Introduction to Finite Volume Methods-II
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Lecture – 28
Temporal Discretisation – IV

So welcome back to the lecture series of Finite Volume and where we will continue our discussion where we left in the last lecture.

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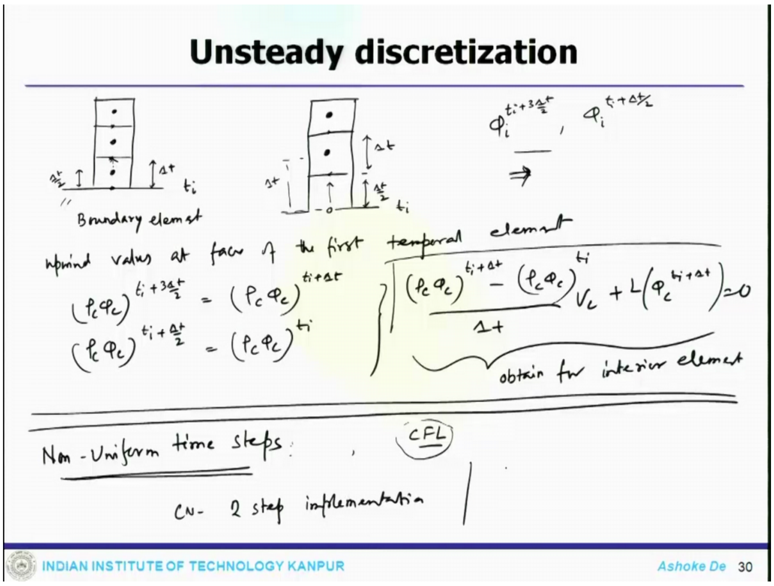

So, if one look at a schematic; let us say we have this and we go top. So, this is the condition this is the cell center value and this is the delta t. So, it is a boundary element; here it is shows the boundary element and this is the cell centre value it moves towards that and this portion is delta t by 2 and this is were t i is now for the.

Any other element if you look at it. So, what that happens that they are at the now face this is the time step delta t and this is delta t by 2, this is also delta t and this is our t i. And that is why for all the interior phases; they are getting a gradient between phi at t i plus 3 delta t by 2 between phi i; t plus delta t i plus delta t by 2.

So, the difference between the two gradients is substantial and any scheme that starts with these gradient will result in large initial error and that will affect the solution and the subsequent time steps. So, this error can be embedded if a grid similar to this one is adopted. In this case the solution of the finite difference and find volume methods will be basically similar as for a regular grid.

So, one important thing is that one can understand that how important is the grid in numerical calculations and then choosing proper spatial and temporal accurate scheme which will also have impact on the solutions. I mean one can say or claim that he is getting a numerical solution done for a physical problem, but that may not make any sense.

Now once you adopt this approach the upwind values upwind values at faces of the first temporal element. So, that are obtained as rho c phi c; t i plus 3 delta t by 2 equals to rho c phi c t i plus delta t and rho c phi c; t i plus delta t by 2 rho c phi c t i; I substitute this one in the discretized equation one get like t i plus delta t minus t i divided by delta t plus L phi c t i plus delta t equals to 0. So, this is the similar one that you obtained for interior element. So, the boundary element thing can be modified so that you get the similar kind of expression for the interior element and then.

Now, the important point is that so far whatever temporary discussion that we have been doing; we are in the framework of the uniform time step. And it may not possibility you know realistic situation that always you get an uniform time steps. So, there could be possibility that you have non uniform time steps. So, if you get non uniform time steps. So, it is I mean very much common in practical applications where the variable time steps are mainly used to reduce the computational cost by selecting a time at every time step and allowing the maximum allowable time step value so that that does not valid the CFL criteria.

So, using the CFL criteria and the every time iteration the allowable limit is decided so that; that can effectively scale down the computational cost by certain factor. Now for example, the first order scheme that we have discussed the discretization typically not affected whether the time step is variably constant. The situation is going to be different when we talk about second order transient scheme, since the use a tensile which involve 2 time step values.

For the case of 2 step implementation like Crank Nicolson; Crank Nicolson has 2 steps implementation. So, where we use 2 step implementation like Crank Nicolson type of skill, nothing changes except that for each of the step if different time step is used. These affects the accuracy as a special derivative affect the accuracy, as the special derivative is no longer at the centre of the temporal element.

For other second order scheme like second order upwind and these the interpolation profile has to be modified to account for the non equal time step. In, what follows in the non uniform transient grid is used in the discretization of the transient and of difference scheme. Now we can see like this non uniform time step calculations in both the context of finite difference and finite volume.

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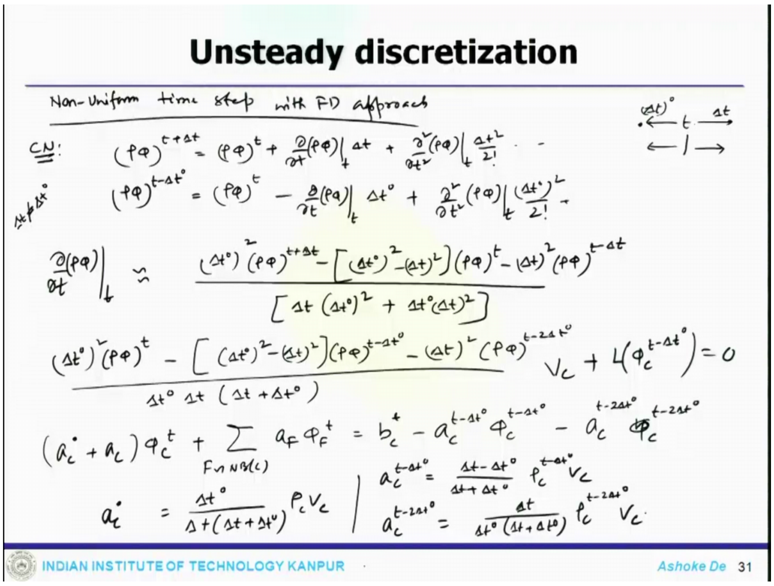

So, now we see the non uniform time steps with FD approach. So, first we start with the Crank Nicolson; so the Crank Nicolson it is what it uses the rho phi at t plus delta t; which is rho phi t plus del del t of rho phi at t delta t plus del 2 t by del t 2 rho phi at t square by factorial 2 and so on.

Now, rho phi t minus delta t which is rho phi t minus del phi by del t at t; it would be delta t naught because here it is a non uniform time step. So, that delta t is not equals to delta t naught. So, we make a difference and then we get to see what happens to the term. So, you multiply it with certain terms and then do some algebraic calculation so that you can obtain the first derivative like del del t of rho phi at time instant t; which can be approximated as delta t naught square rho phi; t plus delta t minus delta t naught square minus delta t square into rho phi t minus delta t square. And rho phi t minus delta t which is divided by delta t delta t naught square plus delta t naught delta t square like that.

Now, one can substitute the expression of this gradient in our discretization equation of the Crank Nicolson now one can get it for non uniform time steps. So, once you substitute this you get square rho phi which is a at that time level t; minus del t naught which is square minus delta t square; rho phi t minus delta t minus del t square rho phi t minus 2 delta t divided by delta t naught; delta t, delta t plus delta t naught where V c plus L phi c t minus delta t equals to. So, here you can see the discretize equation becomes like that. So, this would be now this could be t minus delta t naught and t minus 2 delta t naught.

So and this would be t minus delta t naught because we are seen that positive direction. So, t here when it goes these direction it is a delta t, when it goes direction is a delta t naught; this is current time level previous forward. So, the terms if you expand in the discretized equation; it will become a c dot plus a c into phi c t; summation over F; which will use F phi F t; b c minus a c t minus delta t naught phi c t minus delta t naught; a c t minus 2 delta t naught phi c t minus 2 delta t naught.

Where the coefficients are this guy is essentially delta t naught divided by delta t; delta t plus delta t naught rho c; V c; a c t minus delta t naught is delta t minus delta t naught divided by delta t plus delta t naught rho c t minus delta t naught V c and a c; t minus 2 delta t naught is delta t divided by delta t naught; delta t plus delta t naught rho c t minus 2 delta t naught V c. So, you can get the scheme which is recovered like that.

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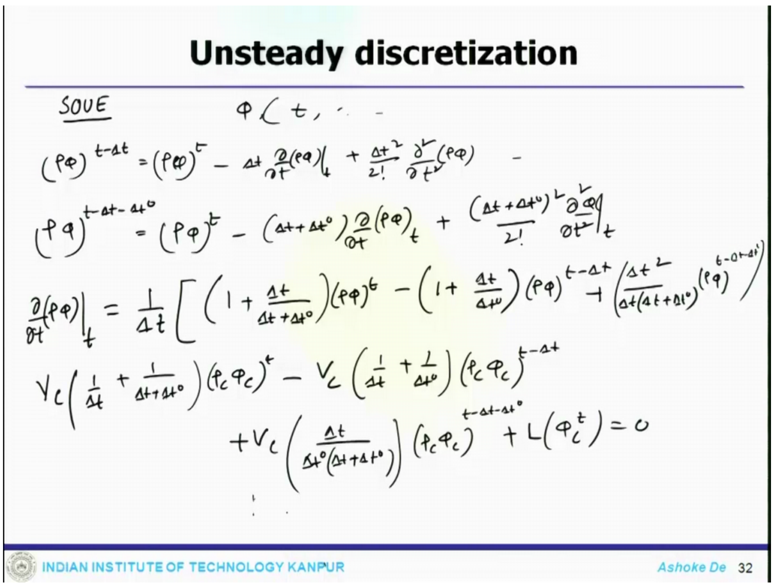

And similarly you can do for SOUE; Second Order Upwind Euler scheme where; where you can use the phi values at t minus delta t t minus delta t minus delta t naught. and so which is a t and other terms.

Now there if you put t minus delta t is rho phi t minus del t t by rho phi at t plus factorial 2 and del t 2; so on. Similarly t minus del t minus del t naught which can be computed as minus delta t plus delta t naught del del t of rho phi at t delta t plus delta t naught square by factorial 2 del 2 phi by del t 2 at t and so on.

Now, you multiplied with some factor and then do the algebra to get essentially the first derivative; del rho phi by del t at t which is nothing, but 1 by del t 1 plus delta t divided by delta t plus delta t naught multiplied by rho phi t minus 1 plus delta t divided by delta t naught; rho phi t minus delta t plus delta t square delta t into delta t plus delta t naught rho phi t minus delta t minus delta t naught.

So, if you put everything back in the semi discretized equation; this we look V c 1 by delta t plus 1 by delta t plus delta t naught. And rho c phi c minus V c 1 by delta t plus 1 by delta t naught rho c; phi c t minus delta t plus V c, del t by del t naught del t plus del t naught rho c phi c which is t minus del t minus del t naught plus L phi c t 0. So, you can actually obtain back the discretized form.

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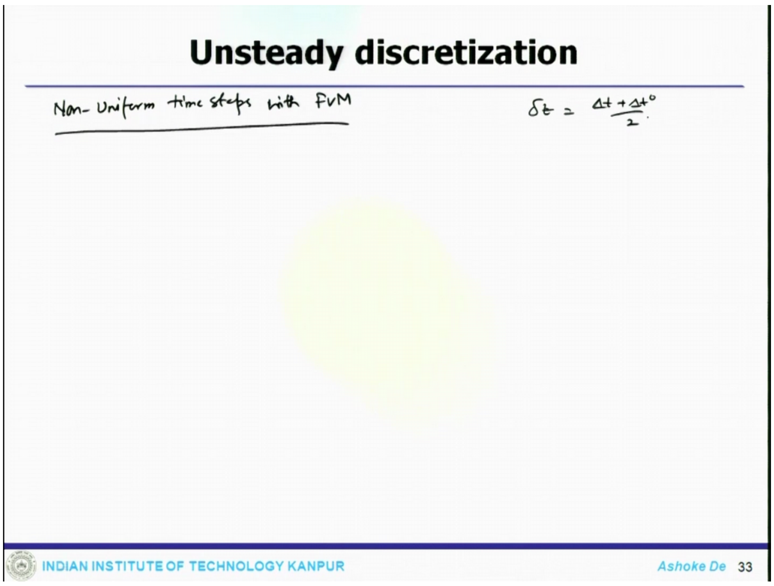

Now sometimes one can use the; now similar thing for the non uniform time steps with FVM and what happens to that? Now sometimes the delta t for this difference can be consider as del t is a arithmetic mean of this non uniform time step that can be consider sometime. But, now similar thing if you expand for the final volume and the thing which will start with the Crank Nicolson.

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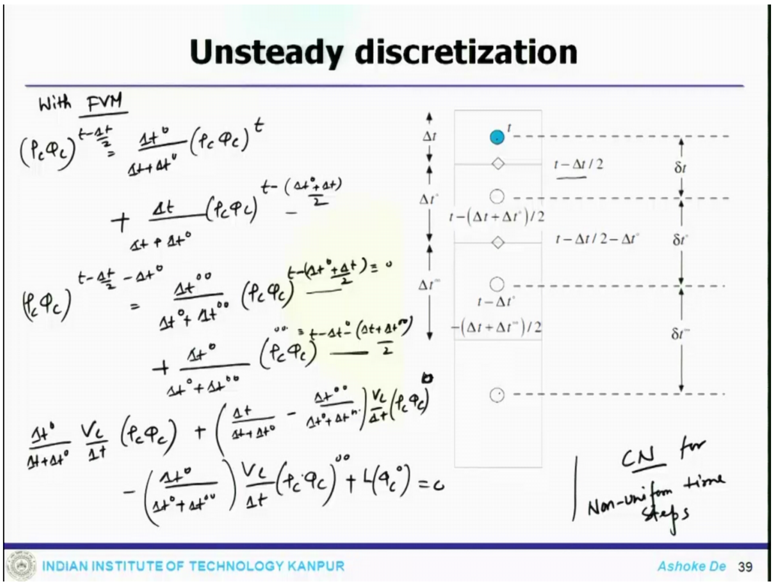

So, let us see what happens for the; this is the Crank Nicolson for non uniform time steps. So, for non uniform time steps; if you write the Crank Nicolson this is with FVM. Now here you see these are the cell centre and this is at that time level of t; so t minus delta t by 2 is this; t minus; so this is delta t this is delta t 0, this is delta t 00.

So, this time is t minus delta t divided by 2; this guy is t minus delta t plus delta t naught by 2 and that is the way you get it. And this is a delta t, this is del t naught del t double naught. Now the term which is rho c phi c t minus del t by 2; now it will become del t naught by del t plus del t naught rho c phi c t plus del t by del t plus del t naught rho c phi c t minus del t naught plus del t by 2.

And the other term which is rho c phi c t minus del t by 2 minus del t naught, which will be del t double naught divided by del t naught t double naught rho c; phi c t minus del t naught plus del t divided by 2 plus del t naught divided by del t naught plus del t double naught; rho c phi c, t minus del t naught minus del t plus del t double naught by 2.

So, once we discretize put this in back in the discretized equation; the discretized equation will look like del t naught by del t plus del t naught V c by del t rho c phi c which is essentially coming from this term t plus del t by del t plus del t naught minus del t double naught divided by del t naught plus del t double naught V c by del t rho c phi c; it is the term which comes from this.

So, one can say that it is t minus these term. So, this can be and the other term will delta t naught by del t naught plus del t double naught; V c by del t, rho c phi c by double naught plus L phi c naught 0. Here this without script stands for the current time step; naught stands from the previous one which is this calculation, double naught stands for this. So, this is equivalent to double naught this is equivalent to naught superscript this is double naught. So, that is the way one can discretized the Crank Nicolson scheme.

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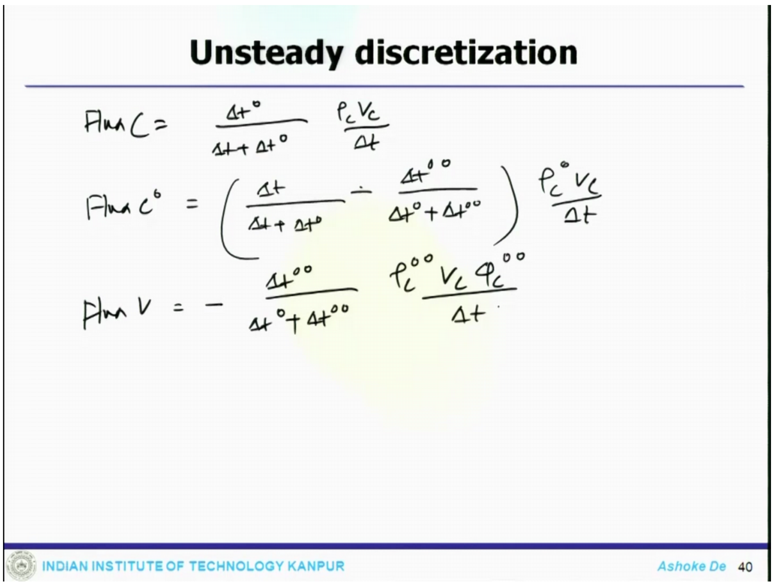

And the fluxes can be written as flux C is del t naught by del t plus del t naught with rho V c by del t flux C naught equals to del t plus del t plus del t naught minus del t double naught plus del t double naught; rho c naught V c by del t and flux V equals to minus t double naught divided by double naught plus rho c double naught V c phi c double naught by del t.

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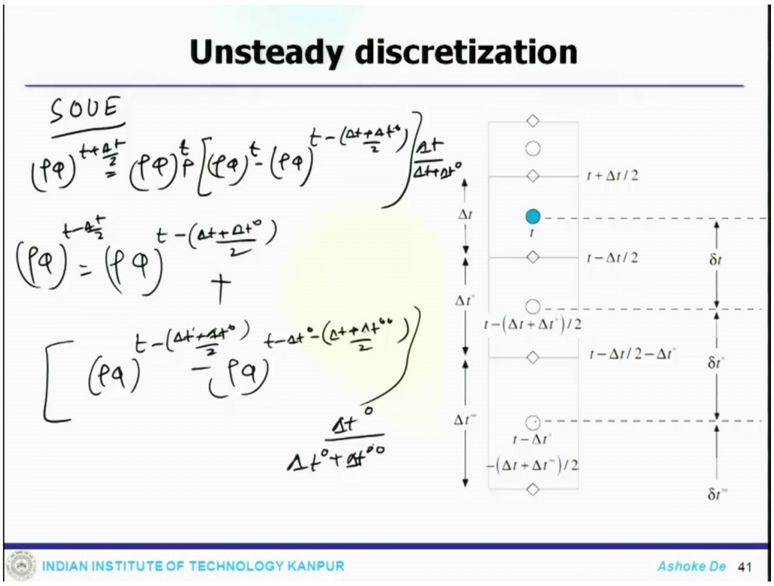

So, similarly one can look at the second order upwind scheme this is SOUE where; also we have the cell centre value this is t, this is t minus delta t and this is t minus delta t. And if you put things back like rho phi at t plus delta t by 2; this would be rho phi t plus rho phi t minus rho phi t minus delta t plus delta t naught by 2 multiplied with delta t by plus delta t naught.

Similarly rho phi t minus delta t by 2 equals to rho phi; t minus delta t plus delta t naught by 2 plus rho phi; t minus delta t plus delta t naught by 2 minus rho phi t minus delta t naught; delta t plus delta t double naught by 2 which is multiplied with del t naught by del t naught plus del t double naught.

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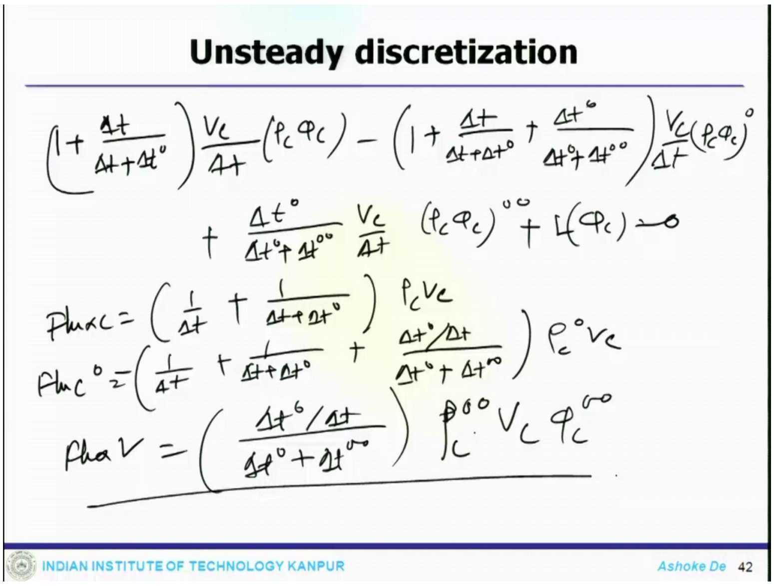

So, if you put things back it will semi discretized equation will look like; del t plus del t naught V c by del t rho c phi c minus 1 plus del t by del t plus del t naught plus del t naught by del t naught plus del t double naught; V c by delta t rho c phi c naught plus del t naught by del t naught plus del t double naught V c by del t rho c phi c double naught plus L phi c equals to 0 where flux C would be 1 by del t plus 1 by del t plus del t naught into rho c; V c.

Flux C naught would be minus 1 by del t plus 1 by del t plus del t naught plus del t naught by del t divided by del t naught del t double naught, which is rho c naught V c and flux V is del t naught by del t by del t naught plus double naught rho c double naught V c; phi c double naught. So, this is how you get all these term and that is how you can get the uncertainty discretization for uniform and non uniform grid.

And so we stop here today and look at the now flow field discretization in the next lecture.