Introduction to Finite Volume Methods-II
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Lecture - 21
High Resolution Schemes-IV

So, welcome back to the lecture series of Finite Volume and what we are in the middle of discussion is the convection diffusion system and then we started looking at the different discretization scheme for the convective flows or rather convection dominated flows. And we started with simple system, linear upwind, second order upwind, then quick form and then we finally, now doing the discussion on the High Resolution Scheme or higher order accurate scheme. And while in the doing show what we are talking now to mapping between the normalized variable formulation and the TVD.

So, both the system or the approach has its own criteria boundedness criteria which we call it the Convection Bounded Criteria that is CBC. So, what we have stopped in the last class is in that condition. Now will move forward and look at that criteria where we stopped.

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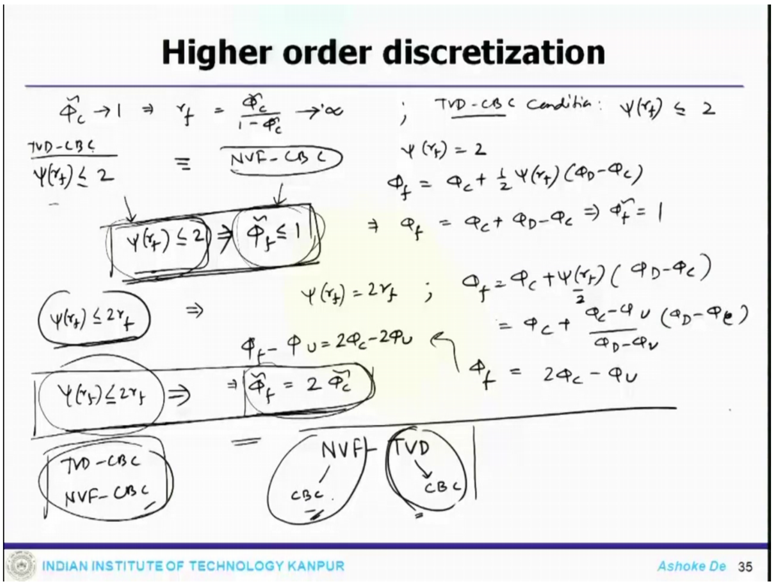

So, what we are doing here? We are if you recall this things, we are trying to find out the mapping between NVF and TVD and both of them are having different boundedness criteria. So, using those two information so, that is what we have obtained like this is for the upwind scheme and this is how the limiter, this things are all discussed just to start with where we stopped so, that we can move forward quickly from here.

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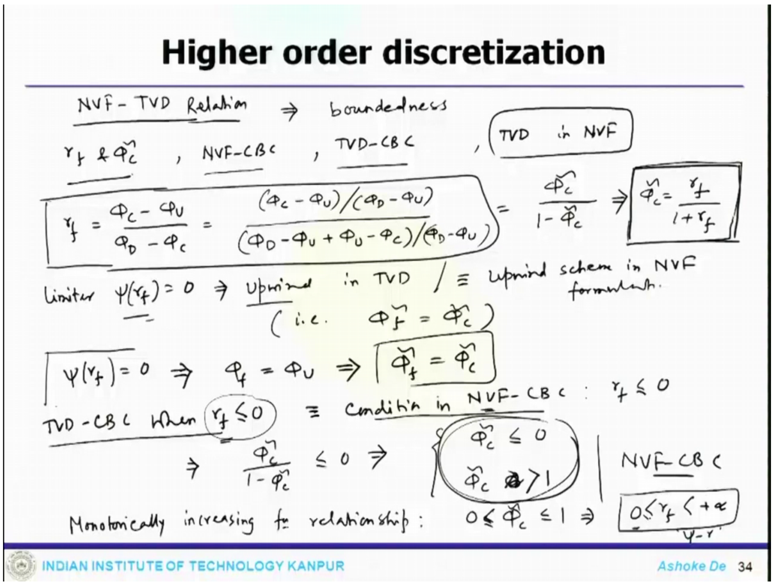

So, we want to find out the relationship where the r f r side diagram in TVD, they look like this. So, this is the expression. Now when you go down to normalize variable formulation and the relationship between TVD and NVF in terms of phi C tilde equals to r f by 1 plus r f.

Now, there are limiters and using that we try to find out the different system. So, one case when r f less than equals to 0. So, similar contribution what would be the condition for the normalized variable formulation and what we found out that this phi C tilde is less than equals to 0 or greater than 0. Then we have another relationship which is essentially the r f 0 to positive infinity and when that happens we find out the second relationship. And the other one which is psi r f less than to lid to the tilde f less than equal to 1 and then psi r f less than 2 r f and the equivalent c in the NVF formulation is this.

So, this is where we actually stopped in the last class. So, now, if you recall and just quickly recollect from the previous discussion, what it is showing that, it is showing the one to one mapping between your NVF formulation and TVD formulation and that is where we are in the business. And what we are trying to find out the restrictions or the boundedness criteria and finally, mapping between these two. So, what it tells you that the NVF-CBC is more restrictive than the, so the this one is the TVD-CBC this criteria is more restrictive than the NVF-CBC and is the only difference between these two formulation.

So, the restriction on this one is more stringent and the other way around and the other this one is slightly lenient. Based on this condition what one can see that TVD-CBC and the modified NVF-CBC would look as we see in this particular picture.

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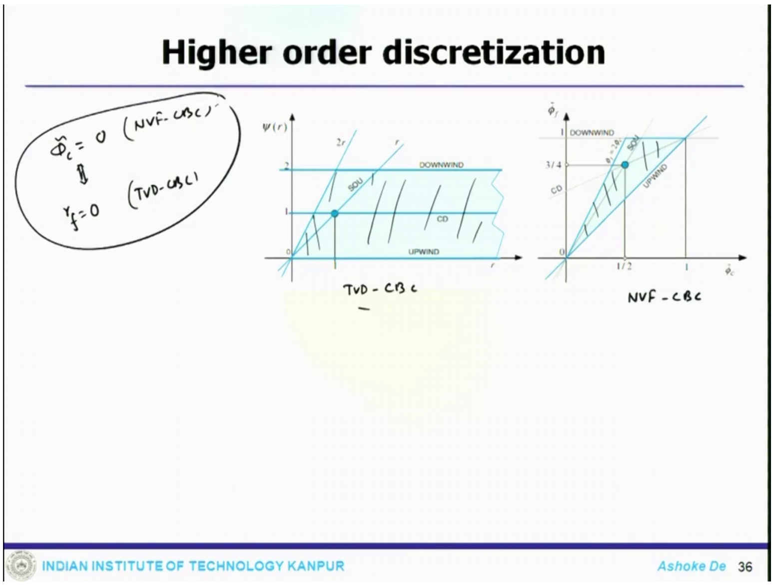

So, this side is the psi r diagram. So, this is my TVD-CBC and this side is the NVF diagram. So, also shows the NVF CBC. Now when you look at that so, monotonicity line is reduced to the upwind and the area where it is between these two. So, that is the line which now going to be the criteria and whereas in this case this is the line which now going to be the region of interest.

Now when you look at the TVD-CBC and the NVF-CBC for condition phi C tilde equals to 0 which is basically in NVF-CBC the corresponding mapping in the TVD-CBC corresponds to r f equals to 0. So, that is TVD-CBC criteria. So, when we see phi C tilde 0 that corresponds to r f 0. So, that also can be seen that when so these criteria would not can be represented by this.

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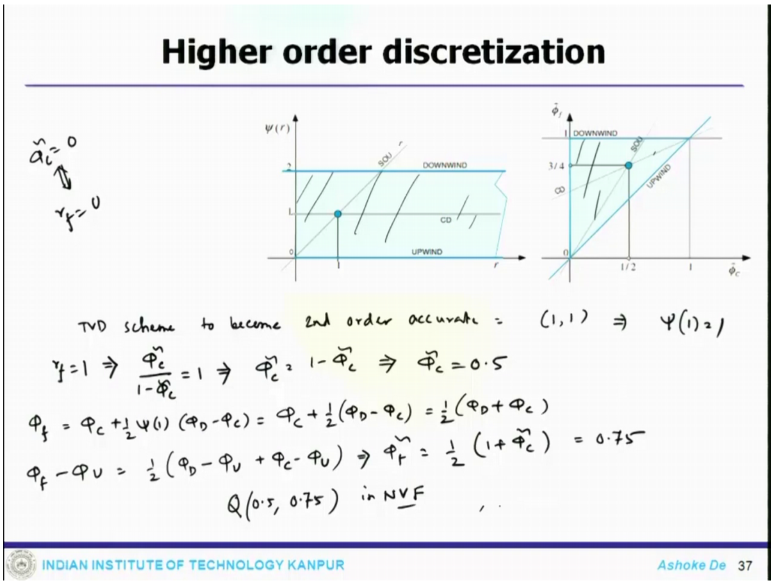

This is the criteria when phi C tilde 0 corresponds to TVD r f equals to 0 and the region will be now bounded like this and this case it will be the up this triangle which will be restricted.

Now, when you talk about the second order accuracy what we have already said that the TVD scheme TVD scheme to become second order accurate, it has to pass through 1 and 1 which corresponds to psi 1 equals to 1. So, this will be the equivalency that one can find out. So, equivalency is that r f equals to 1; that means, phi C tilde plus 1 minus phi C tilde equals to 1 that gets you phi C tilde equals to 1 minus phi C tilde which is phi C tilde equals to 0.5 and similarly phi f equals to phi C plus half psi 1 phi D minus phi C which is nothing, but phi C plus half phi D minus phi C, which is half phi D plus phi C.

Now, if you look at psi f minus phi u is half phi D minus phi U plus phi C minus phi U. So, that gets back phi f tilde equals to 1 plus phi C tilde which is again 0.75. So, if you, now which is the point exactly passing through 0.5 and 0.75 in NVF so, as we are saying that the C D and second order upwind could be the.

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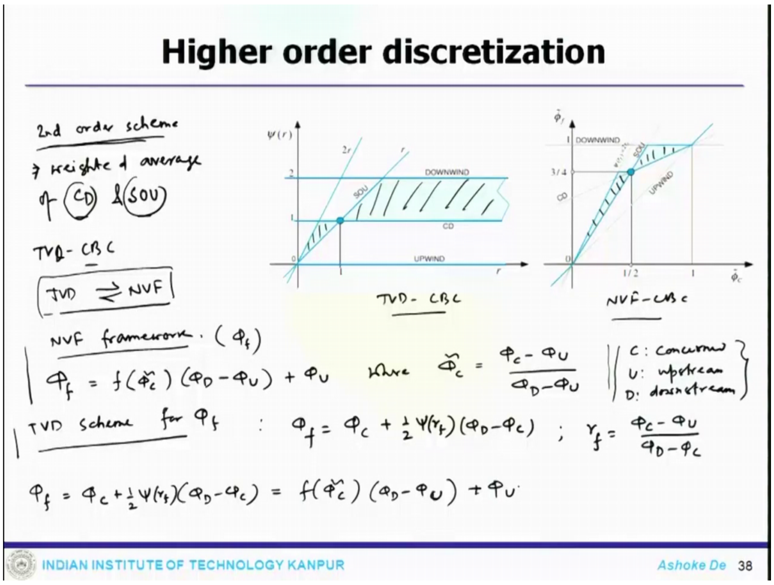

So, which you can see that that the second order scheme can be written as a any second order scheme which can be written as weighted average of C D and SOU, which is central difference and second order upwind scheme. So, the functional relationship should lie between the functional relationship of C D and SOU which in the TVD-CBC TVD-CBC the monotonicity region is now reduce to the upwind line and the area which is shown here which, now reduced to this.

So, this is the condition when one second order scheme can be obtained. And now similar line which you get this is your TVD-CBC diagram, this is your NVF-CBC diagram and the correspondings diagram would become here in this zone.

So, you see how the mapping between these two formulation can be done and the according restriction would turn up in your discretization and then what kind of, what order of a accurate scheme one has to device that will dictate the operational resume where it can be guaranteed that this guy would be the second order accurate. Now this procedure can be generalized for any TVD scheme which is having an equivalent NVF scheme or rather vice versa.

So, TVD and NVF this can be this two formulation can be generalized for those set of schemes which has mapping between the two. So, in doing that so let us start with the NVF framework. So, if you start with the NVF framework and how it goes. So, the here the important point is that to calculate the flux at face f. So, if you want to do that the flux at face f, it would be f phi C tilde phi D minus phi U plus phi U where your phi C tilde is phi C minus phi u divided by phi D minus phi U. Again just to give you quickly C is the cell which is concerned, U is the upstream, D is the downstream.

So, this notation is based on the local flow field direction. The direction where the flow field is acting this will determine which cell to be U and which one is to be D. Now when you this is what you get for the NVF for framework; now when you go down to TVD scheme for finding this phi f, then you get that phi f equals to phi C plus half psi r f into phi D minus phi C where r f is the ratio between two consecutive fluxes. So, this is phi C minus phi U and phi D minus phi C. Now if you equate this two the TVD and NVF equate this two, then what you get? You get phi f equals to phi C plus psi r f into phi D minus phi C equals to f tilde phi D minus phi U plus phi U. Now that is what one can get.

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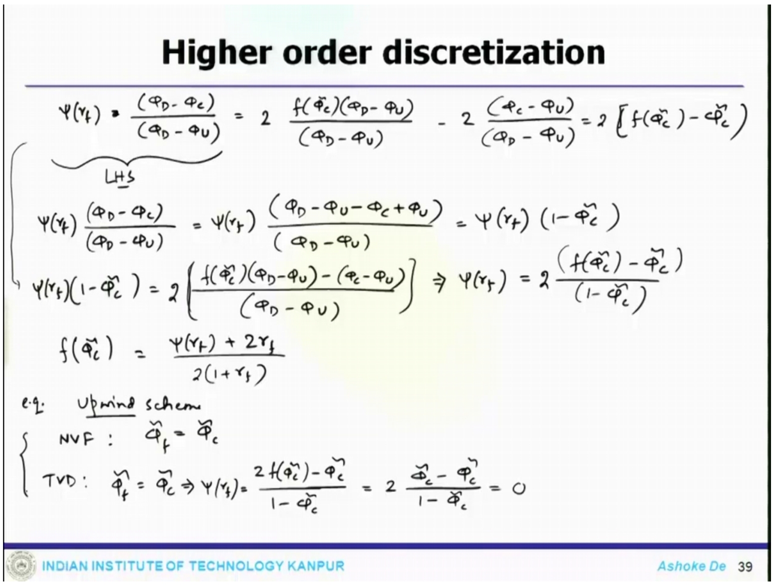

So, from here if you calculate back the psi r f is psi r f into phi D minus phi C divided by phi D minus phi U which is going to be two times f phi C phi D minus phi U phi D minus phi U minus 2 phi C minus phi U phi D minus phi U which is 2 times f phi C tilde minus phi C tilde.

So, the term which is sitting on the left hand side this guy, this is the left hand side. Now this can be now also modified like one can write psi r f phi D minus phi C divided by phi D minus phi U which is going to be psi r f. Now you do some algebraic manipulation here phi U minus phi C plus phi U. So, you add phi U and substrate the phi U, then you get this which one can write psi r f 1 minus phi C tilde. Now what you get is that psi r f into 1 minus phi C tilde equals to 2 times f phi C tilde phi D minus phi U minus phi C minus phi U divided by phi D minus phi U.

So, that is what you get from this particular one, just collect the terms together which is going to give psi r f equals to 2 times f phi C tilde minus phi C tilde divided by 1 minus phi C. So, this is what you can write. Now this one, one can also write phi C tilde equals to psi r f plus 2 r f 1 into 1 plus r f. It is just from here one can right.

Now, the one can see by taking some example for example, you take upwind scheme upwind scheme. Now upwind scheme framework upwind scheme in NVF framework, it provides phi f tilde is phi C tilde. Now its TVD limiter is found like phi C tilde where psi r f equals to 2 f phi C tilde minus phi C tilde 1 minus phi C tilde. So, that is 2 phi C tilde minus phi C tilde by 1 minus phi C tilde which is 0. So, the upwind scheme, you can find out the mapping between the TVD and the NVF.

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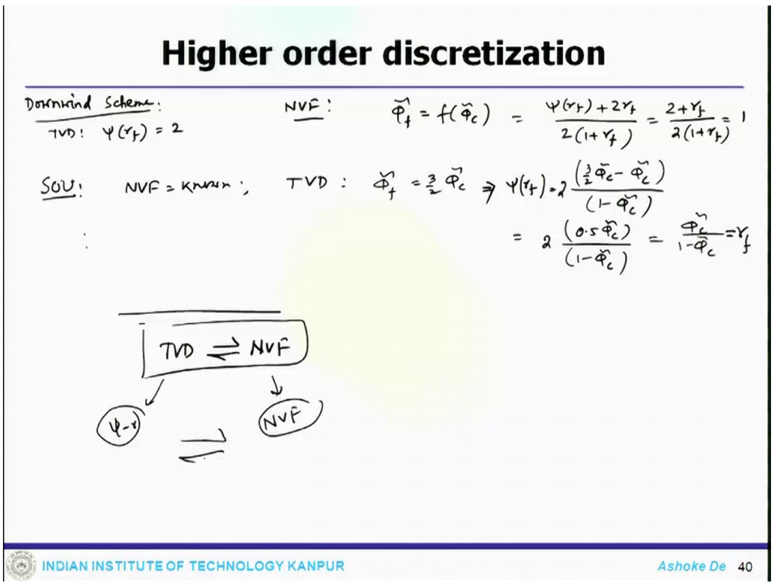

Similarly you can get that for the downwind scheme. If you consider the downwind scheme, then what you get is also downwind scheme. The TVD limiter for that is psi r f equals to 2. So, now, NVF this will be corresponds to phi f equals to f phi C tilde which is psi r f plus 2 r f two into 1 plus r f which is nothing, but 2 plus r f by 2 into 1 plus r f which is 1. Now if you know similarly for second order upwind scheme, if you know the NVF if it is known then one can find the TVD limiter like it is phi f tilde is 3 by 2 phi C that leads to psi r f equals to 2 times 3 by 2 phi C minus phi C 1 minus phi C that is 2 times 0.5 time phi C tilde divided by 1 minus phi C tilde. It is nothing, but phi C tilde by 1 minus phi C tilde nothing, but r f.

So, similarly one can find for other scheme. So, if you know the mapping of TVD, you can find out the mapping of the NVF or the other way round if you find out the NVF, then you can find out the TVD. So, this is the functional relationship mapping between TVD and NVF. So, that is provides a good platform for obtaining and once you look at in the TVD diagram of psi r diagram, the restrictive limit or the boundedness and also the NVF diagram. So, this boundedness will also map 1 to1 accordingly and that has a big role to play while your device in high resolution scheme. Now, we can extend our discussion for the higher order scheme.

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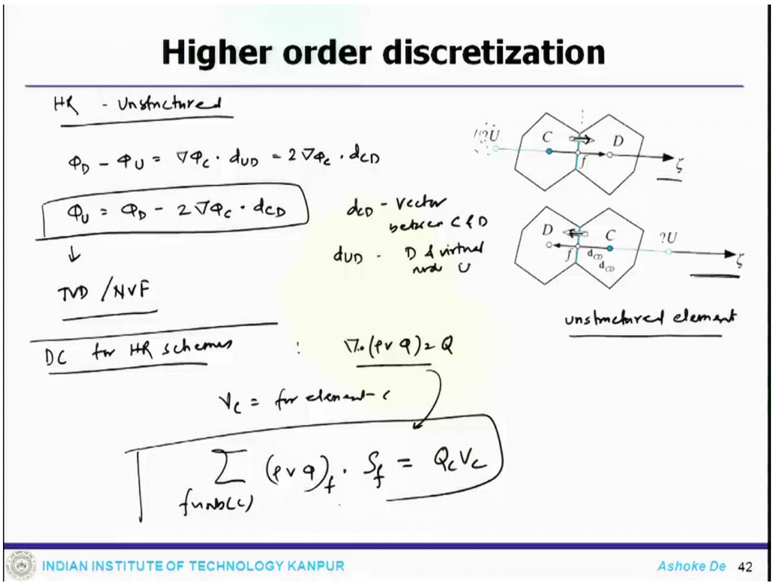

Higher order scheme for the unstructured system. So, this is an unstructured element arrangement. So, one can see that the unstructured elements. So, the cell which we are concerned of the element C and at the face, the local velocity is in this direction. So, the D would be the downstream node and the upstream would be U and then the other way if the local velocity is in this direction and then this should be the D this is C and then there would be a upstream node. Now the important thing that will happen now, you need to transform them to some sort of a co ordinates xi eta kind of coordinates system. Now also what we have already seen that the issue with the unstructured formulation. So, those things also one has to take into account.

Now the point is that so one can very easily assume that U this upstream node can lie on the same line and then one can find out a method for the higher order things. So, if you assume that, that this upstream node lies on the same line which connects this C and D then what one can write that phi D minus phi U equals to delta phi C dot d U D which is 2 delta phi C dot d C D.

So, from here one can calculate phi U equals to phi D minus 2 delta phi C dot C D. So, you can find out that now d C D; d C D is the vector between C and D and d U D is the vector between D and virtual node U. So, as we have said that the U is constructed such that C is taken to the centre of the U and D segment. So, this U is constructed in such a way that it is equidistant from the centre C or it will be on that segment. Now that value with the value of phi U computed the use of either NVF or TVD approach can be framed. So, once you get this, then use the either TVD or NVF formulation similarly like that we did for the structure grid also can be applied to this unstructured grid and then find out the equivalent framework for the.

Now, before moving that so, one important thing to talk about is the default collections for high resolution scheme; so, what is that? Now the implementation of this schemes can only be understood through some sort of an example. Now, one can take some a multidimensional advection equation like del dot rho V phi equals to Q with some sort of a source term. Again this is a conservation equation. Now, what you can write you can have a element volume for V C for element C and then you can apply to this divergence theorem. So, what can be depicted here?

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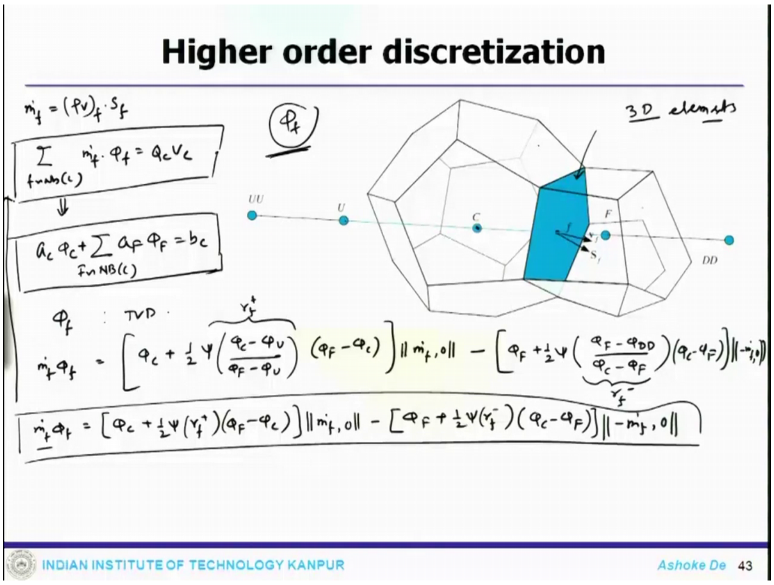

This is the C for multidimensional system. So, it is a 3-D elements in a unstructured grid system. This is the face or the plane which is sort of connecting between elements C and F and this is just C element, U is the upstream of that and the weight is that they lie on this connecting vector of C and F. So, if we take the divergence of these I mean basically and after that surface integral, this equation turns out to be n b C rho V phi at f S f equals to QC VC. So, that is our standard equation so with source term.

Now, mass flow rate at the face m dot f is given as rho V f dot S f and then one can write the equation nbC m dot f phi f QC VC. So, that is the semi discretized form. Now the point is that the value of phi f is obtained by using some sort of an advections scheme discretization that we have discussed already and then the whole system will completely becomes, then discretized system or the algebraic system in a form a C phi C plus capital F NB C a F phi F equals to b C. Since it has some source term the b C is not going to be 0 anymore.

Now, again the difficulty lies in the instability who is arises due to calculation of phi f in terms of nodal value because this is a unstructured grid system. So, when one try to find out this phi f using the nodal value that is where some sort of an difficulties, one can face. Now what kind of difficulties one can face? So, let us say when you try to explicitly derive phi f using the neighboring values and use the TVD framework.

So, if that is the case, then my m dot f phi f, one can write that phi C plus half psi which is phi C minus phi U by phi F minus phi U which is nothing, but your this is my r f into phi F minus phi C multiplied with m dot f 0 minus. Similarly phi f plus half psi, there it would be phi f minus phi D D by phi C minus phi F.

This is again the r f minus and this is the r f plus multiplied with phi C minus phi F multiplied with minus m dot f 0 (Refer Time: 30:16). From there you can get phi C plus half psi r f plus phi F minus phi C multiplied with m dot f 0 minus phi F plus half psi r f minus multiplied with phi C minus phi F minus m dot f 0. So, now, if you substitute this one this case m dot f phi f this guy what we have got into this equation [noise]. So, we will stop here and we will continue the discussion in the next lecture.

Thank you.