Introduction to Finite Volume Methods-II
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Lecture – 25
Temporal discretisation-I

So welcome back to the lecture series of Finite Volume and now in this particular lecture, we will start with a discretisation for the unsteady term. So, far what we have completed? We have completed the diffusion term and then we have looked at convection diffusion system. While doing the convection diffusion system, we have discussed higher order scheme and their approaches, how one can formulate those higher order scheme and then the implementation point of view along with boundary conditions

Now, in today's lecture we discuss the unsteady discretisation. So, that will actually make the platform ready for the fluid flow problem.

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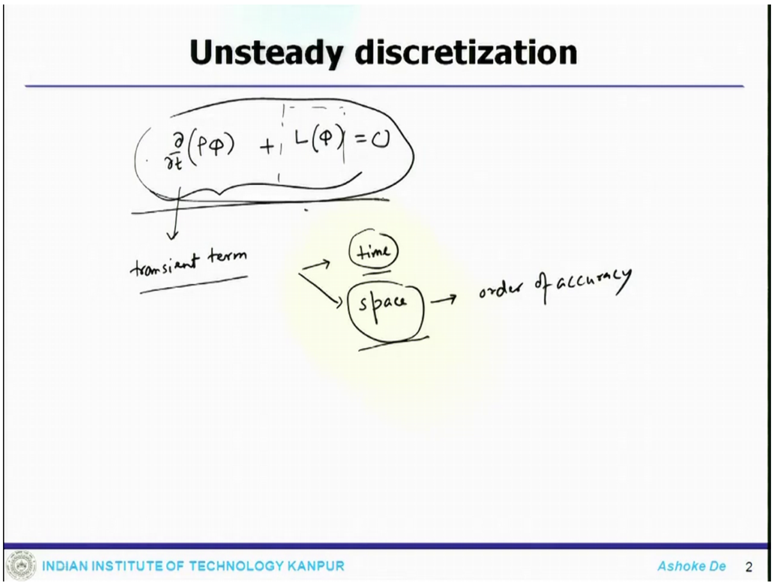

So, in the unsteady discretization what you do you essentially your governing equations that you have which is for any scalar variable which you can say rest of the terms written in one operator. This is how the unsteady equation would look like. Your convection flux, diffusion flux, source term everything is included in this operator.

Now, this is the transient term that we need to look at it or other way one can think that how to try a discretize this transient term all along with the complete set of system. When you say unsteady discretization, it includes discretization both in time and space. So, what we have been doing so far? We are only doing the spacial discretisation so far where we talk about order of accuracy; that means, second order third order or first order of accuracy.

So, that is primarily the spacial discretization; we have not yet discussed anything on time or temporal discretisation. So, now, in today's lecture we will have a look and then we will go on detailed discussion on the temporal discretization along with the spacial discretization. So, how would you start with this particular system?

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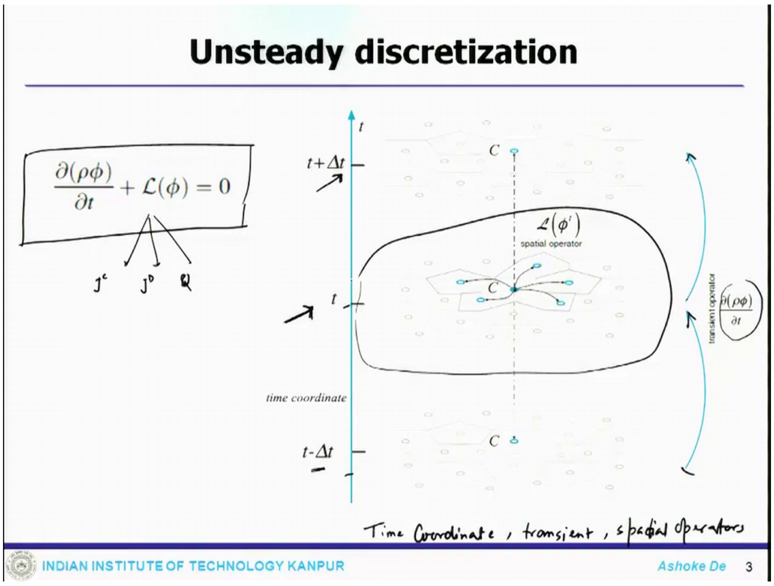

So, let us see first in a picture how the. So, this is the time coordinate essential one can think about this is a time coordinate picture of a time coordinate where you see transient and spacial operator; transient and spacial operator. So, this pictures actually give you the complete idea about the whole system. So, this is an unstructured grid element and as per our notation this element which we are interested in and our complete equation is like these where this guy includes conviction flux, diffusion flux, source term, so everything that includes. So, everything other terms apart from this transient term now has been taken within this operator L phi.

Now, when you look at this particular indexing system the concerned cell is surrounded by this neighbouring cells and when it is surrounded by the neighbouring cell, now this is where at a particular time level t or transient level t. And then when you go to the next time level, here you come at the time level t plus delta t and if you go one level down or previous time step this is t minus delta t. So, you can think about this is how your time coordinate will actually move with your solution; previous time step, present time step, future time step. And this is the transient operator that will work from this time step to this time step and it will see how the solution actually varies with time and then from here this will move to the futuristic timescale.

Now, if we consider this particular element in the time coordinate let say, at time level t and see it I mean expand that system, then this is what it looks like.

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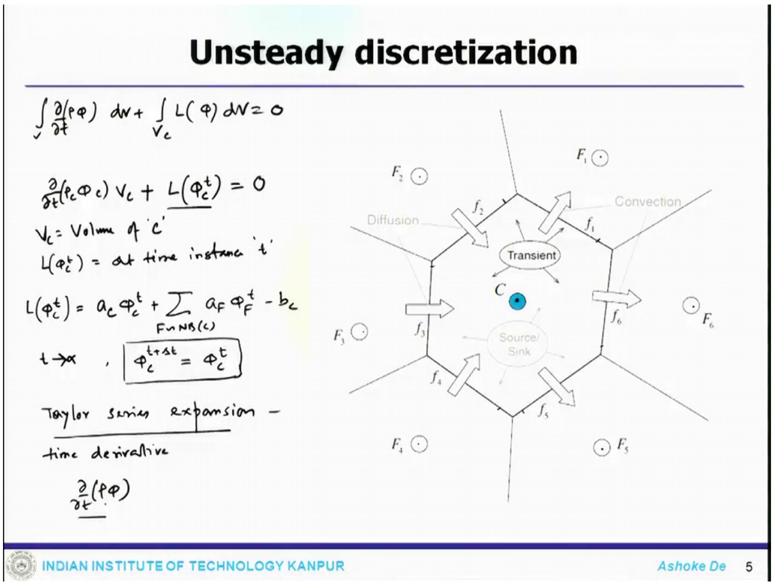

So, here your C and then you have surrounding all the elements the elements like F 1, F 2, F 3, F 4, F 5, F 6 and these are the faces f 1, f 2, f 3, f 4, f 5, f 6 and source term source or sink term and then the transient term. So, across the faces you will have convection and diffusion fluxes which may come in some faces in may come in some faces, it may go out. Now if you do the I mean just over this element C if you integrate with time, then this operator del del t rho phi d V plus V C L phi d V 0. Now the spacial discretisation about this it will get you the del del t rho C phi C into V C plus L phi C at time level t 0.

Now V C is the volume of C and all the operator spacial operator which is expressed at time instant instance t ok. So, that is where the operator lies.

Now, this one can be written algebraically like L phi C t equals to a C phi C t plus summation over F NB C a F phi F t minus b C. Now if t tends to infinity, one can actually retrieve back the steady state discretized equation. Now this is also true when steady state is reached through the time marching that will provide you phi C at t plus delta t is phi C at t. So, that will also tell you that the steady state of the solution is achieved

Now, for the discretization of the transient term, the traditional practice is that we discretize this unsteady term using some sort of an finite different kind of approximation. So, where essentially one has to use typical Taylor series expansion and then try to form the discretize system. So, using the Taylor series expression, you can express your time derivative. So, that is the whole idea.

Now in this way if we integrate this term del phi by del t is integrated over temporary element and transform into the. So, we want to integrate the term del t by del phi over time and then convert them to a face fluxes as a similar fashion what we have done for the convection scheme. Then it will look similar except this is done in a transient coordinate system.

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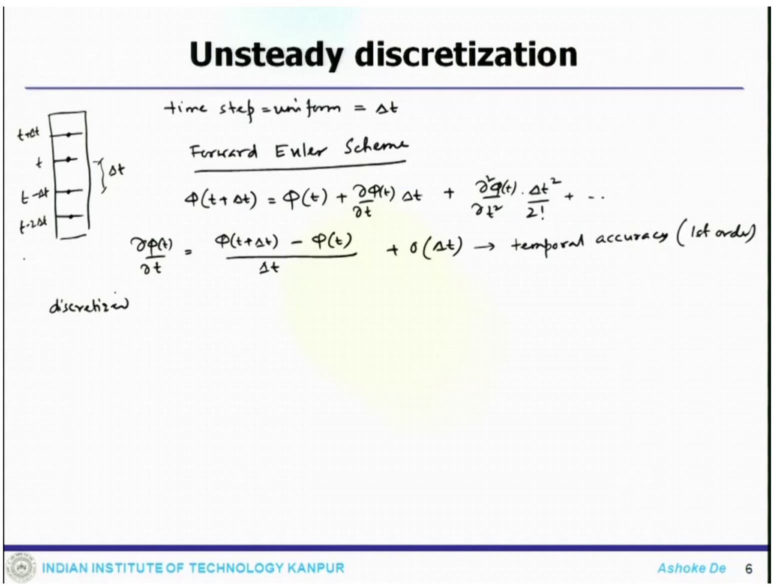

Now, when you look at a structured system let us say, I will look at a structured system like in time coordinate and how things will move? Things will move like this. This is t then this should be t plus delta t this is t minus delta t this is t minus 2 delta t. So, this gap is known as delta t and this is how you can actually this is again in the structured kind of grid system one can think the time coordinate to be a similar to your spacial coordinate. So, it is in the uniform time system.

So, basically the time stepping or time step is uniform which is delta t. So, now one can write for different transient applications let say, if we write forward Euler scheme. So, what we do? The typical transient term let say any value t, some function which is or phi also one can write for the scalar phi; phi at t plus delta t equals to phi t plus del phi by del t into delta t plus del 2 phi by del t 2 into delta t square by 2 factorial and so on.

So, now the first derivative of the del phi by del t if I have to find out. So, that will become phi t plus delta t minus phi t divided by delta t an order of a accuracy also. So, now, this one also get you first order accurate, but temporal accuracy. So, the temporal accuracy is also first order. Now we can use this one for our system and then we can get the discretize equation. Now the discretized equation for the element C that will look like so, if we see this so this is the schematic of explicit Euler.

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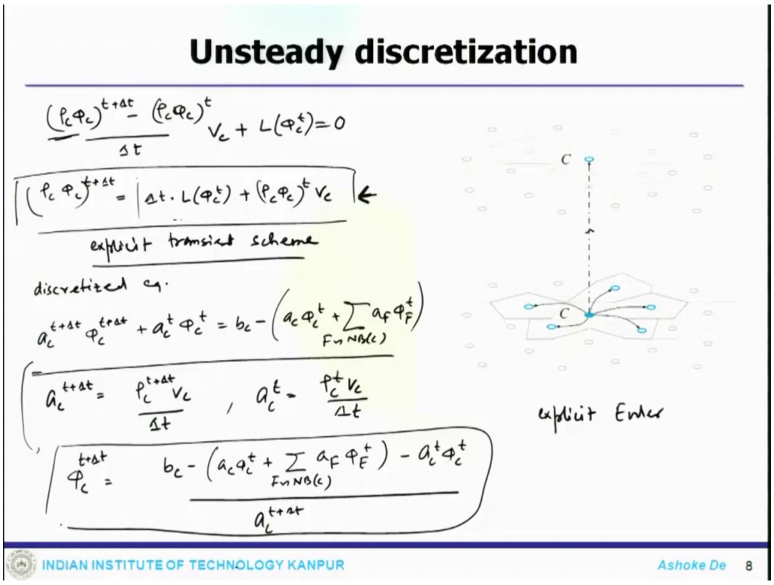

And what is happening from this to this? It moves and discretized equation one can write like rho C phi C t plus delta t minus rho C phi C t divided by delta t multiplied with V C plus L phi C t equals to 0 that is the discretized equation.

Now, which indicates that this quantity rho C phi C at t plus delta t does not require solving a system of equations rather one can directly find out this quantity at t plus delta t. Because everything else if you look at it they are represented at. So, one can just expand little bit and write this is t plus delta t equals to delta t into L phi C t plus rho C phi C t V C. So, everything else in this particular expression is known accept the quantity which one tries to find out at that t plus delta t level.

So, it can be evaluated explicitly using the previous time step value. So, this is known as explicit transient scheme which means I can find out the variable explicitly using the information from my previous time step level. So, this is exactly what is shown here. All this right hand side variables are known from my physical previous time iteration. So, using that information or known values, one can find out the information and that is done explicitly. So, a I mean me one has to I mean one does not need to solve any linear system. So, this has some great advantages like computationally its very cheap. I mean; that means, it will be highly efficient. You can solve this protein point calculation very quickly and you do not need to keep anything in the memory. So, but few of the codes which prefer to use it because, it will have severe restriction from the stability point of view.

Now, once we substitute this things in the discretized algebraic equation. So, the discretized equation will be obtained like a C t plus delta t phi C t plus delta t a C t phi C t b C minus a C phi C t plus summation of NB C a F phi F t. So, that is the discretized system that you can get and where your a C t plus delta t is nothing, but rho C t plus delta t b C by delta t and a C t is rho C t V C delta t. So, the coefficients a C t plus delta t and a C t this comes from the transient formulation and at the different level of these things.

Now, what one can do is that finding the phi C from here using these equation at next time level, one can find out this things very easily. And this equation if you use, you find out the phi C term explicitly which will retain the complete term and one can find out like let us say phi C which will find out V C minus a C phi C t plus summation of F n n b

C a F phi F t minus a C t phi C t by t plus delta t a C t plus delta t. So, this is how you can find out explicitly this expression this is an explicit.

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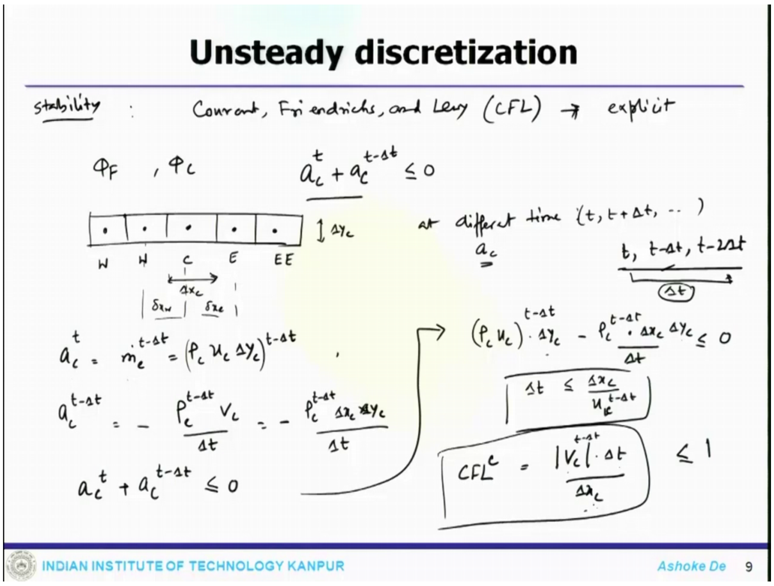

Now one can look at the stability of this particular system. So, the stability condition for this forward Euler is kind of control by the number which is known as Courant, Friedriehs and Leuy. So, this is in short which is common known as CFL criteria. When someone uses explicit scheme the time step is somehow controlled by this CFL criteria and which will provide some sort of an restriction on your delta t using your depending on your grid resolution and the flow condition.

So, in reality the CFL condition can be interpreted simply as one of the basic rules that should be satisfied by this coefficients namely by the opposite sign rule and extended to include the transient coefficients. So, let us say just for a phi F which is consider as a spacial neighbour of phi C and similarly the other elements then one can see that phi the coefficient like a C plus a C. This should be less than 0. So, this is at one is the current time step, another is the previous time step, so, one can use that way. So, that is what one can find out this criterion and we will see in details what it could be.

So, now you can actually represent this things in a expanding the term. So, term if you want to expand that essentially you have to find out the term. You consider a stencil like one dimensional stencil where you have this cells and they are this is C this would be E this is EE WWW uniform tensil. So, this is delta x C and this would be delta Y C and the surface this is a distance which is delta x w and this is delta x e. So, this is a standard notation that we have been using so far in this system.

So, now we have to evaluate this coefficients at different time instance which include t t plus delta t the coefficients of a C that we have to find out. And how do I find out? So, you have to use this using the like a C is sort of m dot e which is rho C u C delta Y C that is the conditions. Now one can use this was the since it is a for forward Euler this could be t minus delta t. So, essentially one can use the time of t t minus delta t t minus 2 delta t like that; so, the in the previous to previous time instances. So, that is how one can use and that this one would be current times step.

So, which includes this one previous time step and the uniformity between that like a spacial coordinate system since it is a uniform. So, here also it is a uniform temporal discretization based on delta t. So, that is how one can find out this whole business. Now a C which is at current time level one can so, the previous times level if you want to find out, this whole system you need to find out from the previous time step that is the condition. And now this you find out the a C at the current time step which is the mass flow rate from my previous time step and then this should be like that. Similarly (Refer Time: 25:27) a C at previous time step which is minus rho C previous time step V C by delta t.

So, that would be minus row C t minus delta t delta x C delta y C divided by delta t. So, that is how you get a current times step which is represented by t and the previous time step t minus delta t. So, now how you find out that this is current time step using the mass flow rate from the previous time step and this is at the previous time step. Now the CFL criteria says that it says a C at the current time step plus a C at previous time step, they must be less than 0 which means if I put this in back here which means row C u C which is t minus delta t delta y C plus or rather minus row C t minus delta t delta x C delta y C divided delta t less than equal to zero. So, delta y C delta y C that goes out row C goes out. So, delta t becomes essentially less than equals to delta x C by u C t minus delta t that is what it becomes.

So, what it provides that condition for the time step. So, one can immediately see the explicit scheme has some restriction on. So, when you have a convection dominated flow, the CFL criteria for convection is magnitude of b C from previous time step into delta t by delta x c. So, that has dependency on the flow field, dependency on the grid resolution. So, these are all what one has to use. So, now and for stability this has to be less than 1 and from there you can find out because you know the velocity condition you know the delta C x C from your grid, then you can always find out the delta t that is the case when we do not consider the this things. So, we will stop here today and we will take from here in the follow up lectures.

Thank you.