Introduction to Finite Volume Methods-II
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Lecture – 18
High Resolution Schemes-I

So, welcome back to the lecture series of Finite Volume Series.

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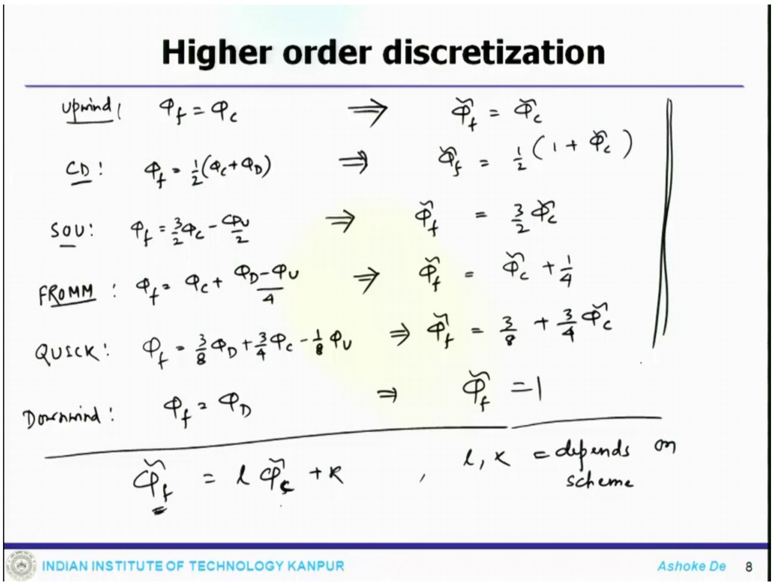

So, if you see, once we plot all these how it looks likes.

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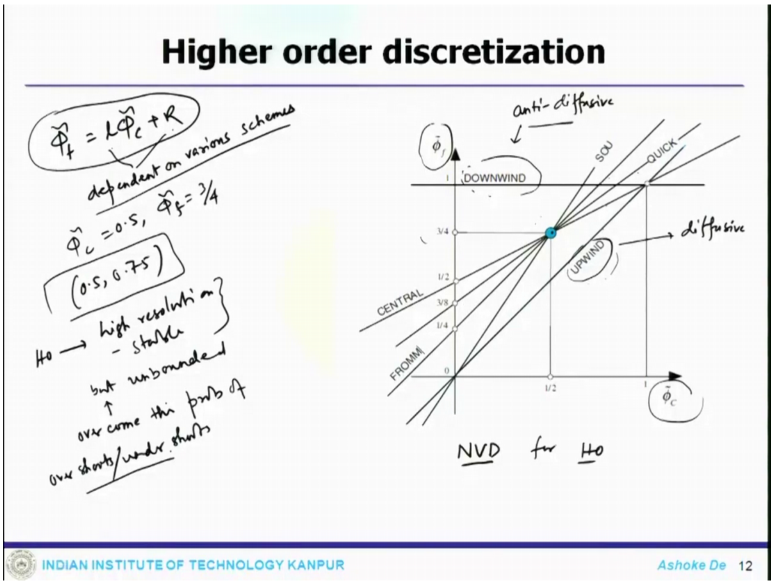

So, this is how it looks like, this is our phi f tilde, this is our phi. So, our phi f tilde is phi C tilde plus k. So, l and k these are depends or dependent on various schemes. So, these are dependent on various schemes. Now, as we said this complete diagram is called the normalized variable diagram or NVD for higher order schemes.

Now, what you can see as we said once you plot them in this kind of a function everything will look like a straight line. So, here is your upwind scheme, this is your central scheme, this line quick scheme, this is Fromm scheme, this is second order upwind scheme and this is what the downwind scheme. So, all the schemes they are, I mean getting a flat straight line and the meeting point for phi C tilde is have an phi f tilde is 3 by 4. So, all of them actually pass through that point 0.5 and 0.75. So, except two schemes, two first order scheme one is upwind scheme which does not pass through this point. So, by 0.5, 0.75 and the downwind scheme these two scheme they do not pass through that point other than that all are higher order scheme like second order and third order scheme they pass through that point.

So, in fact, it can also be shown that the scheme to be second order accurate it has to pass through this point. So, other way round one can think about that any higher order resolution scheme to be at least second order it has to pass through this point. So, this is a unique point and in addition to its slope q is 0.75 then it will be third order accurate. So, this is the point. Now, the upwind scheme is found to be very diffusive while the downwind scheme is very compressive or rather this is anti-diffusive. And this guy is highly diffusive.

So, and therefore, in NVD what we can see that that any scheme whose functional relationship is close to the upwind scheme or diffusive and which one is close to downwind scheme that would be anti-diffusive. So, anything close to this that scheme would be diffusive scheme or in NVD be anything close to downwind scheme that can be termed as anti-diffusive scheme or any other scheme wind pass through this point they will at least have second order accurate and top of that if it pass through also 0.75 it will become the third order accurate scheme.

Now, this higher order scheme, so what we have seen so far these are having high resolution that is one important thing, then this has to be stable, but at the same time as we have been saying this become unbounded, ok. So, you need to overcome this, overcome this problem of over shoots or under shoots, which is essentially the oscillations near the gradient. And that gives the platform for devising this scheme for high resolution scheme.

Now, what one can do the linear numerical scheme? That is monotone that can be the most faster or accurate.

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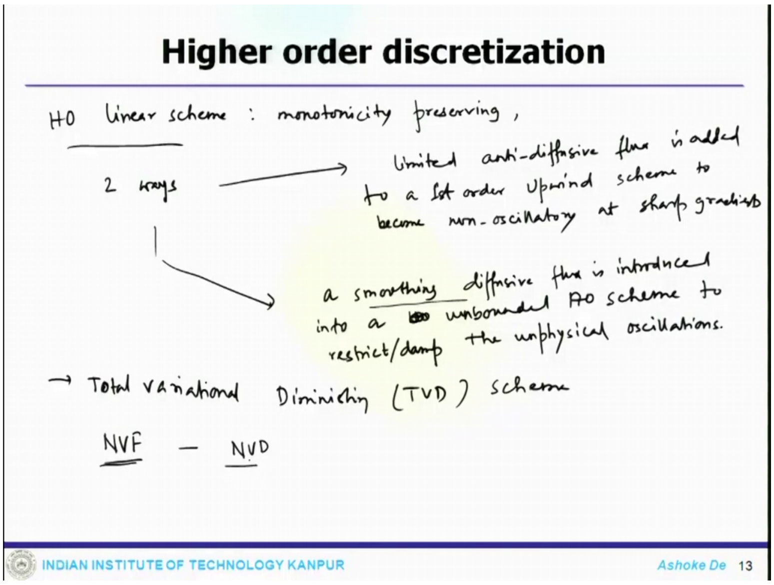

So, what it says that all higher order linear scheme they are monotonically monotonicity preserving. So, one can think about monotonicity preserving number one, and that too the monotonic preserving scheme non-linear limiter function can be used.

So, with these understanding, one can actually device some higher order scheme and that can be grouped in two special categories. So, there are two ways one can obtain. In first approach what one can do? So, limited anti-diffusive flux, anti-diffusive flux which is added to a first order upwind scheme in such a way that the resulting scheme is capable of resolving the sharp gradients without oscillation. So, the limited anti-diffusive flux is added to a first order upwind scheme. So, to become non-oscillatory at sharp gradients. So, problem becomes when there is a sharp gradient which is present in the domain. So, there these upwind scheme actually add so much of diffusion that the results may as out. So, to avoid that some limited amount of anti-diffusive flux can be, that could be one approach.

Other approach a smoothing diffusive flux is introduced into a higher order unbounded or unbounded higher order scheme. So, the second way one can actually add some smoothing diffusive flux to a unbounded higher order scheme to restrict or damp out the unphysical oscillations. So, this is how one can actually move or move forward to have different kind of schemes.

Now, due to their multi step in nature and difficulty in balancing the two fluxes, the flux bending technique tend to be very expensive numerically. And this is why here what will try to do see something with a limiter kind of approach and which will give rise to this scheme which belongs to the class of total variational diminishing, that is TVD scheme. So, this is another class of scheme higher order scheme which also can capture that sharp gradient, but without introducing. So, essentially it means that you use both the advantage of the property of less diffusion and less dispersion kind of system.

So, the composite scheme approach will be discussed and within the framework or normalized value formulation and which can be seen in the normalized value diagram. So, first we will discuss about NVF and NVD, and then we move to kind of approach to get this scheme.

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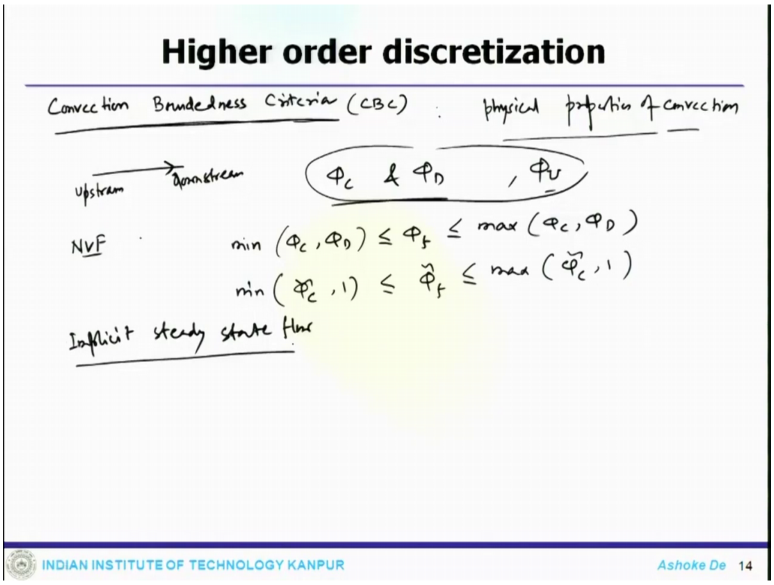

Now, the point curves the convection boundedness criteria, boundedness criteria. So, this is an important criteria which needs for devising this kind of higher order scheme. So, what your numerical scheme is expected to do? It is expected to preserve your physical properties of the phenomena which you are trying to represent or replicate through the numerical approximation.

So, the condition that bounded convection scheme should satisfy can best be understood once we start analyzing the physical properties of convection. So, that what is very important. So, once we analyze the physical properties of convection then only we can understand these convection boundedness because that is what it is going to do which is supposed to represent a physical phenomena through numerical approximation.

Now, convection transport fluid properties from upstream to downstream. Typically, conviction means it goes from upstream to downstream, that is a standard process if the gradient is in this direction the flow moves in this direction. So, the numerical convection scheme should be always upwind biased or else they will lag the convection stability. So, in addition to value at the nodes which are connecting between your phi C and phi D these are the nodes which are connecting the value at the higher upwind node which lies at phi U is also important and to be taken into account when devising this kind of things. So, we have already seen in that normalized value framework that the numerical convection scheme is monotone.

So, monotonicity is one important property and which was kind of one can mathematically think about minimum of these phi C and phi D which is less than your phi f which is also bonded like maximum of phi C and phi D. So, normalizing the above condition which will get you minimum of phi C tilde 1, less than equals to phi f tilde which is max of phi C 1. So, this is called the convection boundedness criteria for implicit steady state flow, implicit steady state flow where you get this.

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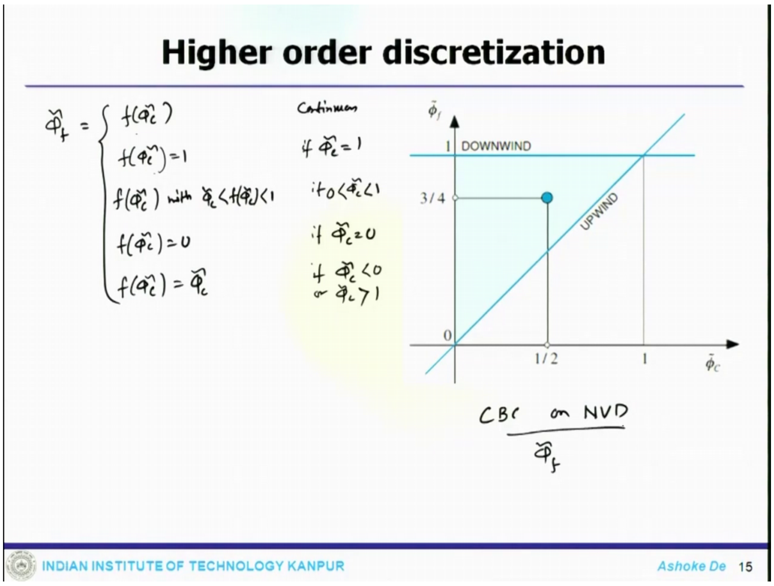

Now, also to get this boundedness criteria we can see in a picture which shows the convection boundedness criterias CBC on an normalized variable diagram, so which shows the region of phi f tilde. Now, what we can see here, here phi f tilde equals to different values. Function of phi C, it could be function of phi C tilde equals to 1. So, when it is function of phi C that means, it is continuous, this case it is if phi C tilde equals to 1; that means it is here, then the function of phi C tilde is 1 other case it could be function of phi C tilde with phi C less than function of phi C less than 1.

So, this is the case where phi C tilde lies between 0 to 1; that means, if phi C tilde lies between 0 to 1 or it could be function of phi C tilde which is 0 and that is if phi C tilde is 0. If it is 0 then the functional form would be also 0; or it could be phi C tilde equals to phi C tilde. And in this case if phi C tilde less than 0; that means, this side or phi C tilde greater than 1 then this would be always phi C tilde. So, you can see this criteria is quite intuitive, and can be also seen in this NVD in the next picture.

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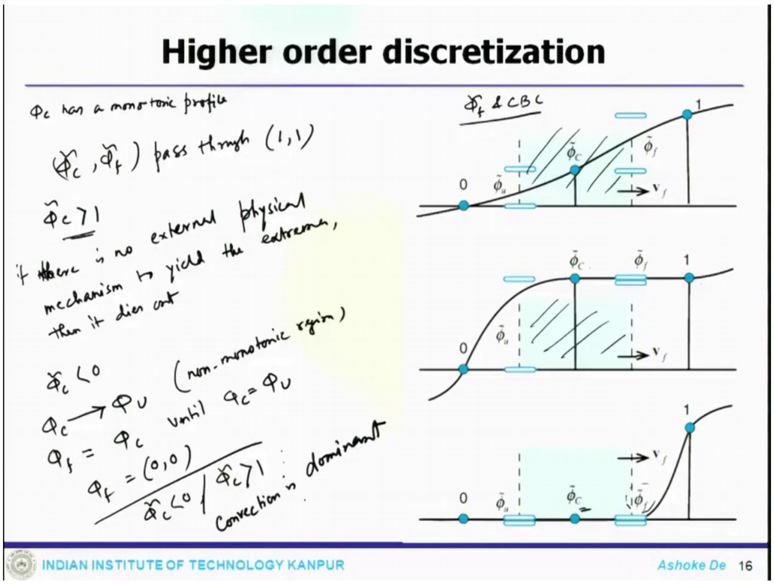

So, what happens, if we see here. Now, this one that importantly it shows the values of phi f tilde and convection boundedness criteria. So, this all these three picture shows that. And how one would interpret this things or what is the physical significance to that? Now, if phi C has an has a monotonic profile, the interpolation profile at the cell surface should not yield any new extrema. So, thus it is constant by the phi values at the nodes of the connecting faces. So, as the phi, as the value of phi C get closer to phi D still it is within the monotonic region the phi f will also tend towards phi D and when. So, this is the case, when phi C actually will get closer to phi D the monotonic profile of phi f also closed towards the phi D.

Or, the other case when phi C tilde or phi C becomes equals to phi D, phi f also becomes to phi D which shows this case; where this guy goes close to phi D that means, the constant value here the flat line this is a monotone profile; that means, there is no extremum at C. And the other case it shows that this guy; so this is the region; this is the region what we are talking when it goes close to phi D phi f also goes close to. So, the condition that phi C tilde and phi f tilde they pass through 1, 1. Now, the other case it could be when phi C is greater than 1. So, phi f is assigned the upwind value of phi C. So, this is the case. When phi C is greater than 1, then this guy will be assigned a value of phi C.

So, this has the effect of yielding the largest conditions or the flow conditions which are actually happening physically in the domain. So, what it tells you that, no if there is no external physical mechanism to yield the extrema that is a source term for example, if there is no external mechanism then it dies out, the extrema actually dies out. So, a similar mechanism takes place when phi C tilde is less than 0; however, one point you can note here when phi C get closer, so phi C goes closer to phi U or get closer to phi U coming from the non-monotonic, non-monotonic region then phi f would be equal to the upwind value phi C, until phi C equals to phi U, implying that condition that phi f passed through 0, 0.

And the other situation where phi C tilde less than 0 or phi C tilde greater than 1, either of these two cases. So, the solution will be in a region where conviction is dominant and upwind f approximation will be an excellent one. So, this is what one can understand that what is happening with different scheme.

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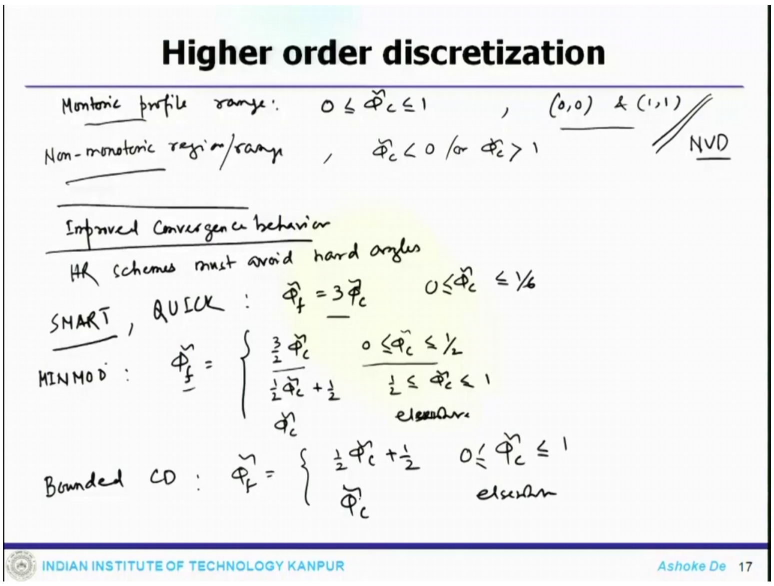

Now, we can device the High Resolution Scheme. So, the High Resolution Scheme the monotonic profile range, so the monotonic profile range is phi C tilde is between 0 to 1 and it pass through 0, 0 and 1, 1 while the remaining with the upper triangular region in this particular picture if you see in this particular picture this upper triangular region, so on the NVD.

So, on the other hand in non-monotonic region, so non-monotonic region or range would be phi C tilde less than 0 or and or or phi C tilde greater than 1. So, this is a non-monotonic range and the monotonic range is 0 to 1. So, these are already shown or we have seen in the normalized variable diagram, and in that case when it is in monotonic range they need to any scheme they need to pass through this coordinates of. So, now, there are number of well-known high-resolution scheme which can be built in this particular fashion, and they can be also seen graphically how would they look like.

So, what is essential is that we need an improved convergence behavior, so that is what is required from this high-resolution scheme. So, the HR scheme should avoid any hard, so HR scheme to have these HR schemes must avoid hard angles. And its profile connection point as well as its horizontal and vertical profiles. For example, if we consider some scheme the SMART scheme this will talk about that how it is devised or squeeze scheme the convert or quick scheme the convection or the convergence can be I mean substantially improved by a minor modification to the vertical portion of its profile such that, if someone uses this equals to 3 phi C like that. In the region, where phi C tilde is 1 by 6 to 0.

Similarly, those changes one can see in those different how they actually improve. So, for example, if you use the MINMOD kind of scheme, MINMOD kind of scheme in that case the phi f equals to it becomes 3 by 2 phi C, when phi C lies between half and 0 it will become half phi C tilde plus half, if it is lies between half and 1 and other places it is elsewhere it is phi C tilde. So, this is the behavior of the MINMOD system.

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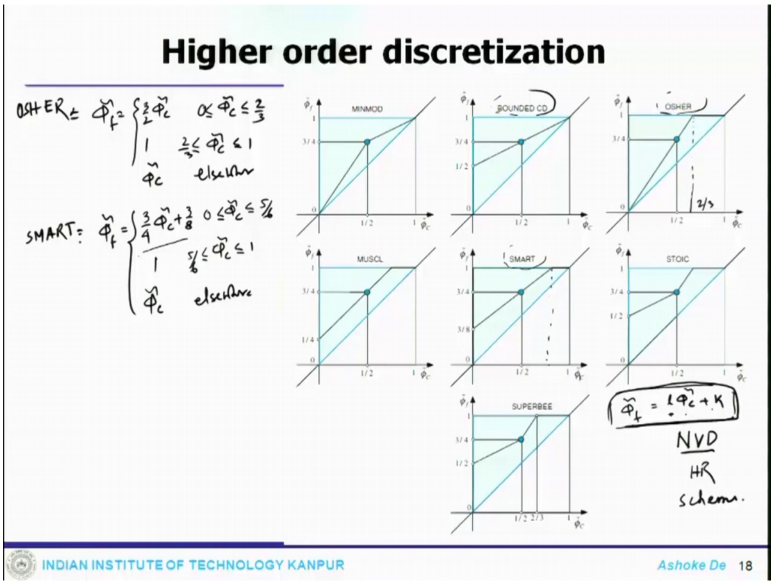

So, one can see that how, this is the MINMOD now again these are the pictures which shows this is again non-normalized variable diagram for different HR schemes. So, one case we can see that the MINMOD case where we have seen that it is the for phase value is 3 by 2 phi C when the phi C tilde lie between 0 to half; that means, 0 to half the phase value is 3 by 2, when it is between half to 1 it is half phi C plus half and elsewhere it is all phi C. So, that is what the MINMOD operator does.

Similarly, if we go to bounded CD, so bounded CD shows that phi f equals to half phi C plus half, if I phi C tilde lies between 1 and 0 or elsewhere it is always phi C. If you see in the NVD this is what the bounded CD does. So, it says between 0 to 1 it is half that is a constant value which is sitting at the phi f plus phi C how its varies, and other than 0 to 1 the value is always phi C which goes as a linear profile.

So, in the NVD if you look at it then similarly this scheme OSHER, that is another higher order scheme. The OSHER says that phi f equals to 3 by 2 phi C; if phi C tilde lies between 2 by 3 and 0 so that means, phi C tilde lies between 0 and 2 by 3, so which is somewhere here. It is only 3 by 2 phi C. And then it becomes 1 if phi C tilde lies between 1 to 2 by 3 and elsewhere it is phi C tilde. So, when it is 3 by 2 by 3 and 1, this is always 1 that is the constant value, so you can see this is the point which is 2 by 3. Before that it was 3 by 2 phi C there is a slope and then beyond that it all phi C that is how the behavior of the OSHER scheme.

Now, the SMART scheme. The SMART scheme if you see here, the smart has phi f equals to 3 by 4 phi C tilde and plus 3 by 8 if phi C tilde lies between 5 by 6 and 0, that means 0 to 5 by 6, 0 and 5 by 6 which would be somewhere here which is 3 by 4, 3 by 8 constant, but then the slope is; so this is the point which is 5 by 6. And if phi C tilde less than 1 between 5 by 6 it is 1, so here beyond that to 1 it is constant value and then you can see it is always phi C where elsewhere.

So, this is another high-resolution scheme which shows up to 5 by 6 this point, the profile follows some linear combination. This is already we have said that all these schemes they can be the functional values of phi f would be represented at a function with some combinations of the linear system which is l and so it can be represented at l phi C tilde plus k. And this k and l value are different for different different profile, as we see the SMART scheme provides this kind of profile, when OSHER scheme provide this kind of; there are others one also there like MUSCL, STOIC and all these will see.

But the point is all these higher order scheme belongs to this kind of profile definition or the normalized flux definition with some factor constant with phi C plus constant. And that is how we are building all this. And now will stop here today and the other scheme will see in the next lecture.

Thank you.