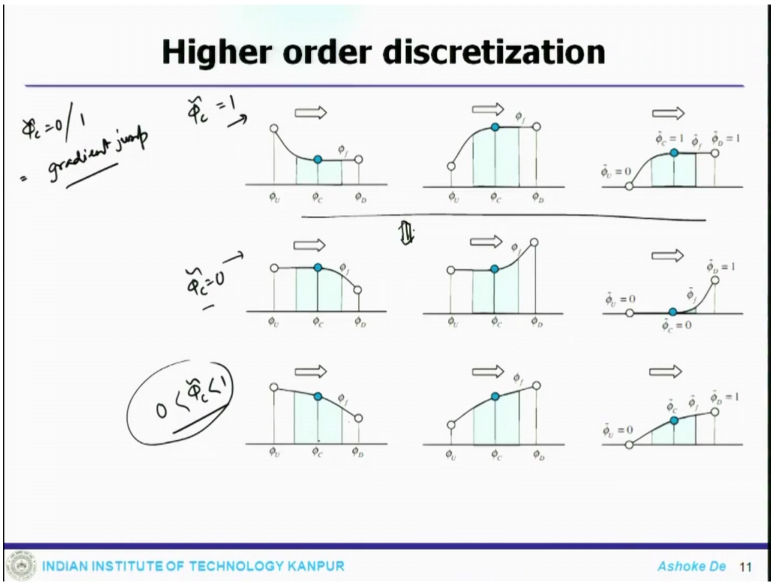

So, when it is between 0 to 1 it shows some monotone profile either monotonically decreasing or monotonically increasing when it is 0 or 1 which indicates there would be sudden jump in the gradient. So phi tilde 0 or 1 it means there is a gradient jump. So, this normalized variable actually shows lot of information, what information it is depend on this normalized value which actually takes into account it is lies between 0 to 1 then the profile shows monotonically profile either monotonically decreasing or monotonically increasing between these points which are used to calculate the flux variable.

If it is 0 then there is a jump in the gradient which one can see if it is also 1 there will be a jump in the gradient. So, these two are sort of equivalent this two picture and if it is less than 0 then either of this C value will have sudden extreme conditions 0 or 1 or if it is 1 or greater than 1 then also it represents.

So, these are the some sort of an equivalency whether it is less than 0 or greater than 1 it shows some extreme point on this. Now since normalization is also useful for transforming this functional relationship or higher order scheme. So, now, one can use this functional relationship for the higher order calculation.

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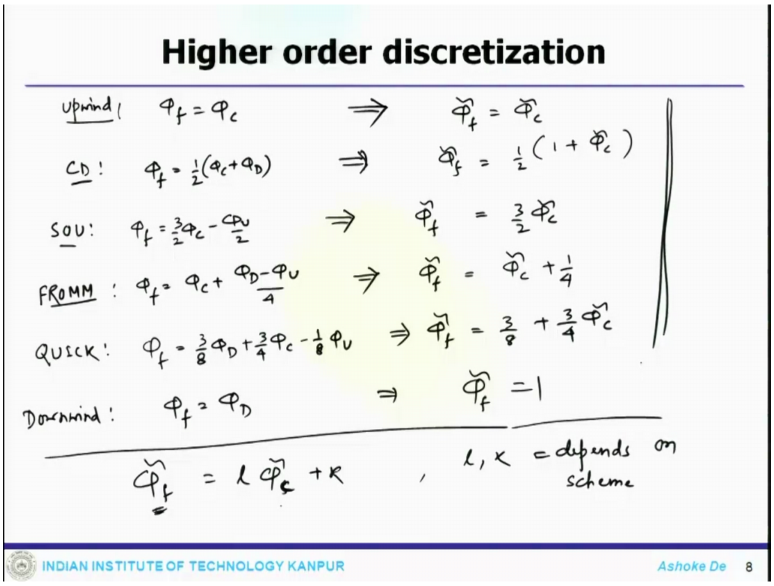

So, which we have already seen that for upwind scheme it was phi f was phi C. So, the transformation will let you have phi f tilde equals to phi C tilde so that is the transformation. Now central different scheme we had phi f half of phi C plus phi D which will transform to phi f tilde which is nothing but half of 1 plus phi C tilde. So, these are functional relationship which is also or which is true for both structured and unstructured grid. And now for different scheme how it transform to this normalized condition. Second order upwind the phi f is represented as 3 by 2 phi C minus phi U by 2 which will get transform to phi f tilde equals to 3 by 2 phi C tilde. Fromm scheme so, that phi f was phi C phi D minus phi U by 4 which will get transformed to phi f tilde equals to phi C tilde plus 1 by 4. Then quick scheme which is phi f equals to 3 by 8 phi D plus 3 by 4 phi C minus 1 by 8 phi U which will get transformed to 3 by 8 plus 3 by 4 phi tilde C and the downwind scheme which phi f equals to phi D here it will be phi f equals to 1. So, all the higher order scheme that are based on this nodal values phi f can always be expressed as a linear function of phi C plus some constant k where the value of l and k depend on depends on scheme ok.

So, the point here is that the phi f tilde this is now one can plot again this phi C tilde in a particular plane. And then the functional relationship of all this schemes will appear as an straight line. So, what we have we can do we can look at this plot a phi f tilde verses phi C. And all this functional relationship for different scheme will turn out to be a straight line. And the resultant plot can be denoted as the normalized variable diagram or NVD on which the functional relationship of all this schemes can be plotted. So, we will stop here and we will continue the discussion in the next lecture.

Thank you.