Introduction to Finite Volume Methods-II
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Lecture – 33
Fluid Flow Computation: Incompressible Flows-III

So, welcome back to the lecture series of finite volume and we are now discussing the discretization of the navier stokes system. And as in the previous lecture I have said that the complexity which is involved in the navier stokes solver is because of the pressure velocity coupling. And since so, far the diffusion system or convection diffusion system that we have discussed, we have not taken into account this pressure gradient term or the rather the pressure velocity coupling. So, that brings that extra bit of complexity while dealing with the navier stokes solver specially in primitive variables.

And what I mean by primitive variable when you stick to the system and solve with velocity component and pressure component density all this. So, you do not transform the system to any other derived variable which is very common in fluid flow problem.

Now while doing that we have looked at that when you store everything all the variables when you store on the cell center, which is a common practice when we have been talking about convection diffusion system that everything has been stored inside the cell centre, especially while talking about the structure orthogonal grid system. And if you stored everything in the cell centre that gives rise to the checker boarding kind of situation. And the remedy to do that that now you just shift by n half of the cell with; that means, now velocity components those are stored at the cell faces.

So, the control volume which will be defined for velocity component that would be the half of each cell and then the scalar variables you store inside the cell centre and that is the where we started doing the calculation. And now we consider a one dimensional system and try to derive the simple algorithm and where you get an equation, a pressure correction equation which actually conserves the continuity or the mass conservation. So, from the pressure correction equation whatever the update velocity field, we will get that will again update the velocity field to re compute the momentum equation.

So, we have just derived the mathematics and now we go back and look at the details of that algorithm.

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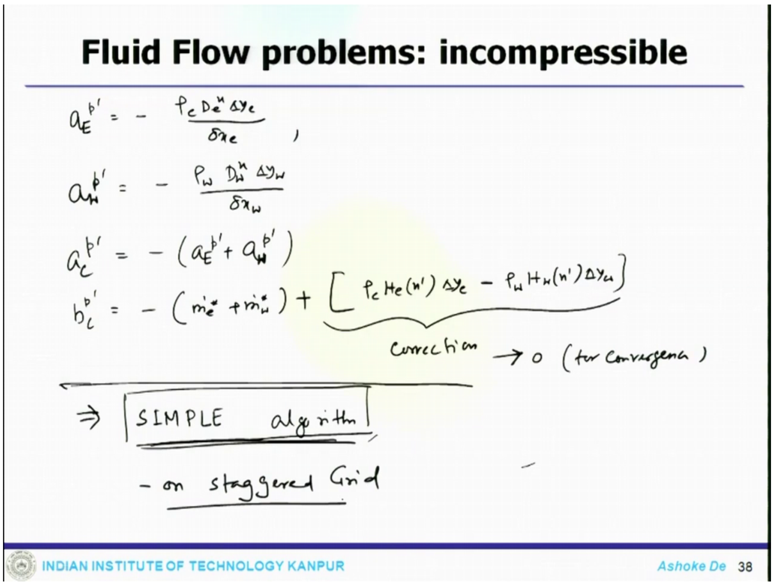

So, let us look where we stopped is that we are in the middle of these things and this is where we derive the mathematics and stopped getting the mathematics. And we got back this equation which is the pressure correction equation. So, if you recall. So, we have gone through this mathematics and got the pressure correction equation; this is what essentially is done in the simple algorithm.

Now we will look at how the algorithm actually works, but still please, keep in mind this is now what we are deriving on staggered grid. So, whatever we have been doing here this algorithm and the corrections term being in the staggered grid system.

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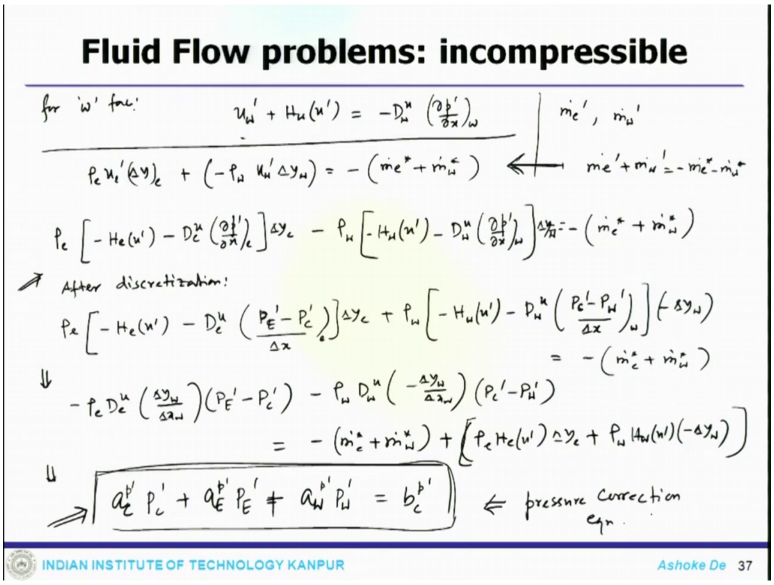

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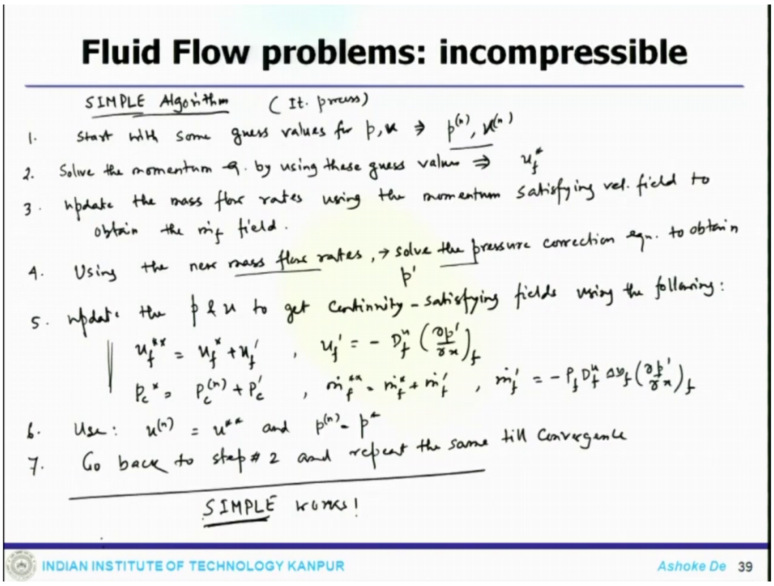

So, how the system would work for the simple algorithm. So, let us look at that the pseudo algorithm how it works. So, this is an iterative procedure to get the solution. So, how you do that? You start with some guess values for pressure and velocity. So, which will allow you to compute p n v n or u n whatever you call it, because in the one dimensional system. So, that is will compute these things at the every location in the domain.

So, that is the guess value or the start value. Now one can solve the momentum equation by using these guess values which will get you the new velocity field which is u f star. Then 3rd you update the mass flow rates using the momentum satisfying velocity field to obtain the m dot f. 4th using the new mass flow rates, you solve the pressure correction equation, solve the pressure correction equation to obtain p prime.

Then Fifth, now you update the pressure and u to get continuity satisfying fields using the following equations which is u f star, star equals to u f star plus u f prime where u f prime would be minus D f u del p prime by del x at f where P c star equals to P c n plus P c prime and m dot f star, star equals to m dot star f plus m dot prime f where m dot f prime is calculated as rho f D f u delta y f del p prime by del x f. Now, 6th you can use U n equals to U star, star and P n equals to P star.

So, you repeat the process like go back to step number 2 and repeat the same till convergence. So, it is essentially an iterative process. So, within that when you in the inside the domain you want to get or achieve the exact pressure and velocity field. So, you start with guess value then next level you solve the momentum equation. And once you solve the momentum equation you get the intermediate fields then you calculate your mass flow rates and all these things to check whether it conserve the mass flow rate or not.

And then using the new mass flow rate which will not be mass flow rate satisfying condition, you obtain the pressure correction equation and the pressure correction equation you solve for p prime. And then once you get the p prime then you update all the variable using this corrections and which will; obviously, satisfy the mass then you again assign those current value for the next level of iteration and you go back till you get the convergence. So, that is how the famous, simple algorithm works.

And this is one of the famous algorithm, which has been implemented in many commercial softwares or the CFD code they use this algorithm for pressure velocity coupling with some level of advancement and that we will discuss as we go towards the end of discussion of this pressure velocity coupling algorithms. So, now, if we go to a two dimensional domain.

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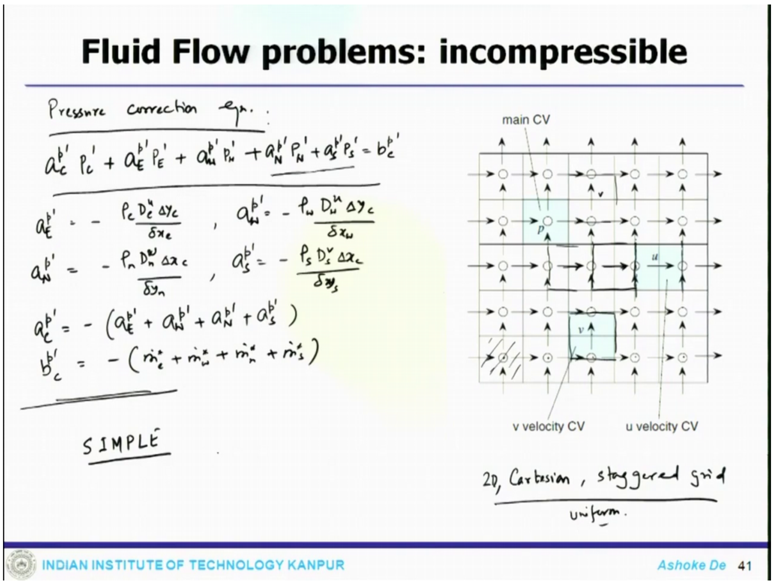

So, what happens there we can actually go back to a system so, where you see this is a Cartesian system 2 D Cartesian staggered grid. So, this is where you start doing the calculations for the pressure and velocity coupling.

So, we first looked at one dimensional. So, one dimensional was nothing, but a special case if you take any of this row where you have only u and v. And now we come back to so, as you can see since it is a staggered grid arrangement all my pressures are stored inside the cell centre. So, these are the control volume, which are going to be taken or considered for pressure. Now when you come down to the u momentum equation the control volume will shift by delta x by 2.

So, this is going to be the control volume for u momentum equation where u is stored at a place. So, this is the control volume for any interior node the u momentum equation. So, these are going to be the control volume. So, you get the u which is stored at the faces. And for v this is going to be the control volume for v. So, if you see v is also stored at the face so, these are going to be the control volume for v, because these are component of v. Now which means your pressure and velocity they are shifted by delta x by 2 and delta y by 2, if you think about its an uniform grid.

Now, this staggered arrangement it just to handle it in a numerical code or inside the code is just nothing, but handling the data structure which means essentially once you keep track of this cell centers and the node you just shift it for one cell or half cell for u and half cell by for v. So, you get a proper control volume for u and v momentum equation. Now what we have got the derivations which presented earlier or we have discussed earlier, they remain same. Now the pressure corrections equation that we will obtain it will have now contribution not only from the x component of the velocity, it will be now from the v component also velocity.

So, if we look at these the pressure correction equation, you can follow similar procedure that we have carried out for one dimensional case and can obtain the pressure correction equation. And the pressure correction equation will look like p prime P c prime plus a E p prime P E prime plus a w p prime P w prime a N p prime P N prime a s p prime P s prime equals to b c p prime. Now if you compare with your one dimensional case, you have now two extra component which will come or the coefficients in the linear metrics the contribution come from the north and south elements where a E p prime equals to minus rho e D e u delta y c divided by del x e.

A w p prime equals to minus rho w D w u delta y c divided by del x w; a N p prime rho n D n v delta x c by delta y n and a s p prime minus rho s D s v delta x c divided by del y s and a c p prime is a E p prime a W p prime a N p prime a S p prime and b c p prime equals to minus m dot e m dot w m dot n m dot s. So, that is what you get the correction equations with all relevant coefficients. Now once, you get the pressure corrections equation then, you have to solve the similar way like an simple algorithm. So, you can still apply the simple algorithm and solve in a similar fashion. So, in this case also you can start with some guess value.

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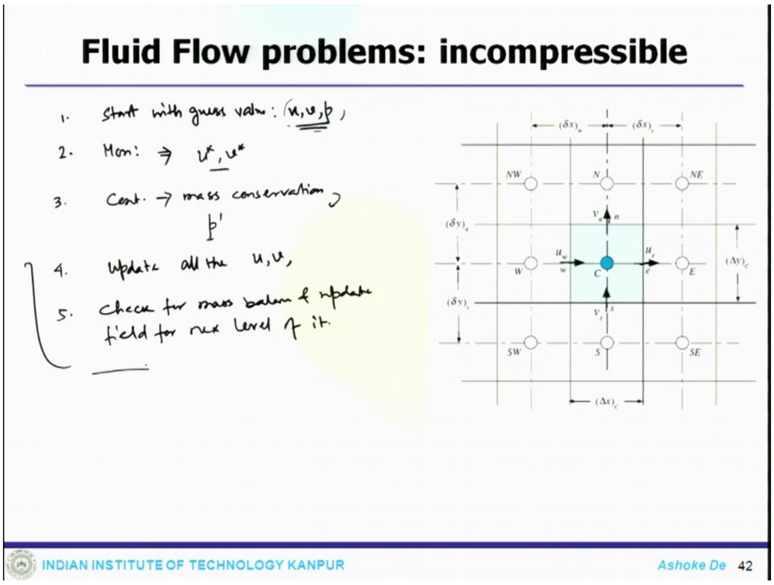

And now what will happen here you start with the guess value start with guess values now this is u v and p. So, essentially instead of u and p, now you have the second component of velocity v and p, then you solve momentum. Once you solve momentum you get all the u star v star and all these intermediate field using the guess value. So, you have to solve two momentum equations and u and v which will be stored at the cell faces. Once you get that then you check the continuity for mass conservation and once it is not there you end up getting the pressure correction equation.

So, you solve for the pressure correction equation and that is what exactly we have obtained here. So, once you solve for the pressure correction equation, you get the new pressure and then you update all the u and v and with that again you check for continuity otherwise. So, again check for mass balance and update the field for next level of iteration. So, you repeat this process and get the solution for that. Now this is what happens when you deal with the two dimensional grid. So, now, from two dimensional to three dimensional the things would change.

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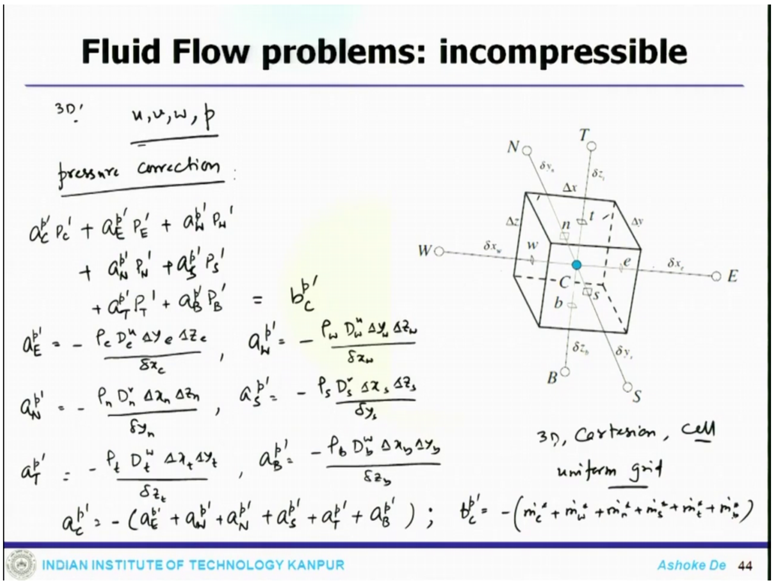

Now, if you go to a 3 dimensional cell. So, this will be a 3 dimensional element. Now this is 3 D Cartesian element there. Now you get 6 faces when 2 D you had 4 face. Now 6, you have 3 D 6 faces east west north south top bottom. So, you got all these faces and the. So, now, in 3 D your fields would be u v w and pressure.

So, once you use this information. So, 3 D also similarly you need to get an pressure correction equation. Now there the pressure correction equation will have more coefficients or your metrics will become a larger in size, what happens you get a c p prime P c prime plus a E p prime P E prime plus a W p prime P W prime. So, that is any direction along the x direction. So, you get this coefficient east west and see now you get for north p prime P N prime plus a S p prime P S prime. So, that will take care the north face and south face and then you get two more component a T p prime P T prime plus a B p prime P B prime which is nothing, but b c p prime.

So, the top and the bottom face. So, top and bottom face and here one can imagine the coefficients would be looking similar as long as we assume it is an uniform grid. So, the coefficients would look exactly similar that we have obtained for two dimensional case. So, let us get that.

So, a E p prime is rho e D e u delta y e and delta z e by delta x c. Now a W p prime is minus rho w D w u delta y w delta z w by del x w, a N p prime equals to minus rho n D n v del x n del z n divided by del y n a s p prime rho s D s v delta x s delta z s by delta y s, a T p prime equals to minus rho t D t w delta x t delta y t. Now this is delta z t a bottom p prime rho B D b w delta x b delta y b divided by del z b. And as usual a c p prime is minus a E p prime a W p prime a N p prime a S p prime a T p prime a B p prime.

So, the negative of all these coefficients and mass conservation would get the source term which is b c p prime equals to minus m dot e star plus m dot w star m dot n star m dot s star m dot t star plus m dot b star. So, that is what you get for all the coefficients in three dimensional system. Now once, we are talking about this staggered grid system, it is not necessarily that always this is going to be advantages there are certain situations where the staggered grid can be also a problematic. For example, I mean, but most of the regular calculations staggered grid is always helpful, but if you try to use for every case that is not possible.

Now top of this, this staggered grid calculations it has more memory requirement, because your data structure or the handling of data structure increases or the size of the data structure increases. And when you move to a non Cartesian system so, this is as long as you are in the Cartesian system, this is a perfect one, one can think about of using this can abort your checker boarding problem.

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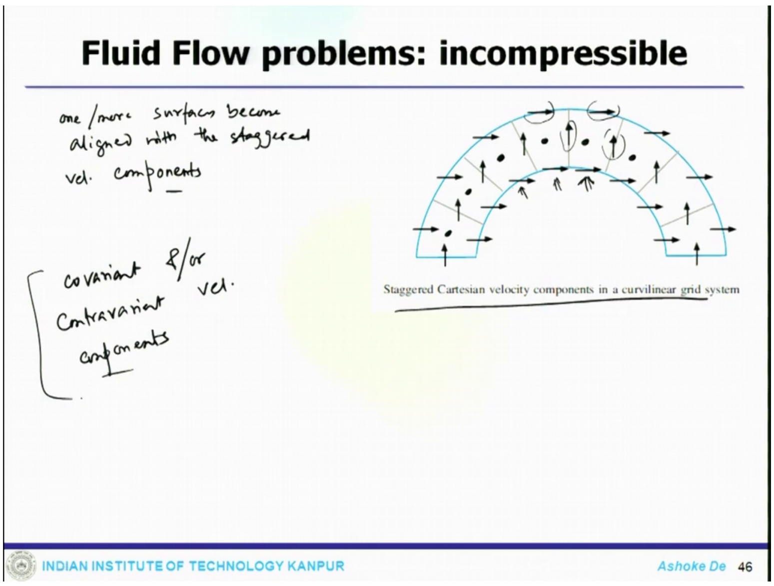

But if you go down to a non Cartesian system for example, in this kind of geometry where there is a u kind of bend. And now you try to this is a curvilinear grid system and the velocity fields are stored in staggered arrangement and this will be our cell centered.

Now, if that is the case then this can be a problem when one or more surfaces become aligned with the staggered velocity component. So, as you can see when you come down to elements like this one, this one and this one. Here, if you see this particular face component of the velocity or the u component of the velocity, pretty much it is aligned with the geometric alignment. Secondly, in the top surface also this guy is pretty much aligned with the geometric alignment similarly for v component of velocity this one and this component they are having problem.

So, this one and this one here this one this one so, they are aligned with the surface curvature and the component. Now one can think about a better alternative to use some sort of a covariant and or, or contravariant velocity components. So, if we use some sort of a covariant and contravariant velocity component then you, you can avoid this kind of alignment with the surface.

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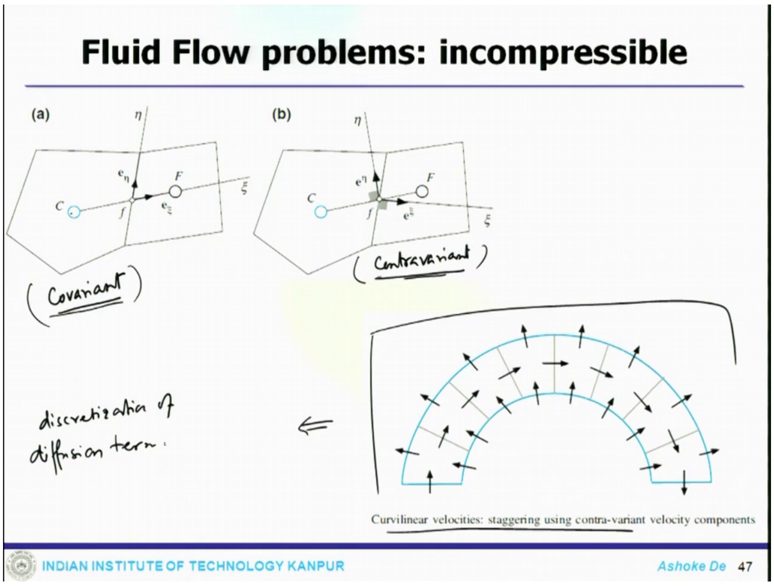

So, this is here you can see this is a example of an cell elements and the components which are shown here these are the covariant component and this is the contravariant component.

So, co and contravariant component, if you do then covariant component is goes in this direction contravariant in the other direction. And then you can have a specific way to calculate this covariant and contravariant component using your transformation system. And if you use the covariant and contravariant in the curvilinear system then this looks a perfect staggering arrangement which does not have any problem or you may not have at all the alignment of the flow field with the geometric curvature.

So, the idea is that whenever you have some geometric complexity or it is not a regular geometry, you can always transform than to a regular geometry and do that or rather your system has to take care of those covariant and contravariant components. Now the thing is that this may lead to some sort of a complications when discretizing the momentum equations in curvilinear coordinate system. So, due to this increased complexity in the treatment of the all the conservative terms.

Now, another option which could be specially this can lead to a problem in the discretization of diffusion term. So, another alternative one can think about to staggered all Cartesian velocity components in all direction. So, as to have all velocity component at all faces.

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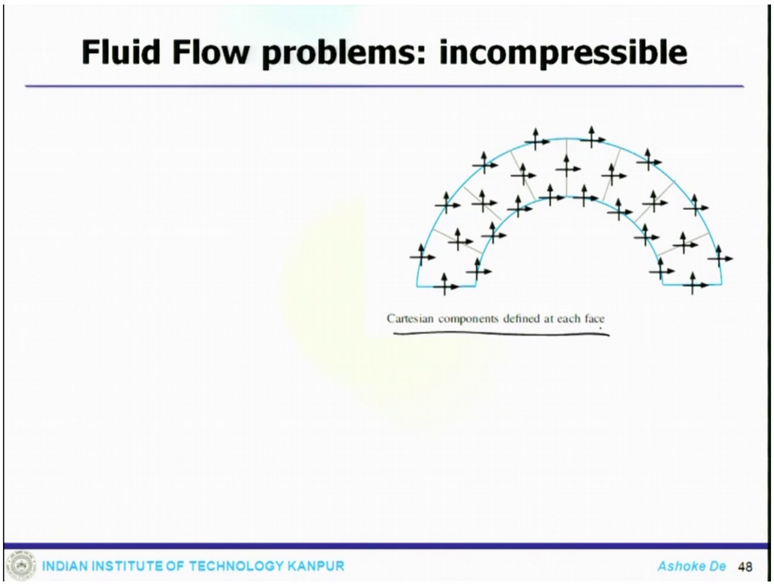

So, that would double the dimension or triple the dimension of the equation like this to be solved. So, we will stop here today and we will take from here in the follow up lectures.

Thank you.