Introduction to Finite Volume Methods – II

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Lecture – 32
Fluid Flow Computation: Incompressible Flows-II

So, welcome back to the lecture series of Finite Volume and where, we will continue our discussion where we left in the last lecture.

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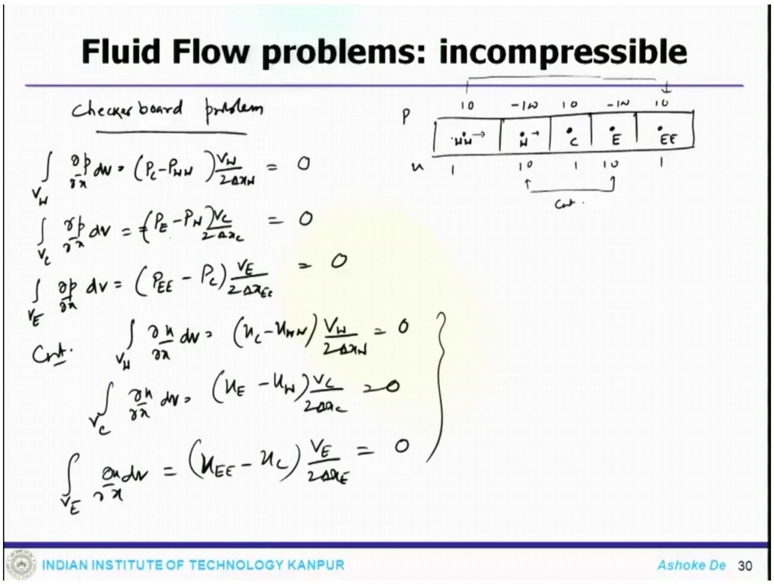

So, how to avoid this problem?

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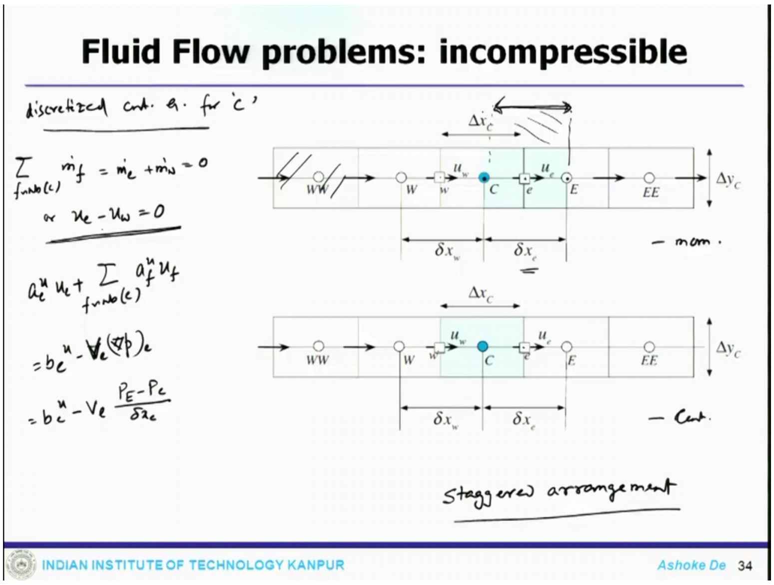

So, one of the solution to avoid this problem is that you use some sort of an staggered arrangement which is called staggered arrangement. As the name suggest you shift the pressure and velocity field and store them differently and how one can see that where you store your velocity field, you do not store your pressure field and where you store your pressure field you do not store the velocity field.

So, this is a typical staggered arrangement this is an staggered arrangement and here if you see this the top picture is shown for momentum and this is for continuity. So, what you do, as you can see that the velocity are typically stored at the face values; so, it is I am interested in this particular cell and the velocities are defined or stored at the faces. So, what helps that when you calculate the fluxes you do not have to do any interpolation straight way you can get the value and the pressure field is stored at the cell centre. So, that way one can avoid the arrangement of the checker board situation which appears in a system.

Now, if you look at the discretized continuity equation for element C, so, what it provides? It provides the summation of faces of m dot f which is m dot e plus m dot w which is 0 or u e minus u w at the face 0. So, you do not need to do any interpolation as I have stored the components at the cell faces, you do not required to interpolate anything here.

Now, top of that the momentum equation is integrated over the elements similar to the element C. Now, what one uses the integration now the integration for velocity component E, the effective cell which is considered is this one which will have an cell length of delta x suffix e. So, this is what is done. Now, you can see your scalars are store at the cell centre for that this is going to be the for scalar this is the control volume and when you talk about the velocity the control volume is going to be this. So, they are shifted by a small delta.

Now, once you write the discretization for this control volume the momentum equation becomes a e u u e plus summation of f around e a f u u f equals to b e u minus V e delta p e which is b e u minus V e P E minus P c by del x e. Now, the pressure gradient is related to the values which is connecting this and this; I mean the adjacent faces of that particular connecting faces. Therefore, the checker board pressure and velocity solution can be avoidable and it can be avoided in your numerical method.

Now, how do you get your pressure correction equation? So, for this particular arrangement whatever we have shown the staggered arrangement where the pressure is integrated over this kind of control volume and for velocity the control volume is modified.

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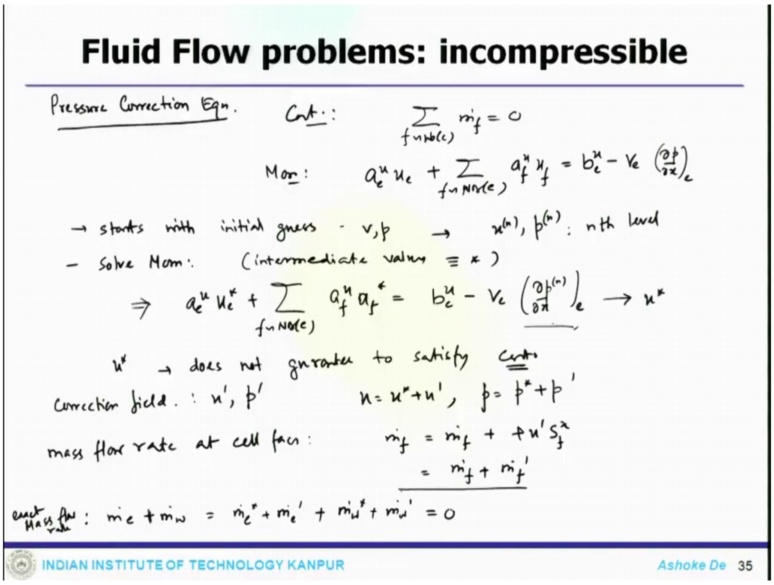

So, the checker board problem I mean the pressure correction equation for this pressure correction equation. So, the pressure correction equation, one can derive which would be the required for your pressure velocity coupling algorithm like simple.

So, we start with the continuity equation from the continuity equation once can get the summation over all the cell is m dot f equals to 0 and the momentum equation get you back a e u equals to plus summation over faces a f u u f equals to b e u minus V e del p by del x at e. Now, the solution actually starts with initial guess for the both for velocity and pressure field. So, once you denote this initial value the solution at the starts of any iteration with a superscript in the tentative velocity and pressure field are given as u n and p n which corresponds to at n-th level of the iteration.

Now, once you have an guess value you can solve the momentum equation to get a value which is actually giving you the intermediate one, intermediate some values which is given as star. So, when you solve that it get you u a e u star plus f ND e f u u f star equals to b e u minus V e del p by del x at n-th data trace e. So, the pressure fill is still based on the values from the previous iterations. So, at the intermediate calculation of the momentum field you use the pressure value which is coming from the previous level iteration which is n-th iteration.

Now, the compute the velocity field u star which actually satisfies this momentum equation, but whatever you get from here the u star, this u star does not guarantee to satisfy continuity. So, this is calculated based on some guess value and then we have used the pressure from the previous iteration. So, it is not guaranteed that I mean most of the time it happens that this intermediate value u star it does not satisfy the continuity equation, but for the whole system one has to satisfy both continuity and momentum.

So, then what is the del here is that there will be some correction field which is added to that u prime p prime which is added to the computed field and then assuming that that will satisfy the continuity equation and if once then my u become u star plus u prime, p become p star plus p prime. Now, the mass flow rate if you calculate mass flow rate at cell faces which will be corrected like m dot f equals to m dot f plus rho u prime S f x which is m dot f plus m dot f correction. So, the mass flow rate is to be corrected.

Now, in such that the exact mass flow rate satisfy the continue equation. So, if you assume in that fashion the it will like that m dot e plus m dot w which is our exact mass flow rate which will be now m dot e star plus m dot e prime plus m dot w star plus m dot w prime which would be now 0, which will satisfy this things.

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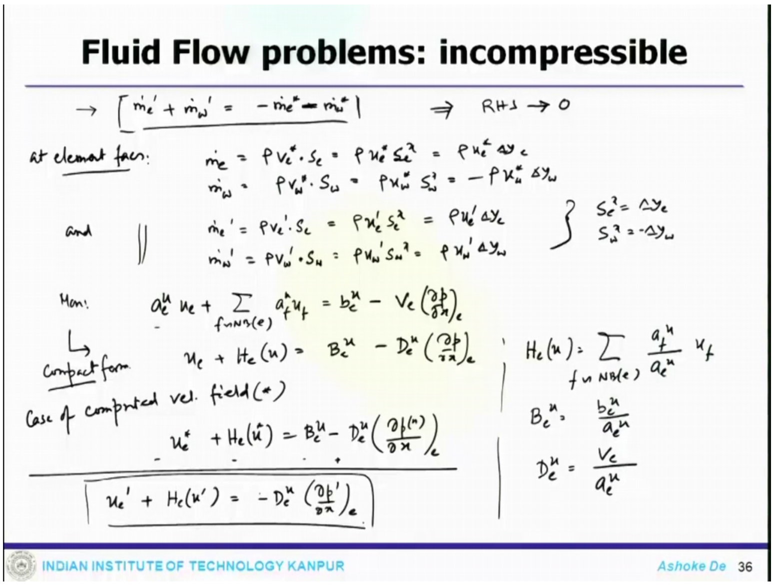

Now, one can rewrite that in the fashion that m dot e prime plus m dot w prime equals to minus m dot e star minus m dot w star. So, this is an interesting form of continuity equation. So, with the corrections or rather this equation represents the correction term for continuity equation. So, once you calculate the mass flow rate so, when there is no correction needed theoretically the right hand side should be tending to 0. So, that means, no correction is needed. So, whatever is the intermediate value calculated that is the exact value and that will happen when you iterate the solutions in over different level.

So, one can think about this mass conservation error of the current field so, that drives the correction field. So, the mass flow rate and the mass flow rate corrections at element faces; at element faces; if you write m dot e is rho V e star dot S c which is rho u e star S e x which is rho u e star delta y e and m dot w rho V w star dot S w rho u w star S x w minus rho u w star delta y w. So, and m dot e prime equals to rho V prime dot S e which is rho u prime S e x which is rho u prime delta y e.

And similarly the corrections for west face it will be rho V w prime dot S w which is rho u w prime S w x rho u w prime delta y w. Now, where your S e x is delta y e, S w x is delta y w minus delta y w which is used because we are in the one dimensional stencil like that. So, as long as you are in this kind of a one dimensional stencil this is what you can use.

Now, the pressure field does not appear neither this equation nor the equation that we have written here. So, now, you have to bring out the pressure field into the system. So, one can process to start writing the momentum equation which is this equation. The momentum equation for u face is a e u plus summation of at a f u f u b e u minus V e del p by del x at e. So, one can start writing if I write this equation in compact form; compact form it can be written u e plus H e u equals to B e u minus D e u del p by del x at e.

Where, you have H e u is summation of f N minus e a f u divided by a e u u f, B e u source term divided by a e u and D e u is V e divided by a e u. So, its essentially you are dividing by this coefficients and then rearranging the term and writing in this fashion.

Now, for the case of computed velocity field which is the star value. So, one can write u e star this star B e u minus D e u del p n by del x at e. Now, what one can actually do subtract this equation, from this one to this one. If you subtract the equation then what it gives? It gives an compact equation in terms of u e prime plus H e u prime equals to minus D e u del p prime by del x equals to e.

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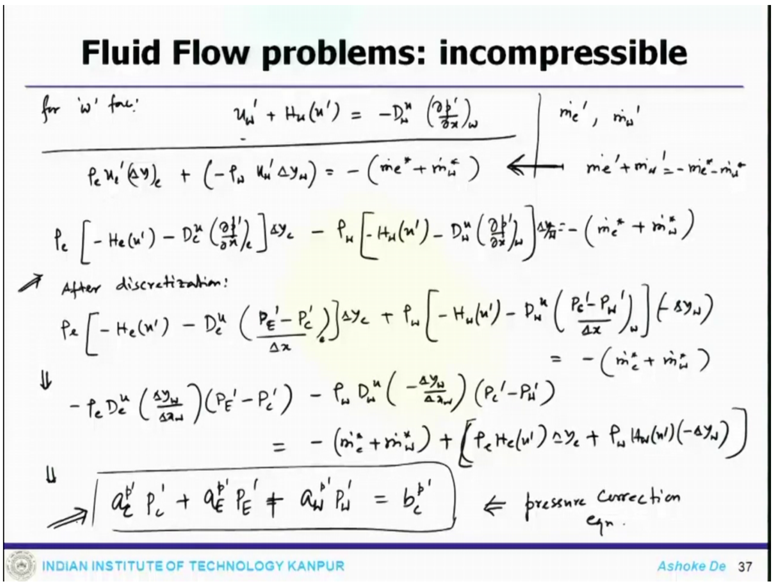

So, that is an equation which is exactly similar approach one can have adopt for w face and write that u w prime plus H w u prime minus D w u del prime by del x w. Now, if you use this and combine with the continuity equation, whatever we have got like m dot e m dot w prime these things if you put in the continuity equation and so, that is one component and whatever we have obtained here these things m dot e prime m dot w prime if you put those things in the continuity equation. So, that is getting u rho u u e prime delta y e plus minus rho w u w prime delta y w equals to minus m dot e star plus m dot w star.

So, the equation which was used here it is essentially m e prime plus m w prime equals to minus m dot star minus m dot w star. So, that is how it gets you back to the modifier. Now, the you we can replace this discrete equation of u e prime and u w prime that we obtained just right now and put it back in this equation what it gets you rho u minus H e u prime minus D e u del p by del x at e delta y e minus rho w H w u prime minus of that minus D w u d prime by dx w which is nothing, but m dot e star plus m dot star w.

Now, here the pressure field is appearing in this particular equation. The pressure field is appearing as an diffusion term. So, which after discretization so, after discretization what one can obtain is that rho u minus H e u prime minus D e P E prime P c prime divided by delta x multiplied with delta y e minus rho w. So, this could be written as H w u prime minus D w u P c prime minus P w prime by delta x at west this is at east. So, here there is a multiplication of delta y w minus delta y w equals to minus of m dot star e plus m dot star w.

So, you further expand this one and one can write rho e D e u del y w by del x w P E prime minus P c prime minus rho w D w u minus del y w by del x w P c prime minus P w prime equals to minus m dot star e plus m dot star w plus rho e H e u prime delta y e plus rho w; rho w H e w u prime. So, this will become minus delta y w. So, which is essentially you get this.

And the correction term in a discretized form one can think about it is a C for p prime P C prime plus a E p prime P E prime plus a W p prime P W prime equals to b c p prime. So, you get an discretized equation just like your diffusion system steady state diffusion system with source term and now, this is in pressure correction equation and the beauty of this discretized system if you compare with other one the expression look exactly similar. Where the differences would come? The differences would come in the definition of the source term.

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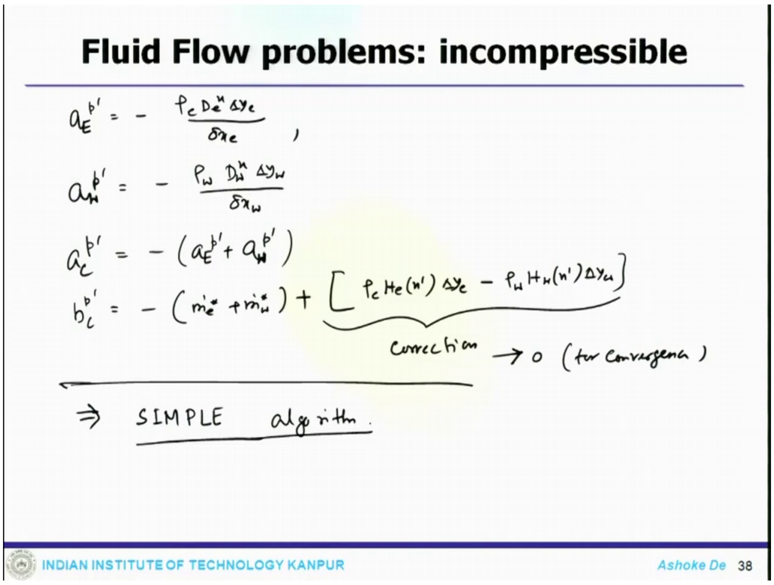

Here like coefficient term a E p prime is minus rho e D e u delta y e by del x e and a W p prime equals to minus rho w D w u del y w by del x w. You get a C p prime minus a E p prime plus a W p prime and your b C p prime is minus of m dot e star plus m dot w star plus rho e H e u prime delta y e minus rho w H w u prime delta y w.

So, this particular term in this involve the corrections. So, this is the corrections and this will become 0, when there would be no correction or the solution is convergence for convergence. So, that there will be no effect on the solution. So, the different approximation of these terms in this particular algorithm is essentially lead to the so called simple algorithm. So, this is the simple lead to the algorithm called simple algorithm and we will discuss the algorithm in details in the next lecture.

Thank you.