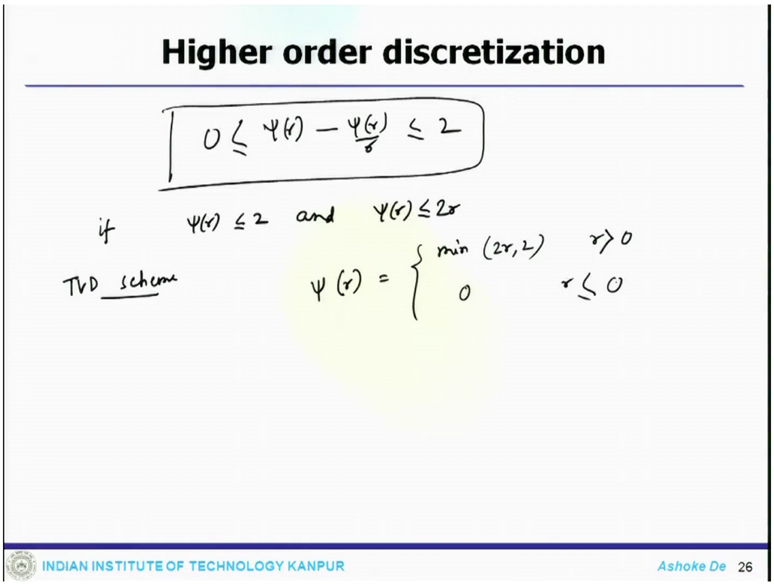
Introduction to Finite Volume Methods - II
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture – 20
High Resolution Schemes-III

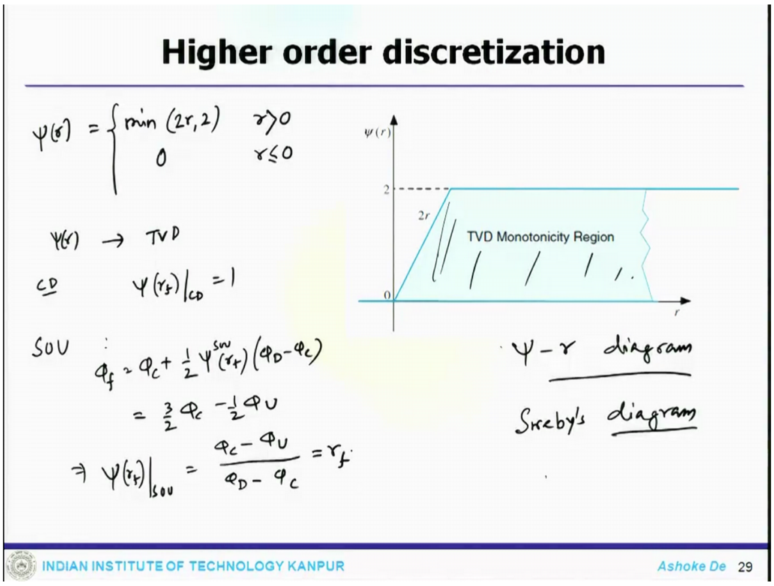
So, welcome back to the lecture series of Finite Volume. And what we will continue our discussion where we left in the last lecture.

(Refer Slide Time: 00:21)



So, in addition to psi r greater than 0 a condition also impose that psi r require by psi r 0 for negative values of r. So now, this condition will be satisfied if psi r less than equals to 2 and psi r less than equals to 2 r. So, combining all of these to produce a TVD scheme a criteria similar to that CVC or CBC can be developed where one can write psi r equals to mean of 2 r and 2, when r greater than 0 and 0 when r less than equals to 0. So, that is the condition that one can device and this condition can be seen in the psi r diagram. This is called the psi r diagram.

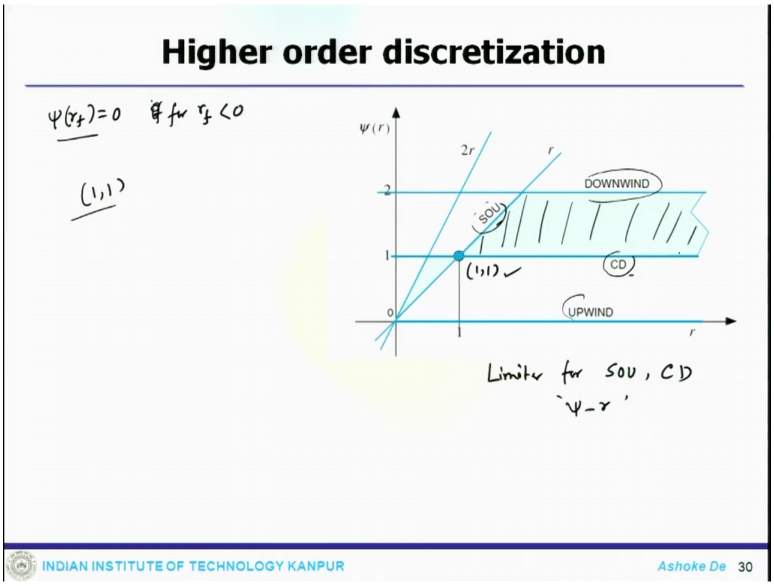
(Refer Slide Time: 01:24)



And this is the region which is TVD monotonicity region where it is satisfied, this is also called Sweby's diagram. So, this satisfied this criteria of psi r equals to mean of 2 r and 2 and 0, where r greater than 0 and r less than 0. So, these are the zone which is TVD monotonicities. So, if you look at this the simply you can grasp the formulation of TVD scheme. Any flux limiter psi r which is formulate, I mean which lies within the TVD monotonicity region will yield an TVD scheme or this diagram is very similar to that NVD diagram, Normalize Variable Diagram that we discussed earlier.

Now, the limiter for all these schemes presented so far can be derived and their functional relationship can be also drawn on this particular diagram. For let say very specifically limiter of central difference scheme can be easily obtained, where psi r f for CD scheme would be 1, while same thing for second order upwind scheme can be computed as phi f equals to phi C plus half s r f phi D minus phi C which is 3 by 2 phi C minus half phi U. So, which gets you back for second order up wind it is phi C minus phi U divided by phi D minus phi C nothing but r f. So, this can be displaced here.

(Refer Slide Time: 03:59)

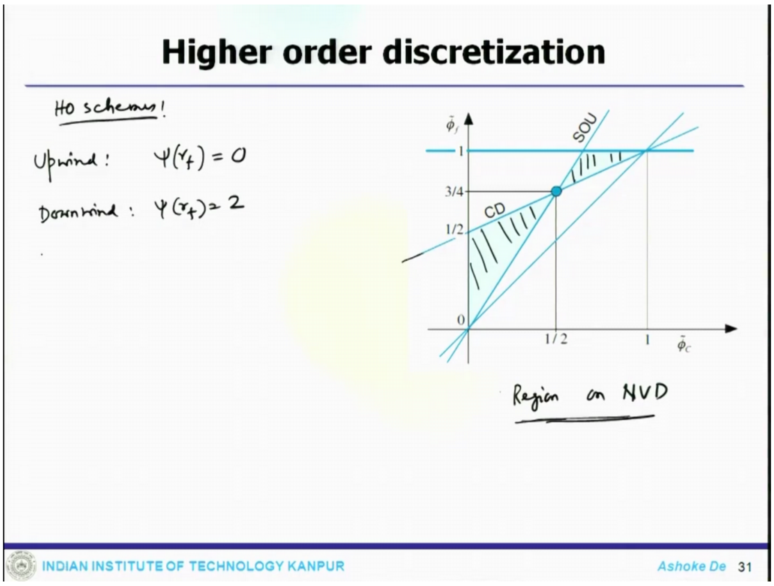


This is the diagram, so which shows the limiters for second order upwind and CD in this psi r diagram. So, one can see that psi if from these diagram you can notice if psi r f equal 0 for r f less than 0. So, r f less than 0, psi r f is 0, the second order accuracy is actually lost at extrema of the solution.

Now, both these second order upwind and CD they are actually in by nature second order scheme and accurate, and which can see clearly pass through the point 1, 1 both these and this, but neither the downwind scheme or upwind scheme pass through these point. So, also one can; so, thus it is very clear that the scheme to be a second order accurate it has to pass through these coordinate 1, 1. And the limiter should lie in the region which is bordered by second order upwind and the CD scheme.

So, all the scheme which has to pass through or which will which is suppose to pass through 1, 1 and lie within this border they would be of second order accurate and so the corresponding region one can show in the in NVD.

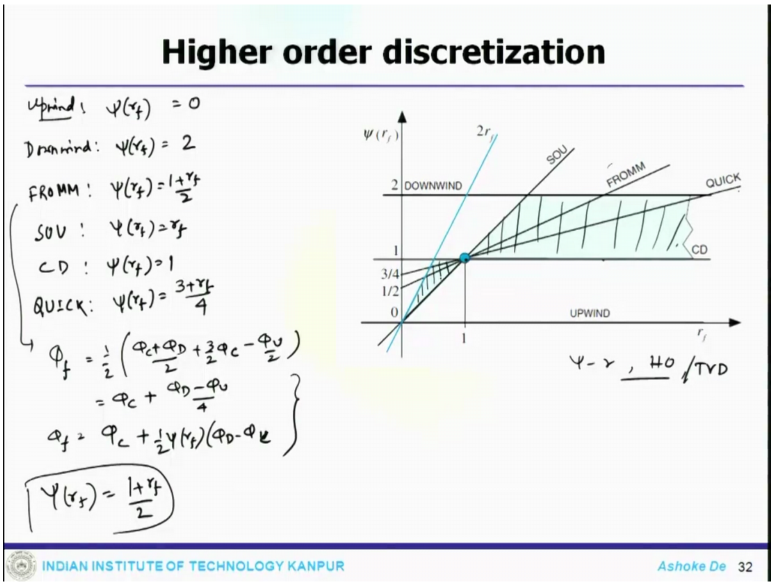
(Refer Slide Time: 06:10)



So, if you look at this is the region on normalize variable diagrams. So, here we are trying to make a one to one correspondence between NVD and this psi r diagram, because psi r diagram is going to get you the TVD different class or variant of TVD screen. Now, here if you put the monotonicity region on a; for second order scheme lies in the shaded region which is bounded between the CD and the is SOU. So, this is the zone which is actually going to be the second order accurate scheme or that will provide the second order accurate scheme.

Now, once we adopting this approach and the procedure which we followed for the second order upwind scheme, the functional relationship of the limiters for any higher order scheme can be formulated. For example, this is the higher order schemes functional relationship. So, we can say upwind scheme the psi r f is 0.

(Refer Slide Time: 07:57)



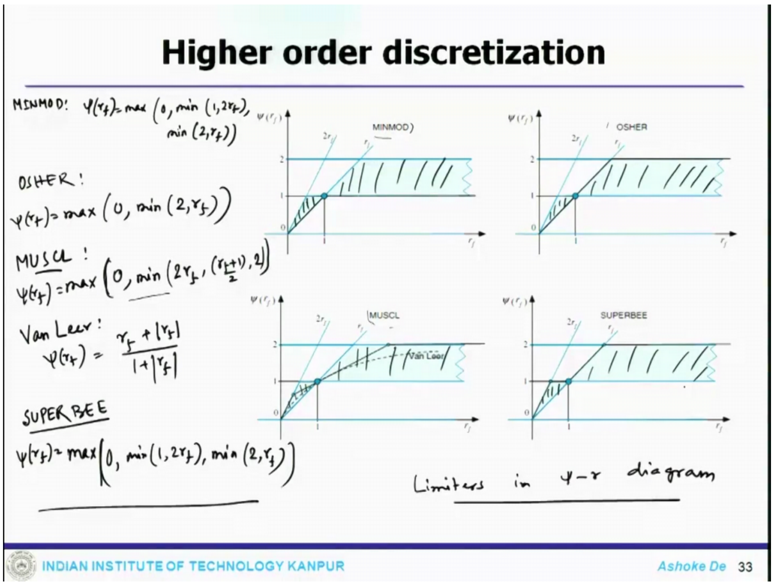
Now, for downwind scheme psi r f equals to 2, and then we see this we can see these things in the psi r f diagram here, which nicely shows the this is the region where all the monotonicity preserving scheme can be found in these particular shaded region. And this is the psi r diagram for higher order scheme and TVD monotonicity preserving scheme.

So, here if you get as we said the upwind scheme will provide you psi r f equals to 0 that is the upwind scheme which lies here. Now, for downwind scheme the psi r f is 2 which is lying there this is the constant line then from scheme the psi r f is half of r f by 2. So, that is what the from scheme second order upwind. So, psi r f is r f, so that pass through the 0 0 and 1 1, then CD where psi r f is 1, that is where it is constant, and then you have QUICK which is psi r f equals to 3 plus r f by 4. So, this is the line for QUICK.

Now, the form scheme is essentially the average of CD and second order upwind scheme and its function all relationship can be presented like phi f equals to half of phi C plus phi D by 2 plus 3 by 2 phi C minus phi U by 2 which is nothing but, phi C plus phi D minus phi U by 4 and phi f equals to phi C plus half psi r f phi D minus phi C which leads to that psi r f equals to 1 plus r f by 2. That is what you get. Now, all these are shown here. So, the limiting psi r f of various schemes are also given, and the schemes which pass through these and lie within the shaded region they will give you different kind of higher order TVD based scheme.

Now, another thing is that many TVD scheme which are there available in the literature the limiters for a number of them, we can see and the functional relationship of their limiters can be also presented.

(Refer Slide Time: 11:18)

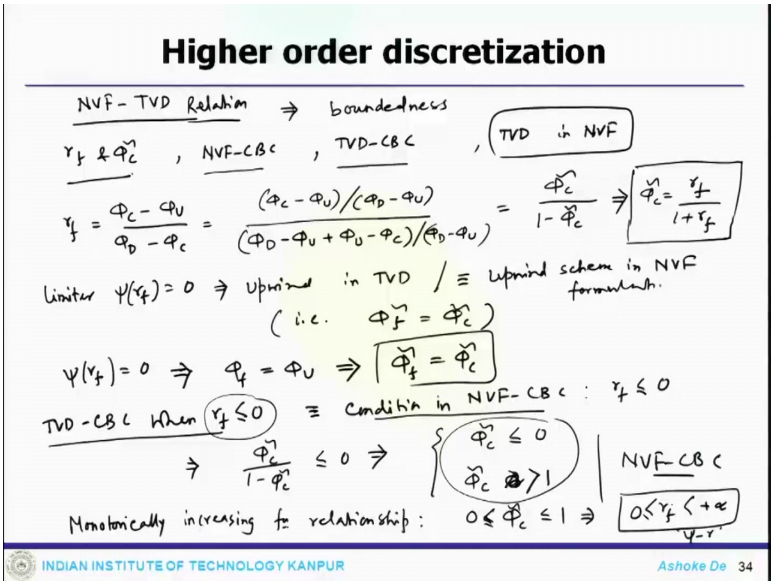


So, here we can see some of these limiters like MINMOD limiter, OSHER limiter, MUSCL limiter. So, these are all shown in the psi r diagram. So, these are limiters in psi r diagram. So, this is what it shows. Now, how one can actually get this functional form for all these. So, for let us say we start with the MINMOD limiter. For MINMOD limiter psi r f is between maximum of 0, minimum of 1 and 2 r f comma minimum of 2 and r f. So, that is how it takes. So, it is the maximum of 1 of these 3 and there either it could be minimum on 1 or 2 r f or minimum of 2 and r f. So, that is where the MINMOD limiter is picked up.

Then similarly OSHER limiter OSHER the psi r f is maximum of 0 minimum of 2 and r f. So, it is taken the 0 and the minimum between 2 and r f. If you look at the MUSCL, so MUSCL it is psi r f equals to max of 0, minimum of 2 r f comma r f plus 1 by 2 comma 2. So, this is between 3 parameters. That is how the MUSCL limiters work and in between that there would be van leer limiter. So, the Van-Leer limiter which is very popular limiter in the high speed flows or the compressible formulations this is used. So, that is Van-Leer. And then SUPERBEE, SUPERBEE which is psi r f equals to max of 0 minimum of 1 2 r f minimum of 2 comma r f. So, this is how one can define this limiters and they are presented in this diagram. So, these are the preserving calculations for this.

So, all these are the shaded regions here these are going to provide that TVDs kind of scheme, this, then here, this, this, so this all these shaded regions are going to provide the TVD based scheme. Now so, once you find out this psi r relationship and one can actually form these higher order TVD based scheme which are going to be monotonicity preserving and we have seen what are the limiters for that.

(Refer Slide Time: 15:58)



Now, the second is that we can find out the NVA and TVD relation which is going to be very important. So, that both NVD, NVF, and TVD formulation what it offers is that the one unique thing is boundedness. So, through different ways so now, one can achieve this relationship by first derive a relation between r f and normalized phi tilde then we can compare between NVF and CBC, with the TVD CBC. And then finally, can find out the general transformation that allows the functional relationship of any TVD schemes to be written in NVF. So, that will allow to write that down. First, we need to find out this then use the information of NVF CBC relationship, along with the information of TVD CBC we can actually find out this information.

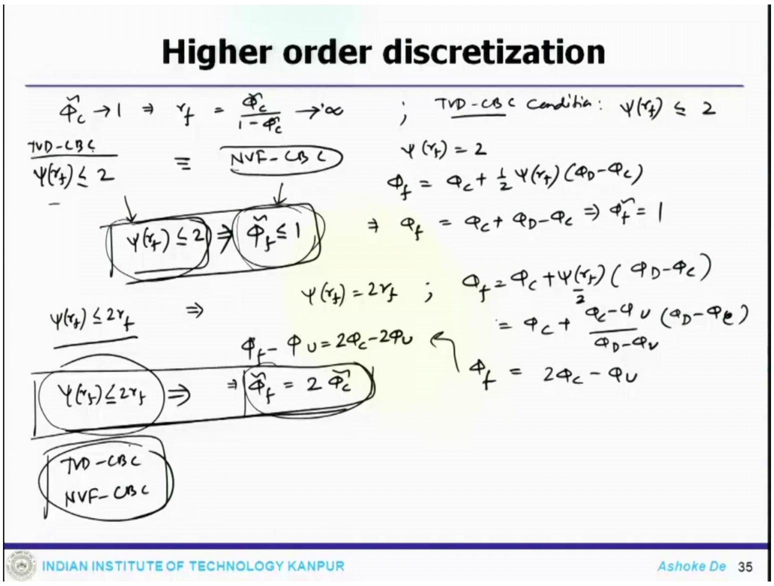
Now, so how we proceed about that? To proceed first we need to find out the relationship between r f and phi C tilde. So, the definition of r f says it is the ratio between 2 consecutive gradients which is phi D minus phi C. Now, one can write that phi C minus phi U divided by phi D minus phi U and denominator you can have phi D phi U plus phi U minus phi c. So, what it does? It just divide, by both numerator and denominator by phi D minus phi U which will cancel out. And also these phi D minus C you add phi U and then subtract the same amount. So, it is a algebraic manipulation which will lead to phi C tilde by 1 minus phi C tilde. So, that essentially provides phi C tilde equals to r f by 1 plus r f. So, that is the relationship that one can obtained between phi C tilde and r f.

Now, using this information of linear scheme one can be compared in to different frameworks. So, the limiter, the limiter psi r f equals to 0 which actually represents the upwind scheme in TVD formulation is also equivalent to the upwind scheme also equivalent to the upwind scheme in NVF formulation. So, that is where that means, this will tell you phi f tilde is equivalent equal to phi C. So, these follows the fact that psi r f equals to 0; that means, phi f equals to phi U which in other way around phi f tilde is phi C tilde.

So, the upwind scheme is imposed as a limit for the TVD-CBC when r f is less than 0 and the similar or equivalent condition in NVF-CBC. So, this is the condition for the TVD-CBC and this is the condition in NVF-CBC is r f less than equals to 0. So, that tells us phi C tilde 1 minus phi C tilde less than 0 that is phi C tilde less than 0 or phi C tilde greater than 1. So, these also represent the condition for imposing upwind scheme in the NVF-CBC.

Now, in addition to that on NVF-CBC the functional relationship has to increase monotonically in the region where, so the monotonically increasing functional relationship has to satisfy these condition phi C tilde lies between 0 to 1. So now, if you recall your psi r diagram this extends the region in psi r diagram. Now, r f goes to plus infinity and it is between 0. So, so this is this is the condition you get in psi r diagram.

(Refer Slide Time: 22:51)



So, now, the phi C tilde tends to 1 which will get you r f equals to phi C tilde by 1 minus phi C tilde which tends to infinity or positive infinity. Now, further we have this TVD-CBC condition which is psi r f less than equals to 2. So, the psi r f less than equals to 2 it is in TVD-CBC this equivalent condition in NVF-CBC would be psi r f equals to 2 which will get you back phi f equals to phi C plus half psi r f into phi D minus phi C; that means, phi f equals to phi C plus phi D minus phi C which means phi f tilde is 1.

So, psi r f less than equals to 2 gets you back in NVD-CBC less than equals to 1. So, whatever the functional the conditions here in the TVD-CBC, equivalent conditions one can find out in NVF-CBC. Now, if this condition is satisfied inside as TVD scheme, this condition needs to be satisfied in NVF conditions. So, the last condition which is imposed on the TVD CBC is that psi r f less than 2 r f, then the equivalent condition here one can find out equals to 2 r f.

Now, phi f equals to phi C plus psi r f by 2 into phi D minus phi C so that will get you phi C, minus phi C by phi U phi D minus phi U multiplied by phi D minus phi C that is 2 phi C minus phi U. Now, that is what you get phi f minus phi U equals to 2 phi C minus 2 phi U, it is an algebraic calculation. And that get you phi f tilde equals to 2 phi C tilde. So, the condition which will satisfied here psi r f less than 2 r f by are equivalent condition is phi f tilde equals to 2 phi C. So, this is slightly more restrictive than the NVF-CBC and is only difference between 2 formulation; now, based on this condition that TVD-CBC and the modified TVD-CBC, and the modified NVF-CBC would look and can be seen in this particular diagram.

So, this is the condition and we have imposed all these conditions of the TVD-CBC one by one, like psi r f less than 2 r f that is one condition in TVD-CBC. So, equivalent condition NVF-CBC is phi f, when the psi r f less than 2 the equivalent condition gets phi f tilde less than 1 and previously, we have got r f less than equals to 0 or r f less than 0. Then, this is what you get or TVD NVF CBC get you this information.

So, there are conditions or equivalent conditions between the TVD-CBC and NVF-CBC. So, you got three different conditions here one is this condition, this is one, then you get the information of this and then you impose TVD-CBC this condition equivalent condition in NVF-CBC, then we have imposed other TVD-CBC condition and equivalent conditions in the NVF-CBC. Now, we can see how these different mapping or one to one mapping can be plotted in the diagram, and that we will see in the next lecture. And, we will stop here today and continue from there.

Thank you.