Introduction to Finite Volume Methods-II
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Lecture – 23
High Resolution Schemes-VI

So welcome back to the lecture series of Final Volume and as we recall, we are just looking at the default correction approach using two different method. One is the downwind waiting factor method which we have discussed and we have seen the shortcomings of that method when from a multidimensional system to one dimension system when we go we can easily or could easily see what is the problem with that approach and there could be some remedies. And now what we are going to discuss in this particular lecture is the NWF which is the Normalised Waiting Factor.

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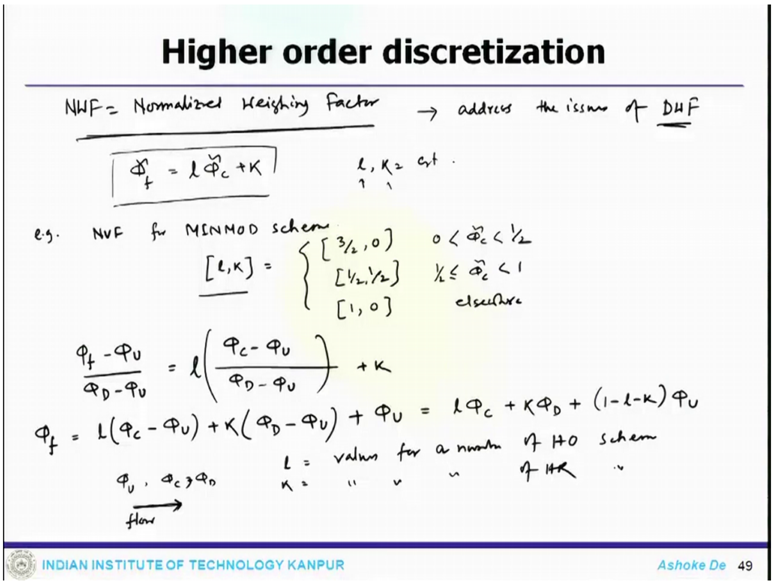

So, that is what we are going to talk on in this lecture. Now we have seen that if issues with the DWF. Now we will similarly formulate the NWF and here in this case the one would try to so this one actually try to address some of the address the issues of DWF method so, that we can have in better platform or framework.

So, it operates by linearizing the normalised interpolation profile such that phi f tilde would be looking like l phi C tilde plus K. So, that sends sort of a linear region and l and K these are on constant that represents the slope; l represents the slope, K represents the intercept of the linear function within any interval of phi f with the number of interval depending on the HR scheme which is being used. So, this is an exact representation for nearly an all HR scheme that we have already discussed earlier.

Now, for example, in normalised variable formulation for mean mode scheme, if you look at that the values of l and K that we have obtained there is like this 3 by 2 0 for range where phi tilde goes from half to 0, then half half where phi C actually goes between half and then 1 0 which is elsewhere. So, these are the values of l and K which we have already derived while looking at the HR scheme.

Now second step is to rewrite this ratio phi f minus phi U divided by phi D minus phi U that ratio one can write l multiplied by this factor and then plus K. So, that is what one can do which can be transformed to a system where one can write the phase value phi f equals to l into phi C minus phi U plus K into phi D minus phi U plus phi U which is l phi C plus K phi D plus 1 minus l minus K phi U.

So, where phi C is the cell constant, phi D is the downstream node, phi U is the upstream node along the flow direction. So, this will be the flow direction. So, that is how the representation of the nodes which are being used and this is pretty much consistent throughout our lectures what we have been following with the notation system. Now l and K these are the values l n for a number of higher order scheme and this is the values for a number of HR scheme. So, which we can see for different kind of scheme what are the values.

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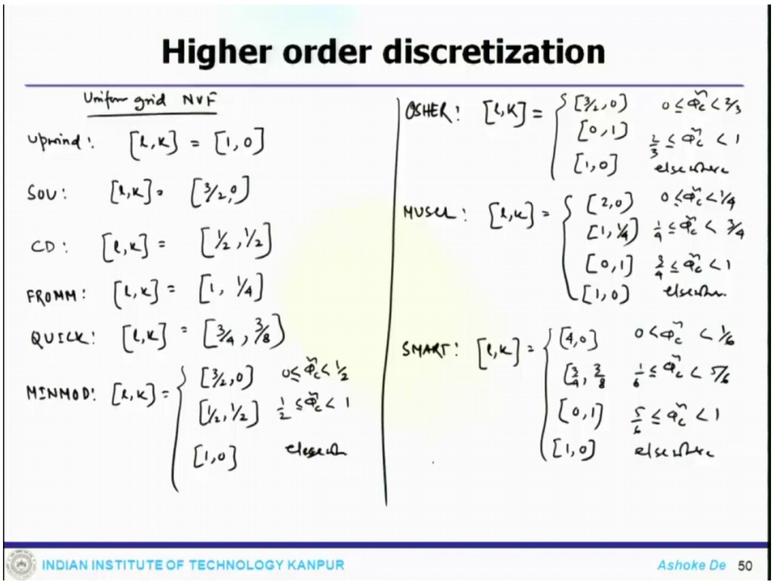

So, let us say this is again in uniform grid with NVF formulation, then what one can see for upwind l and K. This will have values like 1 and 0. Now second order upwind which will have l and K is 3 by 2 0, central different scheme this value would be half half. FROMM so, this should be 1 one-fourth. Now when you look at quick the value would be 3 by 4 3 by 8. Now we have already seen for mean mode, the value has an different segment one is 3 by 2 0 it could have half half, it could be 1 0. This is the range where phi C less than half greater than 0 phi C less than 1 greater than half and else square.

So, similarly for OSHER the l f value l K value will have four again three segment. One is 3 by 2 0 0 1 1 0 here the phi C goes between 0 phi C tilde goes between 1 to 2 by 3 this is elsewhere. MUSCL scheme it will have a segment of 4. So, 2 0 1 one-fourth 0 1 1 0, this is phi C will lies between 0 to one-fourth when one-fourth to three-fourth, three-fourth to 1 elsewhere. And similarly the SMART scheme where this will also have segment of 4, 4 0 which will get you 0 3 by 4 by 3 by 8 0 1 1 0 this is 1 by 6 to 5 by 6 5 by 6 to 1; this is elsewhere.

So, one can see that different l y K value can be obtained on a uniform. Now for unstructured grid this upstream node or U is virtual so, the 10 term involving phi U is treated as a treated in deferred correction fashion, but the resulting source term which would be for unstructured.

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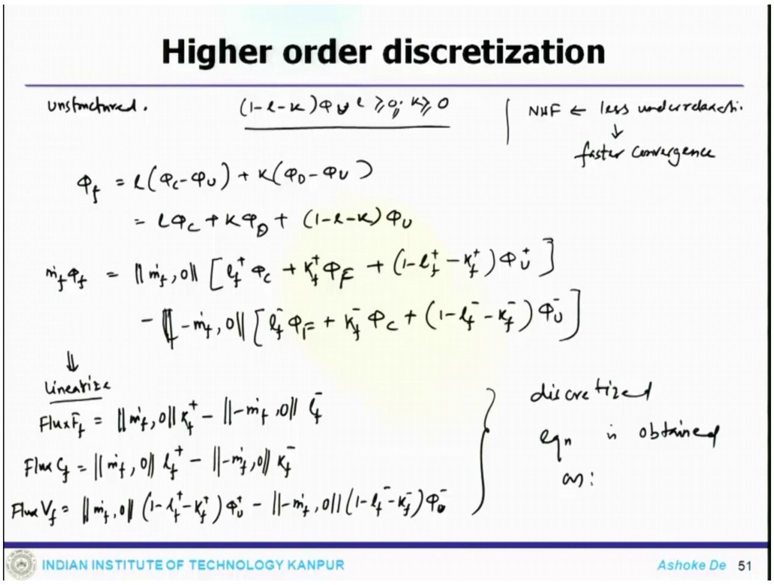

The resulting source term which will be in form of 1 minus l minus K phi U is smaller than 1 would be obtained with the standard deferred corrections. So, so, that is what it would give in your unstructured system. So, the NWF requires less under relaxation than the standard deferred correction method. So obviously, if you required less correction NWF requires less under relaxation which will lead to faster convergence. So, if you have that now we will start from our equation which we are dealing with this phi f. So, the phi f that we are having is l phi C minus phi U plus K phi D minus phi U which is finally, written are l phi C K phi D 1 minus l minus K phi U.

So, we start with that for a general case multidimensional grid m dot f phi f can be written as m dot f 0 l f plus phi C plus K f plus phi f then 1 minus l f plus minus K plus phi U plus minus m dot f 0 l f minus phi f plus K f minus phi C 1 minus l f minus K minus phi U minus so, this plus and minus goes in the. So, if you linearized if you linear rise, then you get flux effect m dot f K plus minus m dot f l f minus flux C f m dot f l f plus minus m dot f K f minus and flux b f m dot f 0 1 minus l f plus K plus phi U plus minus m dot f 0 1 f l f minus K f minus phi U minus. Then once you substitute all this things the discretized equation is obtained as a C phi C plus F n b C a F phi F equals to b c.

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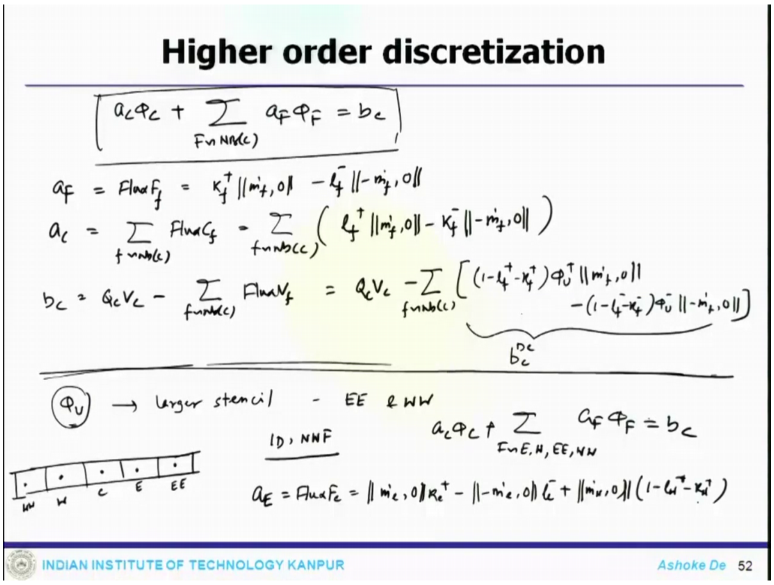

So, system looks similar, but the differences would come in the coefficients like here it would be flux F f which is K f plus m dot f 0 minus l f minus m dot f 0 a C which is flux C f n b C l f plus m dot f 0 minus m dot f 0. And your b C is Q C V C minus summation of this fluxes flux V f which is going to be Q C V C n a b C 1 minus l f plus minus K f plus phi U plus multiplied with m dot f 0 minus minus 1 minus l f minus K f minus phi U minus minus m dot f 0. So, this is the term which is due to deferred correction or the source term due to deferred corrections. So, that is how in your NWF formulation the discretised variable would look like and they would appear in the discretized system.

Now, this 1 was initially developed. There is a small thing which is associated with this one. This was initially developed for unstructured grid and with is formulation which allows a full implicit treatment of the HR scheme. But what is happening the full implicitness of the method on unstructured grid of phi U which is being an actual node in the computational domain that can be resolved in the algebraic equation. So, that has now a larger stencil that includes the for notes EE and WW. Now for the one dimensional system like what we have done for the - D system C E W E E W W. For 1 D system, this again we can simplify this particular set of multidimensional formulation to 1D; 1D NWF formulation and their it will be a C still phi C plus F goes from E west EE WW a F phi F equals to b C. So, where your a E would become flux F e which is m dot e K e plus minus m dot e l e minus plus m dot w 1 minus l w plu K w plus.

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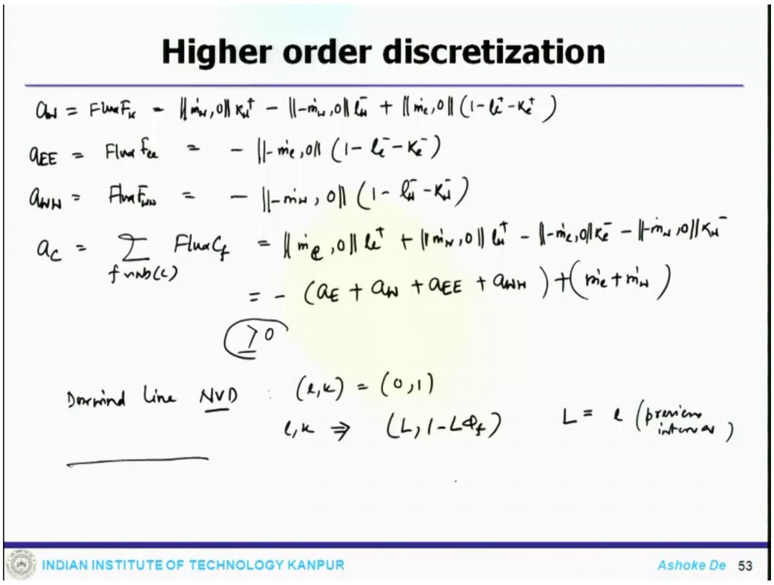

Similarly, the a w is flux F w which is m w 0 multiplied with K w plus minus m w 0 multiplied with l w minus plus m dot e 0 multiplied with 1 minus l e K e plus a EE which is going to give you the flux f e which is minus m dot e 1 minus l e minus K e and a WW flux f ww which is minus m dot w 1 minus l w minus K w. So, a C is summation of n b C flux C f which is m dot e 0 l e plus m dot w 0 l w plus minus m dot e 0 K e minus m dot w 0 K w minus.

So, that is nothing, but your minus a E plus a W plus a EE a WW plus m dot e plus m dot w. So, that is what you get now what that does this n w formulation that HR schemes l is greater than that of K. So, provided there is a narrow region of NVD close to the downward line and that will see how it happens. The second thing is that the value of a C is always positive and that is why so, a C is always positive and that makes the system is a stable system or the instability does not arise.

Now along the downwind line of NVD where your, so downwind line NVD now variable diagram my l n K it should be 0 1 a a value of 0 for a a C coefficient is obtained. So, in this case the L and K can be set to capitalise 1 minus l phi f where l is usually said to the value of l from the previous interval of this scheme. So, what it brings down to the moral of the thing is that NWF to be much more robust than the DWF scheme because it always guarantees this coefficient a C to be positive which does not lead to any instability.

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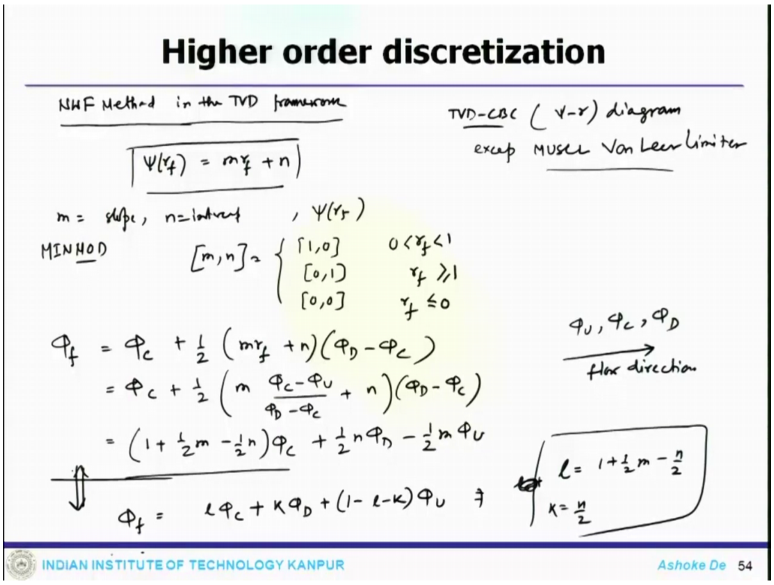

Now, the previous derivation was based on the NVA formulation. Now we look at this NWF method in the TVD framework. So, now we want to see so, what happens that accept that MUSL value are limiter all the limiters of the TVD scheme actually will be presented along a straight line in the TVD CBC or psi r diagram. So, an one can write that psi r f is a m r f plus n all of them except MUSCL Van Leer limiter. All other limiters will follow some sort of a straight line there. Now m and n these are constant, one is the m is the slope, n is the intercept of the linear function and depend on their geometric equations with in a interval of psi r f.

Now, the number of intervals it depends on the high resolution TVD scheme. Now for example, let us see what happens to this particular equation if you equate with the MIN MOD scheme. So, for MIN MOD scheme the value of m and n would be 10, 01 or 00 because in this range r f is less than 1 greater than 0, r f is greater than equals to 1 r f is less than 0. Now once we use this psi r f definition and then we try to evaluate the phase value like phi f equals to phi C plus half of m r f plus n phi D minus phi C. One can do little bit of algebra here which would be half of m phi C minus phi U divided by phi D minus phi C plus n and phi D minus phi C which will again lead to 1 plus half m minus half n phi C plus half n phi D minus half m phi U.

So, again phi U phi C phi D it will follow the flow direction. So, the values of m and n, one can find out all that for the different different schemes. And also so, the form one thing that you can obtain like this particular expression when you has the same form as earlier expression like; if you write in this form phi f equals to l phi C plus K phi D 1 minus l minus K phi U.

So, if you compare these two what one can write is that l plus l equals to 1 plus half m minus n by 2 and K equals to n by 2. This is what it gives you back, so once you look at the similarity. So, we will stop here and will continue the discussion in the next lecture.

Thank you.