**Introduction to Finite Volume Methods-II**
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**Lecture - 24**
**High Resolution Schemes-VII**

So, welcome to the lecture of this Finite Volume Method. Now, for different schemes for the values of this.

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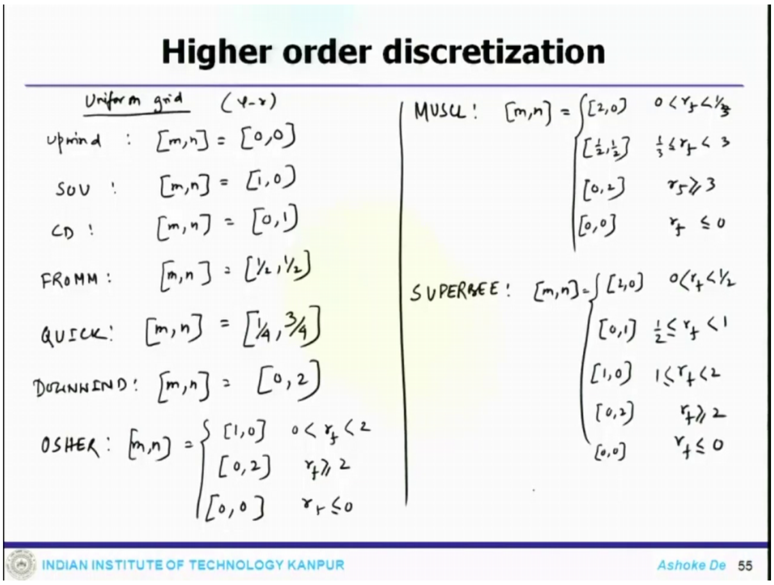


So, again this will be on uniform grid and what you get uniform grid the psi r diagram then the upwind scheme that has m and n equals to 0 and 0. Second order upwind scheme which has 1 0, central difference scheme has m n equals to 0 1 from m has m n equals to half half then you can have quick which has the m and n 1 by 4 3 by 4 downwind scheme which has values 0 2. Then you can have the other like osher where the m n has 3 segment one is 1 0, 0 2, 0 0 which is r f less than 2 greater than 0, r f greater than equals to 2 r f less than 0.

Now, muscl the m n has 4 different segment 2 0, half half, 0 2, 0 0 where r f goes between one-third 0 r f goes one-third 3 r f greater than equals to 3 r f less than 0. And super bee which is given as in again 5 segment. One is 2 0 0 1, 1 0, 0 2, 0 0 where r f less than half greater than 0 r f less then 1 greater than half r f less than 2 greater than equals to 1 r f is greater than equals to 2 r f less than equals to 0.

So, this approach is similar to your in n v f n w f can. So, does an approach which is similar to that n f v f n w f can be implementation of the similar t v d based in w f. So, one can implement in the similar fashion.

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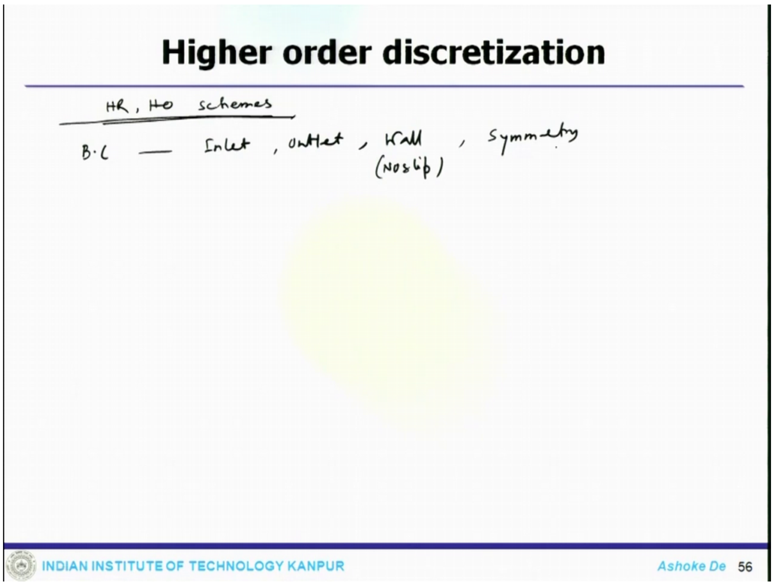


So, that actually talks about different class of higher order high resolution schemes and their improvement and then looking at the corrections. Now the point comes in use this things final in the convection term because we are in the framework of convection diffusion system, the boundary condition becomes then important element. Now the boundary condition for the convection terms are generally much simpler than for the diffusion term; why? Because that typically the boundary condition which you encounter are inlet where some velocity would be given outlet wall, where no slip boundary condition would be given; that means, then symmetry.

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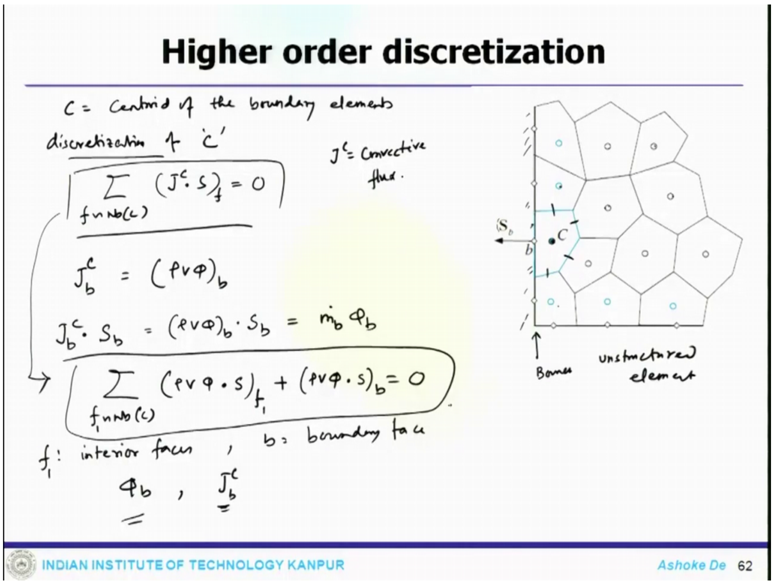


Now, typical boundary elements which one can see, now here in this particular unstructured element this is an unstructured elements you can see the boundary elements. So, this side is the boundary side, this particular this is the boundary and all these elements are exposed to this or rather this face which are exposed to the boundary face. So, this all these elements are exposed to the boundary face; now you have the discrete values of 5 which are stored at the sale centers. Now, but one has to calculate this S b denotes the boundary wall vector, surface vector b is the boundary face.

Now, this C is the centroid of the boundary elements and so this face b which is actually exposed to the boundary. So, an S b which is the surface vector pointing outward. So, the discretization of this particular cell would now become for any dimensional system in a generic system it will become if n b C J C dot S f equals to 0; C stands J C stands for convective flux only. So the interior faces are also discretized using the similar expression, but they are independent. So, any interior faces are also can be discretized using the same expression, but they are free from this boundary elements.

Now, here one face is exposed to the boundary. So, this flux which is shown here it has to be properly represented for this boundary. Now when you talk about that particular flux at boundary b, one can write this should be the convective flux of the component of the flux this is rho v phi at these boundary faces. So, such that this J b C dot S b which will become rho v phi b dot S b and one can say at this face it is the flux m dot phi b.

Now, if you implement this in this discretized system so, this will become now surface integration of rho v phi dot s at faces plus rho v phi dot S at b equals to 0 where, f refers to the maybe one can say this is f 1. So, f 1 refers to the all interior faces for this particular element C this one this one this one and this one are the interior faces and 5fth face is the boundary face.

Now, the specification of the boundary condition involves, either you specify the value or unknown boundary value phi b or one can specify the boundary flux. So, depending on that this particular discretization equation or discretized equation needs to be modified. If the boundary value is known that is what we call it is a Dirichlet kind of boundary condition and that we have already seen how to implement that kind of boundary condition. So, if the surface flux is provided then that is known as Neumann conditions or the gradient conditions; now that is the formulation at any inlet boundary.

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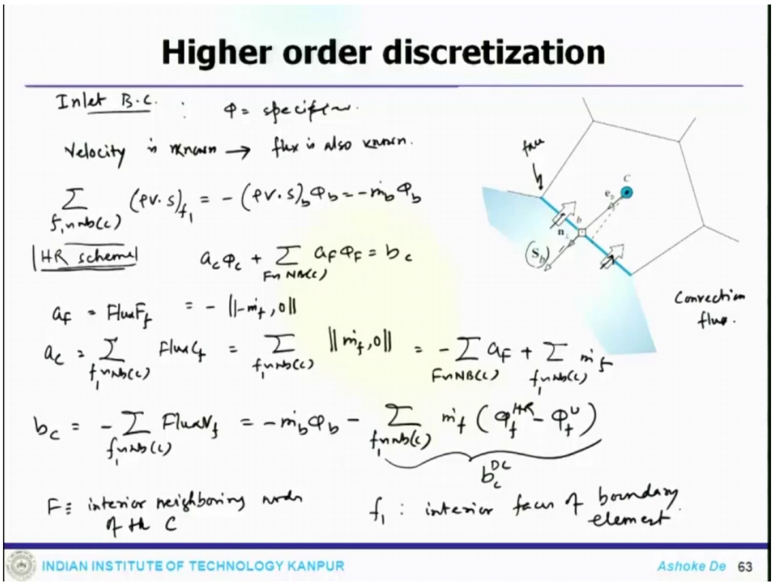


Now, will look at the inlet boundary conditions; so, inlet boundary conditions. So, this is for convection flux. So, that is the boundary face and the inlet through this face goes in this direction and normal is this direction, this is the surface vector and the connection between C and b is this line along e b and this is perpendicular to that. So, this already we have considered that slightly the centroid is slightly or.

So, here in this domain you can say that phi is either specified since, the inlet boundary condition for convection its dependent on the velocity field; so, most of the time the velocity field is known. So, which means if the velocity is known that leads to that the flux is also known, if you know the velocity then you can also say the flux is known.

Now, once you look at the discretized equation this would be f 1 n b C rho v dot s f one equals to minus rho v dot s v phi b which is minus m dot b phi b. Now in any high resolution scheme high resolution scheme is used to discretize the convection flux at the interior spaces. I mean assuming that one has used the HR scheme for the interior faces, then is implemented via default collections approach then the modified algebraic equation for the boundary element could be written as a C phi C plus F goes over the element equals to b C

So; that means, my a F is flux F which is minus f dot 0 a C is summation over all the faces of flux C f which is going over all the faces m dot f 0. Now one can write minus capital F NB C a f plus f m dot f and then b C equals to minus flux v f which is m dot b phi b minus f n b C m dot v minus phi f HR minus phi f up winds. So, this is my term due to deferred corrections, where f refers to the interior neighboring nodes of the C and here one can say this could be also f 1 then that makes life simpler what we are doing. This f 1 is the interior faces of boundary element so, the.

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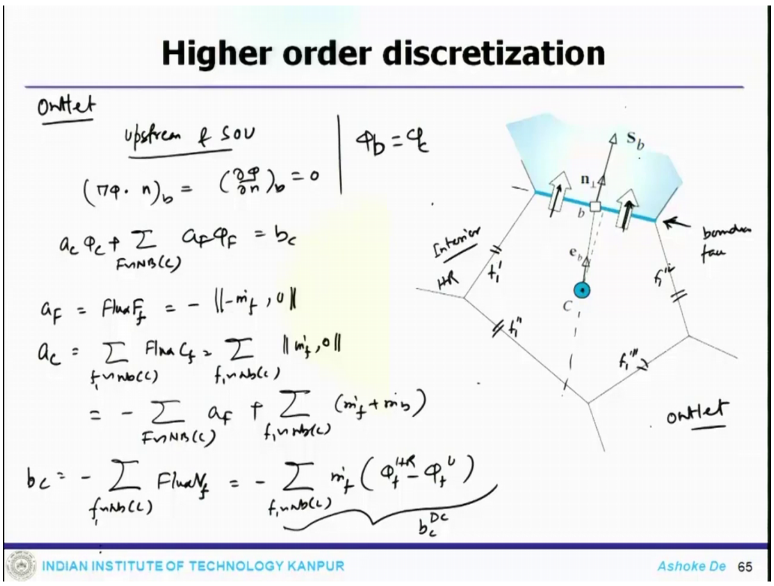


Now, that is how you implement the, now if you go to outlet boundary condition. So, that is in outlet; so, outlet the flow goes in the outward toward that and this is a cell C. So, outlet no information from the downstream of the boundary grid point is available so;

that means, if this is the boundary face then you do not have any information from this side, everything whatever is interior one has to use that to get this. Now because of this directional phenomena the phi value at the boundaries highly dependent on the upstream location; because at the outlet the flow has to go outward. So, everything will be heavily dependent on the information available at the upstream locations.

So in fact, the up wind and the second order upwind scheme for example, it does not do not require any information at the outlet since, all its value can be expressed as a function of the value at upstream. So, if you look at pure ups of first order up stream and second order up stream they use all the information from the points which are sitting ahead of it does not require anything at the downstream information. But the treatment that has to be proven to be very effective at the outlet boundary condition is to assume that high profile is either fully developed which is equivalent to that del phi dot n at the boundary should be 0; that means, no gradient at the boundary, usual practice at the outlet is to apply the upwind scheme where phi d phi b should be equals to phi C

Now, similarly you discretize the convective flux or for the interior elements we are using these are interior elements if you use for high resolution scheme then the modified algebraic equation for the boundary element would be using some deferred correction. So, the equation will look a C phi C plus F NB C a f phi f equals to b c. Now here again the a F would be flux F f which is minus m dot f 0 a C is f one n b C because f 1 only.

So, this is the face which we are interested because that is the boundary face. Now F 1 a C will get information for all the interior faces these are the so, this is all are belongs to interior face for example, this is f 1 prime, f 1 double, f 1 triple f 1 fourth. So, 4 interior faces are there where these fluxes need to be calculated which will become capital F NB C a F plus f one goes n b C m dot f plus m dot b and b C which can be represented at f 1 N b C flux b f which is minus f 1 N b C m dot f phi f HR minus phi f upwind which is again the boundary condition, a boundary value for the deferred corrections.

Here also f 1 refers all the interior faces of the boundary element and C is the element which is concerned or the owner and f is all other elements or the neighboring elements and the boundary its says the gradient is 0 and then. So, you have looked at the inlet, we have looked at the outlet.

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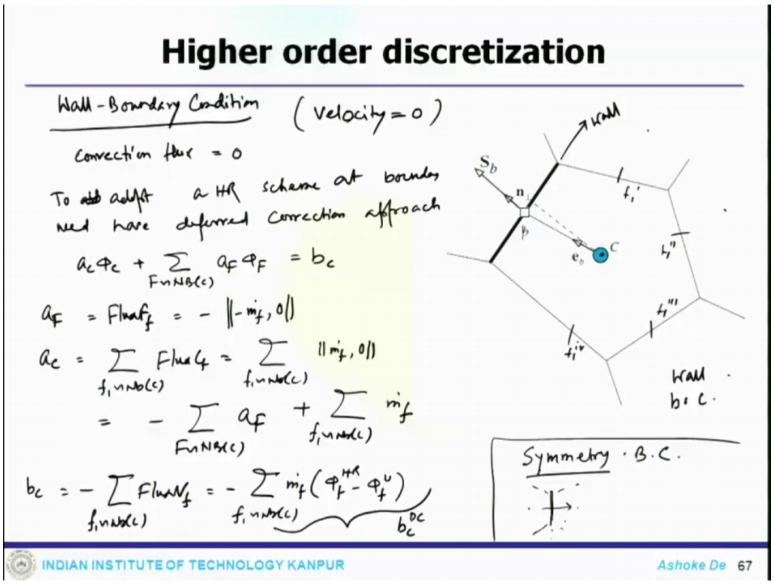


Now, the other one which we can see is the wall boundary condition. So, the wall boundary condition which is essentially at the solid wall typically fluid flow problem it is a no slip wall, which means the this is the surface which is the wall. So, it is a wall boundary condition, again this is the cell C interior cell and this is the wall.

So, wall boundary condition actually no slip boundary condition says velocity components would be 0 at the wall would be 0; that means, whatever velocity components if it is a multidimensional system you have u v w they would be 0. So, a such convection flux; that means, this will lead to convection flux to be 0 and that does not appear in the algebraic equation. Now again similarly if you are using for any interior nodes the high resolution scheme or HR scheme then when you come down to boundary you need some corrections and or rather to adopt a high resolution scheme at boundary need to have deferred correction approach.

. So, the modified algebraic equation for the boundary element this C which will look like a C phi C plus f goes over NB C a F phi F equals to b C. So, the coefficients they would be now can be evaluated where a F equals to flux F f which is minus m dot f 0 a C which is summation over f 1; f 1 means all other interior faces. So, this could be f 1 1 f 1 2 f 1 3 f 1 4.

So, therefore interior faces which can be used for this calculation, which is summation over f 1 goes which is m dot f 0. So, if you rearrange that so you get back the summation over neighboring cells with a F plus F one goes over the interior faces m dot f and the right hand side term would be minus f 1 goes over all these faces V f which is f 1 N b C m dot f phi f HR minus phi f upwind which is again the term arises due to deferred correction. And f 1 here which also refers to the all the faces belongs to the interior faces and C is a cell which is concerned or the owner cell and a for all neighboring cells

Now, there could be one more condition which you cans look at is the symmetry boundary condition. In symmetry boundary condition which means the if this boundary is a symmetry boundary of a element nothing crosses across this boundary. So, no flow across that boundary; so, whatever is here it should be exactly similar to that side. So, it is treated in a similar fashion to the wall boundary condition where the flux, convection flux normal to the symmetry boundary said to be 0.

So, 3 different kind of boundary conditions we have looked at it in respect to the higher order discretization. So, when we started with the convection discretization we had looked at all these inlet boundary, outlet boundaries, symmetry boundary, wall boundary conditions , but that was with respect to our initial discretization scheme.

Now, what we have looked at here is that for interior node we are using the higher order schemes or higher resolution scheme so; obviously, when you come down to boundary you do not have the information from the other side. If this is the boundary you do not have the information from this side. So, to apply the HR scheme you require some sort of an modification to that which is a default corrections and which will come as a source term in the discretize equation. So, that objective like the application of h r scheme to be consistent with your all interior discretization we looked at at the different boundary conditions. So, the first one we looked at inlet, where you come across all this coefficients and you can see the corrections which arise due to the deferred correction

Similarly, you come to outlet where no flow process the boundary and final equation. So, the source term will have a some deferred correction. So, that allows you to and the third one at the wall boundary where the no slip boundary condition is satisfied what means the conviction flux is 0 also come across this deferred correction. So, you can be consistent in application of the HR scheme at interior node and the boundary node and that concludes the discussion on the HR scheme. So, we will stop here and the next class will take it up from there.

Thank you.