Introduction to Finite Volume Methods-II
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Lecture – 30
Discretisation of the Source Term, Relaxation and Other Details-II

So, welcome back to the lecture series of Finite Volume.

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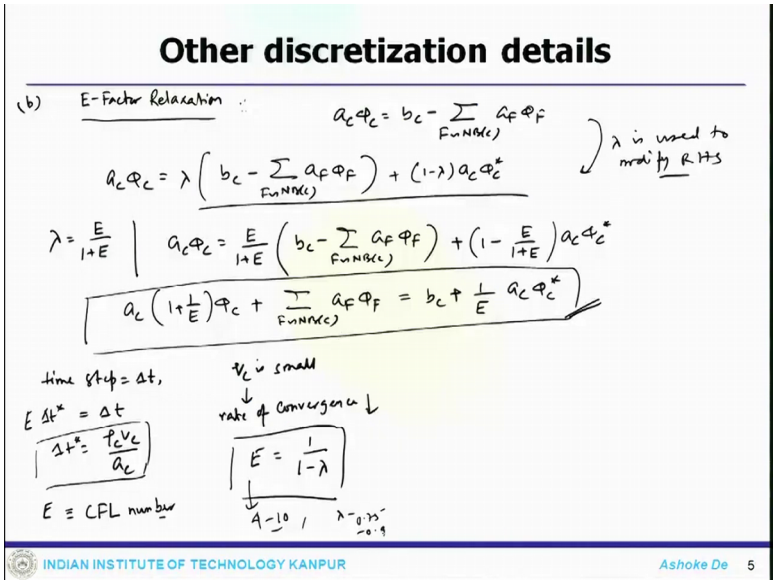

And where we will continue our discussion where we left in the last lecture; so, these are two different things.

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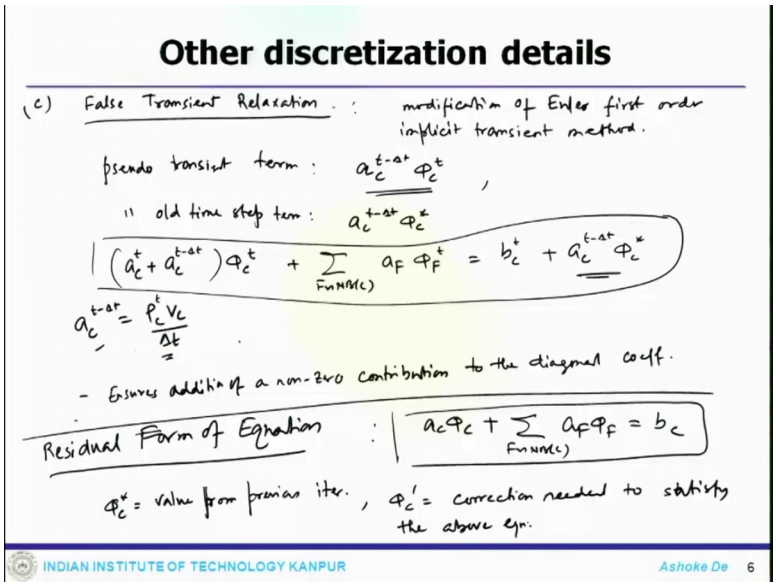

Now, the third one we can talk is the false transient relaxation. So, in this case the false transient relaxation method it is the modification of the of Euler first order implicit transient method. So, its essentially obtained by modifying the Euler first order transient method. So, where the previous iteration values are used are used instead of the old time step values.

So, as in Euler method the diagonal dominance of the algebraic equation is increased through the addition of the pseudo transient terms which is in terms of the that pseudo transient term which was using as a c t minus del t phi c t like that. So, to the diagonal coefficient and the pseudo old time step term which are pseudo old time step term was used at a c t minus del t phi c star to the right hand side.

And with these modification one can get this ac plus ac this is t this is t minus del t multiplied with phi c at time level; then F N B C; a F phi F at t equals to b c at this level plus a c t minus del t and phi c star, where is a c t minus del t computed rho c; b c by del t.

So, it is a basically an equal to the transient coefficient which is obtained from the first order implicit Euler discretization. Now rho density V is elements cell volume and this; now here as you can see the large values of delta t, the added term which is it is very much this is the term which is added is negligible and the under relaxation effects are also quite negligible. So, the solution of the equation is same as the original under

relaxed one, but if you have a very small delta t; then the term which is here becomes quite large and this becomes the dominating over other terms. And the solution is heavily under relax duty due to small change in this value of phi c from to phi star.

Now, in addition to all these the solution to advance consistently over the inter domain the false transition method ensures the addition of non zero contribution to the diagonal coefficients. So, it ensure ensures addition of a non zero contribution to the diagonal coefficient, even in extreme cases where the diagonal coefficient is 0. So, in other way one can think that there is no generic rule for assigning optimum under relaxation factor as the values are used one case may not work properly for the another case. So, this is one of the issues with the; now we will look at the residual form of the equation.

So, residual form of equation; so the we have a discretized algebraic equation and which is written in a very standard form that is a c; phi c plus summation of a F phi F equals to b c; so this is a standard form. Now here it can be written in correction or residual form by rearranging the terms so that the; so that the solve for the correction needed to satisfy the equation and the way it is done. So, we say that phi c star is one value which is a value from previous iteration number 1 and then the phi c prime which is the correction needed to satisfy the above equation.

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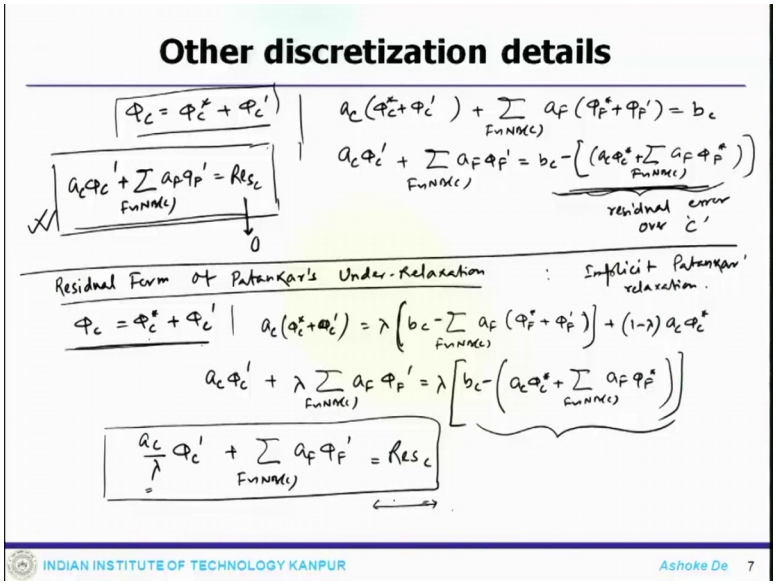

So, which means I can write phi c equals to phi c star plus phi c prime. Now, with that you can put back in the actual equation and this will become a c; phi c plus phi c star I

mean prime plus star plus summation F N B C which will become a F; phi F star plus phi F prime equals to b c or you can rearrange this. If you rearrange this will become a c phi c prime plus f which is a F phi F prime equals to b c minus a c phi c star plus summation over a F; a F, phi F star; so that is what you get.

So, the equation system gets modified and the right hand side you can see this is an error due to residual and if you say that is the residual over element C. Then this one can write a c phi c star plus F N B C over a F; phi F prime equals to residual c. It is a residual error over C for a particular element this is the residual error and then you can write that.

So, this gets modified in this particular passion; now then what one can do? If your solution actually converges theoretically this guy should be 0, then my correction would be nothing. Once this gets zero this will the correction would be 0, then one can actually get the exact solution or reaches towards the exact solution.

Now, mathematically this is equivalent to the equation that we started solving. So, equivalent to this particular equation, but these particular one; the one which we obtained for the correction has certain numerical advantage. Now in these form what happens the numerical errors; during the solution of the equation are slightly less than those associated with the standard form; that means, the previous form which is a standard form. And for cases were small variation are expected for large values of phi; so that is an important advantage that you have once you write.

Now, in the residual form also we can use Patankar under relaxation. So, one can think in the residual form of Patankar's under relaxation. So, what we have seen that relaxation factor; now that now residual form of the it is implicit Patankar relaxation; Patankar's relaxation which you can derive from the equation or the standard equation of this. So, that is our standard equation and one can derive that in using Patankar relation. So, what you can do is that here also we have phi c which will be corrected at phi c plus phi c prime.

So, this is from previous situation as we have used in the residual form and this is the correction that is there. And if you put in the standard form of the equation which is a c phi c summation over element a phi F and b c. The modified equations in Patankar forms it becomes like a c phi c star plus phi c prime equals to lambda b c minus summation

over F N B C; a F phi F star plus phi F prime and then we have 1 minus lambda a c phi c star.

So, we are using the standard Patankar notation and then also trying to implement the residual form and then the system of equation will become like that. So, which you can simplify and simplify to a c phi c prime plus lambda summation of F a F; phi F prime equals to lambda which is b c minus a c phi c star plus summation of a F; phi F star; so; so right hand side essentially shows the residual of the original equation.

And now if this is the residual error over c; so it can be reduced down to a c divided by lambda phi c plus summation of N B; a F phi F prime equals to residual of c. So, that is the residual form of Patankar under relaxation, where we have used this value of lambda and lambda is an relaxation factor that what one uses to get the solution done. And while doing this under relaxing actually the equation it modifies only the diagonal coefficients; so which improves the rate of convergence.

Now, any iterative solution process it is quite important that one should be able to determine whether the solution considered is a good enough or when the error can be estimated to below certain tolerance or what precision the I mean error limit to be satisfied.

So, having the tools to answer any of the questions is an important ingredient for any CFD solver. So, it is what we have been or rather our focus in and around that that end of the day you must have an good CFD code; which can handle all these issues that we are discussing in this lecture.

So, it should be I mean accurate, it should be stable, you get quick solution, it should be applied to I mean multidimensional complex problem. So, all this kind of features and once as we are moving ahead with the discussions, you can see each and every term has his own property each and every phenomena they are treated differently. When they are put together in one single system, it becomes quite complex to deal with.

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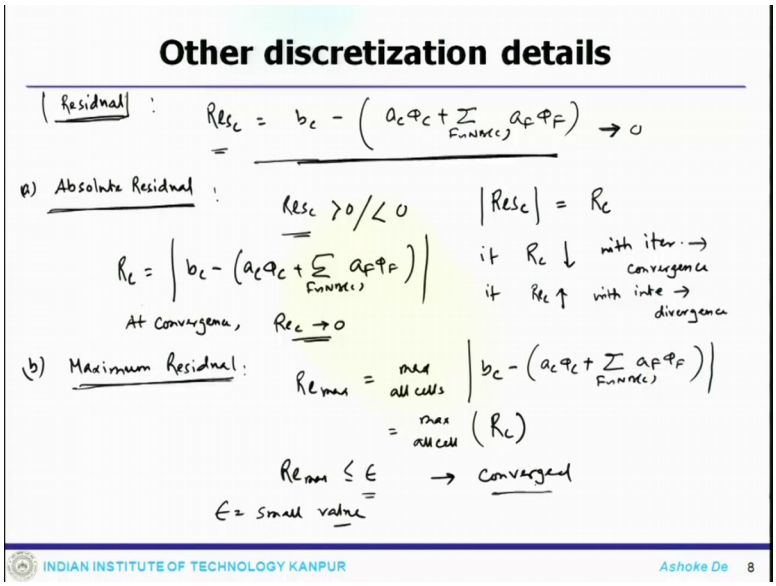

Now the other things that I would like to touch upon is the residual which one can think about what is that residual that we have been obtaining for a particular cell. We are writing that residual c is b c minus a c phi c plus summation over N B C; a F phi F.

Now it is very much clear that when the solution is reached and the converged completely and the gurney equations are satisfied; these guy should tend to 0. So, this is what it theoretically supposed to do or it is supposed to behave in that fashion, but what happens if you have issues with your numerical scheme or discretizations and other implications or implementation, then you may not get this.

And that is where iteratively you keep on updating and trying to reach towards the convergence. Now when you talk about the residuals different code or different implementations; those are possible and people can interpret in a differently. And let us look at that one thing you can say one has to look at the absolute residual and what is absolute residual?

Now, as we have defined in this particular equation the residual maybe this Res c could be positive or it could be negative. Since the sign is immaterial the absolute value of the residual which is denoted by this and it can be decided whether the solution. So, the absolute value of the residual Res c is essentially the R c; one can represent and then one can write the R c is nothing, but b c minus a c; phi c plus F over N B C a F phi f.

So, this absolute value can determine whether the solution has converged or not. If R c decreases with iteration then the solution will be converging. So, that leads to the convergence; otherwise if the absolute value increasing if absolute value is increasing with iteration; then that leads to divergence.

So, your convergence or divergence of the system is kind of dictated by this absolute residual and this is one has to and theoretically when you reaches towards the convergence this guy. So, at convergence this R c should be tending to 0 and then only it is guaranteed that you are getting a solution which is close to the converge solution of the actual solution otherwise.

So, this has a quit important value and when you are using some sort of a iterative solver this is one of the criteria the user has to satisfied or rather it has to be implemented inside your CFD code such that when you are checking your errors you has to has to have you have to check this error. Where this can give you an clear indication which direction your solution is heading.

Whether it is heading towards the stable solution or it is heading towards the; I mean unstable solution because that is how one can see the code the behaviour of the CFD code. Now the other way one can represent the residual as the I mean instead of looking at the absolute residual one can say I would be happy to look at the maximum residual; so, that is a another way of looking at it.

And when you say maximum residual the maximum residual is the way it defines that if you say then R c max is maximum of all cells of this magnitude b c minus a c phi c plus summation over all the neighbouring elements; so, this essentially max overall cells, so that is R c. So, this tells you that if your maximum residual which is say R e max less than equals to some small value, then the solution must have converged; so that epsilon here is a small value that would be provided to your inside your code to check.

So, one way to other way one can also think about that instead of looking the absolute residual; one can think you may look at the maximum residual. Then you calculate for Res c which stands for one particular cell then you go over complete domain and look for each and calculate this absolutely residual for each cell and after doing that calculations you check what is the maximum value of that residual? So, which then you put an criteria if that maximum residual inside the domain; that means, that is the what

cell which is providing you the maximum residual whether it has fallen down or gone down the below a specified small value.

If it is so, then you can estimate that you are heading towards convergence otherwise you may think of that this may lead to the divergence.

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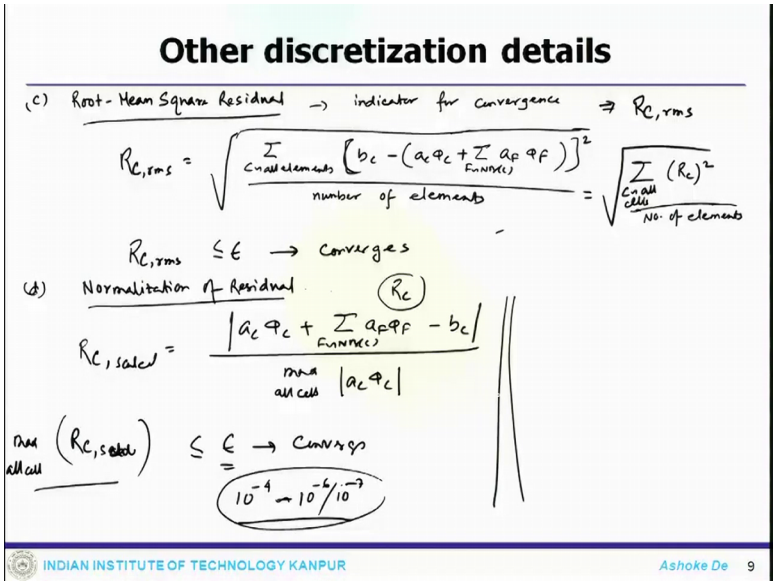

Now, alternatively only one may think about that root mean square residual. So, that is another parameter which can be an indicator for convergence. So, that one can look at it and how you represent that? You can represent that R; now R c and some sort of an rms. So, mathematically if you write this could be written as R c rms equals to square root of summation over, all elements b c minus a c phi c plus F N B C; a F, phi F and this will go over all the cells and square of that and divide by divide by number of elements.

So, you go over all the cells, take the sum of individual cell residual. So, in our terminology one can write which is nothing, but summation over all cells and this could be R c square; divided by number of elements. So, you can calculate individual cell residual and then take the square of that and get an sum over all the cells and divide by the number of cells and the square root of that. So, for convergence criteria; one has to put R c rms should be below and specified non then solution is converging.

Now the last option which is possible is the normalizing the normalization of the residuals; so normalization of residuals. So, the it is possible that the absolute residual is

a strong function of phi. And therefore, the different variables can result in a different R c. So, which makes is too difficult because if you have a multi variable system or degrees of freedom are different let us say velocity or the scalar; they can lead to different value of R c.

So, it is very difficult to see or put an criteria on the convergence or not. So, in that cases or such cases; it is better to look given inside which can be gain through the scaling the different residuals by dividing them by their respective maximum fluxes. So, recalling the a c; one can divide that R c, which is scaled equals to magnitude of a c, phi c plus summation over N B C a F phi F minus b c divided by maximum of all cells a c, phi c.

Now these case this converge when the maximum value of this scaled which is R c scaled or maximum of that for all cells less than a specified not then it is convergence. So, it is very common that these epsilon or the small value order of minus 4 to minus 6, minus 7 or something like that.

So, it is a very small value which is specified; so to get this calculation done or the get an criteria on the residual. So, one can see that you have can define absolute residual like we have done that. You can have a maximum residual of all the cells you can look at the roots mean square of the residual or you can normalize and this is quite advantageous when your degrees of freedom are high.

That means, you have a velocity you have some scalar and their residuals are different. So, in that case it is a good idea to look at this kind of scaled residual and put an criteria on the scaled residual to obtain the convergence criteria. So, we will stop here and then we will start the fluid flow problem in the next lecture.

Thank you.