Introduction to Finite Volume Methods-II
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Lecture – 22
High Resolution Schemes-V

So, welcome to the lecture of this Finite Volume Method and where we will continue our discussion where we left in the last lecture.

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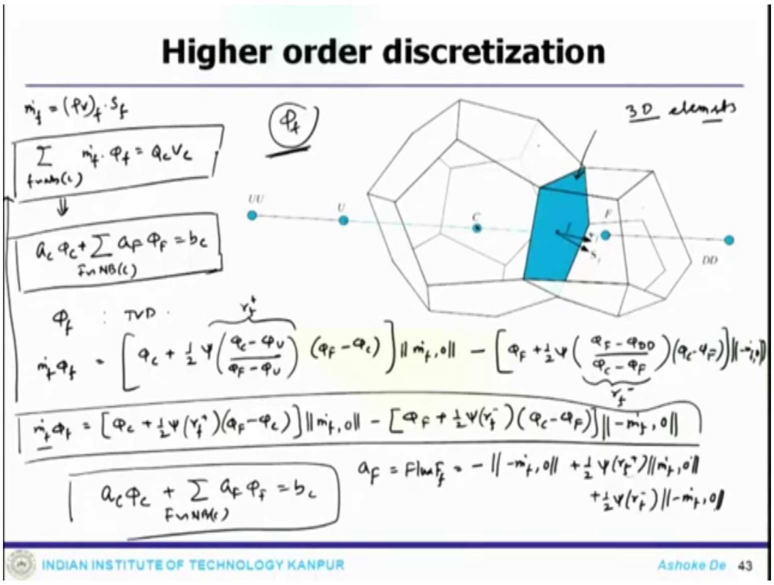

So, what you get essentially combining that to you get the discretized form as a c phi c plus summation of a F phi F equals to b c, where your a F is the flux F f which is minus m dot f 0 plus half psi r f plus m dot f 0 plus half psi r f minus minus m dot f 0.

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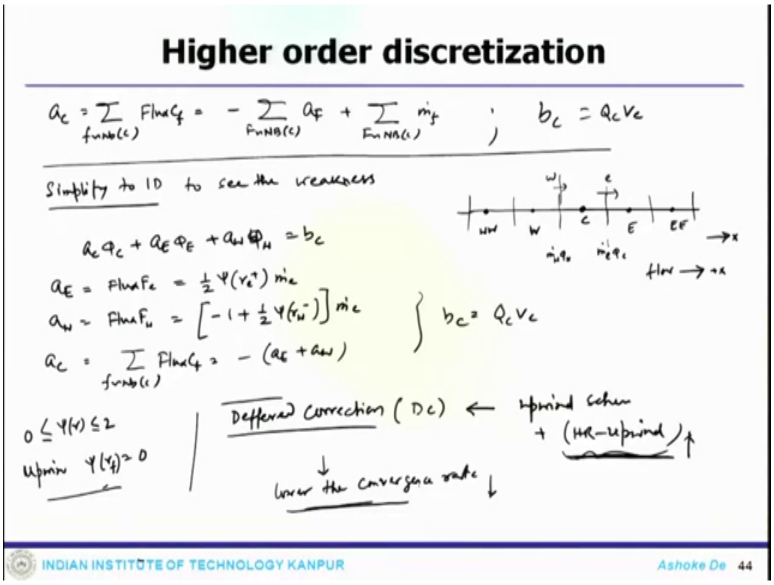

Now, a c would be summation of F nbc which is flux C f, minus F NB C a F plus F NB C m dot f and b c is Q c V c. Now to see the weakness in this formulation so, what one can do, we can this is a multidimensional discretised system, now to see the weakness one can actually simplify to problem to a 1 dimensional system.

And then we can try to look at that what happens when so, if you brings down to or simplify to 1 D to see the weakness so, the stencil would look (Refer Time: 2:56) so, this my c so that is the cell faces this would be E E E W WW and the face this is the east west.

So, here it would be m dot e phi e m dot w phi w. So, if you come down to that and assume the flow to be in this direction positive direction positive x, then this equation becomes a C phi C a E phi E a W phi W b C. So, where you can see a E is flux, F e which is half of psi r e plus m dot e, a W is flux F w which is minus 1 plus half psi r W minus multiplied with m dot e and a C is summation over all these faces which is nothing, but a E plus a W and; obviously, your b C is Q C v C, now what you can see now the range of psi r is 0 to less than 2.

So, the a and a W coefficients will be opposite signs except for the upwind scheme where upwind scheme where your psi r f is 0. So, that is violating one of the basic rules for stability and causing convergence difficulties of the iterative procedure. So, approach if you adopt in the n v f formulation that will lead to the same short comings or the similar observation. Now, what is the remedy? So, the remedy is that which is kind of one can do is that deferred correction. So, this is similar to that we have discussed earlier so, for the higher order scheme also one is to do some sort of a deferred corrections.

So, in which the coefficients will be based on upwind scheme, while the difference between HR and upwind scheme would be so, HR and upwind scheme the difference would be added as a source term. So, this would be added as a source term in the discretized equation and the formulation the coefficient should be based on the upwind scheme.

Now, this defect correction procedure is quite simple as we have discussed to implement, because can be used in face values calculated with upwind scheme and that calculated with the HR scheme. Now, the another issue is that it is easier to implement, but it at the same time it also lower the convergences rate. So, the convergences rate of the iterative solvers also get is affected. So, that effect can be easily estimated on an NVD formulation of the difference between the upwind line and that of the chosen for HR scheme in the NBF formulation. So, if that difference is the HR high resolution scheme and upwind scheme it becomes larger, then as it goes up the convergence also goes down.

So, if as long as this difference is not too much, the convergence will also not be that bad or would be affected. So, that has also kind of opened up an area, where scientists and engineers or other researcher, they have put lot of effort to find out a scheme which will be free from this kind of issues. So, that gives us to discuss about two different things, which was developed on based on this technique to overcome the reduction in the convergences, which is associated with deferred corrections.

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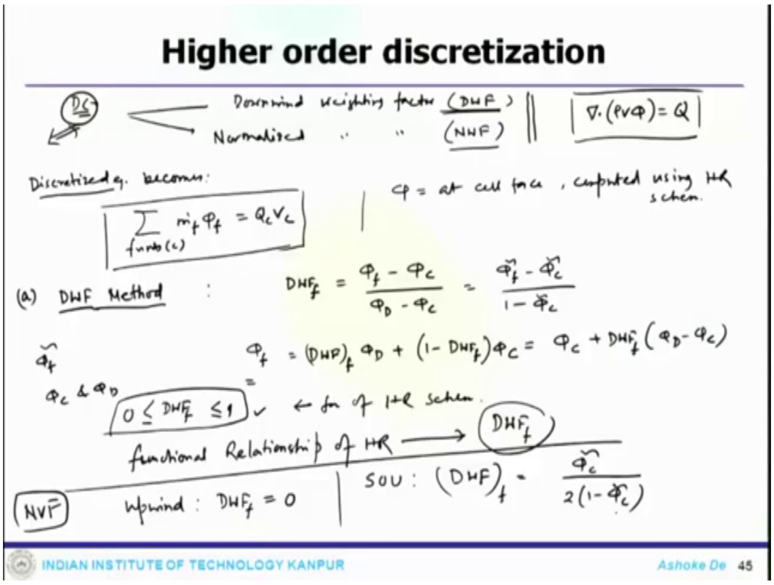

So, there are two approach one can adopt, one is the downwind waiting factor that is DWF method, or normalized waiting factor NWF. So, to avoid the problems which are associated with the deferred corrections one just to improve the convergence and other stability limits. So, there are two methods one can adopt, one is based on the downwind waiting factor called DWF method, another is normalized waiting factor which is known as the NWF. Now, the implementations with respect to the equation system which will be del dot rho v phi equals to Q.

So, the convection with source term. So, we will do the implementation based on that, and now once you start with that equation and your discretized equation becomes like summation of a f NB c m dot f phi f Q C V C. So, this phi values are at cell faces and computed using HR scheme. So, the idea is that one can implement this idea in more effectively in the calculation. So, that is the whole idea now to do that that is our discretized system which we are dealing with now we will first look at approach a that is downwind waiting factor method or DWF method. So, using in that method so the DWF factor which could be defined as phi f minus phi C divided by phi D minus phi c which is phi f minus phi c divided by 1 minus phi c.

So, you can rewrite this with the face value like phi f equals to DWF factor multiplied with phi D plus 1 minus DWF f into phi c which is phi C plus DWF factor multiplied with phi D minus phi c. Now, which actually re distribute the HR scheme estimate to phi f or the normalised value of phi f tilde, between upwind and downward nodes ok. So, the effect is reduced stencil for the discretised coefficients.

Now, if the phi f this value is computed using HR scheme, which lies between phi C and phi D the value of these DWF factor is always between 0 to 1. So, that is the restriction that you get. Now, instead of computing the DWF directly or explicitly which uses the computed value of phi f the DWF factor can be expressed directly from the functional relationship of HR scheme.

So, one can use the functional relationship, which we have already established of HR scheme to find out this DWF factor. So, instead of doing that or instead of calculating that using the calculated or computed face value one can do that.

Now, also and how one can do that one can look out look for this different functional values in, now we stick to in the normalised value formulation here, now look at first upwind scheme. If you look at a upwind scheme the DWF factor would be 0, if you go to second order upwind scheme the DWF factor would be represented at phi c tilde minus 2 into 1 minus phi c tilde.

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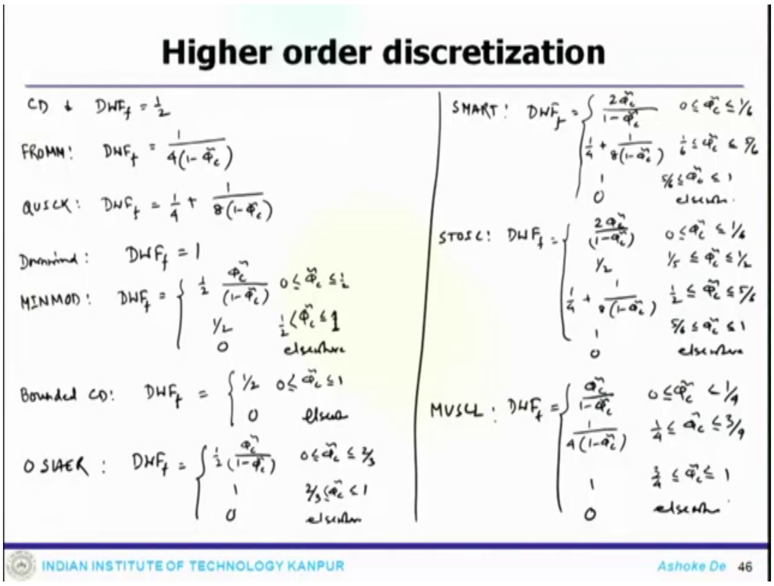

So, similarly like CD 1 can get this DWF factor is half, if you have from then this DWF factor is 1 by 4 into 1 minus phi c tilde, quick this DWF factor is 1 by 4 plus 1 by 8 minus phi c. Now, DWF factor for downwind scheme would be 1, if you use mean mode then DWF factor will have 3 different segment like half of phi c tilde 1 minus phi c tilde that is for phi c lies between half and 0.

Once phi c lies between half and 1 that is half another is 0 elsewhere. Now, bounded CD which is DWF is half and or 0, it is elsewhere where phi c tilde lies between 1 and 0.

Now, similarly one can write for OSIAER where DWF will have 3 segment 1 is half phi c tilde 1 minus phi c tilde which is phi c tilde less than 2 by 3 0 1 0, where to buy 3 1 and elsewhere, then you get smart which is DWF factor is in 4 different segment 1 is 2 5 tilde 1 minus 5 tilde which is the case, when phi c tilde lies between 1 by 6 and 0, then it is 1 by 4 plus 1 divided 8 to 1 minus phi c tilde. So, that is the phi by 6, then 1 0 when it is lies between 5 by 6 and elsewhere.

Similarly, stoic which will also have 4 segment for this factor first one is 2 phi c minus 1 by phi c tilde, where phi c tilde is 1 by 6 0, it is half which is 1 by 5 half 1 by 4 plus 1 by 8 into 1 minus phi c tilde which is 5 by 6 1 and 0. So, that is one and elsewhere and the muscle scheme, where the factorial have 4 segment again. First segment is phi c tilde by one minus phi c tilde, which is lies between 1 by 4 and 0, then 1 by 4 into 1 minus phi c tilde, where it is lies between 1 by 4 and 3 by 4 and lastly, where it is 1 elsewhere this is 1 this is 0.

So, that what one can see the different representation of this DW factor, when you look at now this is in NB a formulation, because it is based on phi c.

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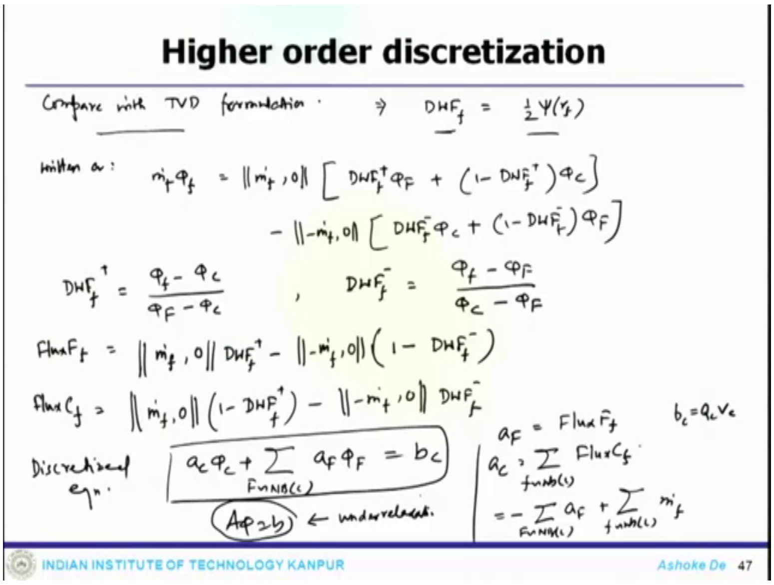

Now, if you comparing the TVD formulation. So, if you compare with TVD formulation so, what happened that this DWF factor would be half of psi r f. Now, the coefficients option from this implementation would be diagonally dominant, if you use this downwind factor based on the TVD framework, they will become diagonally dominant and that is why this should be quite stable.

Now, for the completeness of the sake of completeness the analysis of the implementation, DWF factor can be written as m dot f phi f equals to m dot f 0 DWF f plus phi F plus 1 minus DWF f plus phi c minus m dot f 0 DWF f minus phi c plus 1 DWF minus f phi F, with DWF plus equals phi f minus phi c phi F minus phi C and DWF f minus which is just like your r f and r f minus, where the ratio between the consecutive fluxes this is f minus if F divide by phi C minus phi F.

So, now the flux coefficient can be written as my flux F f is m dot f 0 DWF plus minus m dot f 0 DWF minus, it is 1 minus DWF f minus and flux C f which is going to be m dot f 0 1 minus DWF plus minus m dot f 0 DWF minus.

And the discretized form of the equation, discretize equation would be a c phi c plus 8 F n BC a F phi F equals to b c, where again your a F is flux F f a c is summation over faces flux C f which is minus capital F with a F plus summation of small f with m dot f and your b c is Q c V c.

Now, if you look at this coefficients here they can actually end up giving you an highly unstable system of equation. So, this linear system which will get back here, this will be quite unstable and it requires quite a bit of under relaxation under relaxation to get the solution done. Now, this you can see or one can demonstrate by looking at a quick.

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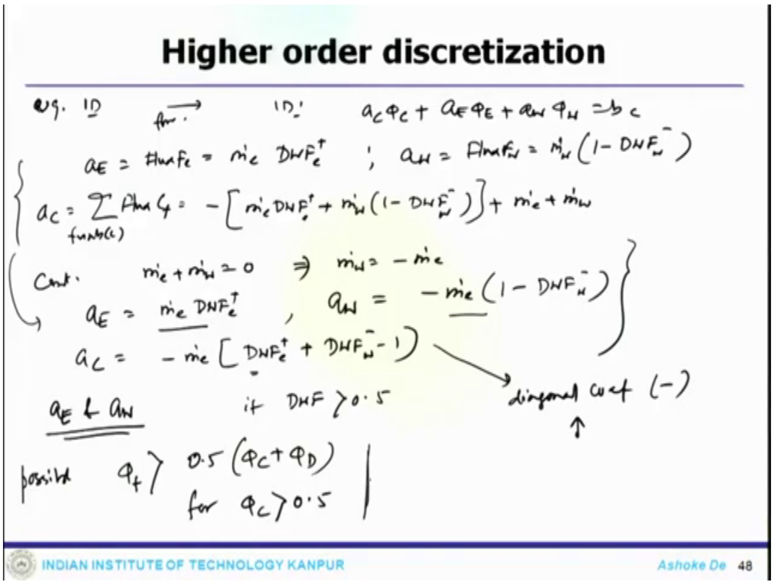

For example 1 D example, now without loss of generality if you assume this is the positive flow direction then the 1 D system would become a C phi C plus a E phi E plus a W phi W equals to b C, where your a E is flux F e which is m dot e DWF plus e, then your a W is flux F w, which is m dot w 1 minus DWF minus F w and a c is summation over faces. So, which is m dot e DWF plus e plus m dot w 1 minus DWF minus w plus m dot e plus m dot w.

So, the continuity equation provides you m dot e plus m dot w equals to 0, in other words m dot w equals to minus m dot e. So, this coefficients now they will become a E equals to m dot e DWF plus e a W will become minus m dot e 1 minus DWF minus w and a c which will become minus m dot e DWF plus e plus minus w minus 1. So, that is the simplified system for 1 D where you get this.

Now, here if you look at this a E and a W they are of opposite sign, this is positive and this is negative, this is a serious violation to one of the basic coefficient rules. Moreover the values of DWF factor that are larger than 0.5. So, if DWF factor which is greater than point the diagonal coefficient of a c becomes negative. So, this the diagonal coefficients will be negative in sign and which will actually can be a problem to solve through the iterative process. So, these would occur when your phi so, this is possible when phi f greater than 0.5 of phi C plus phi D, which is a situation common to all high resolution scheme for phi C greater than 0.5.

In fact, that DWF moves much of the HR flux influence on to the downwind value causing this problem. And the situation can be resemble in effect that central defence scheme. So, it can be some sort of recovery can be made using that kind of things. So, now, in the next lecture we will look at the other formulation. So, we will stop here and we will continue the discussion in the next lecture.

Thank you.