Introduction to Finite Volume Methods - II
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Lecture - 34
Fluid Flow Computation: Incompressible Flows-IV

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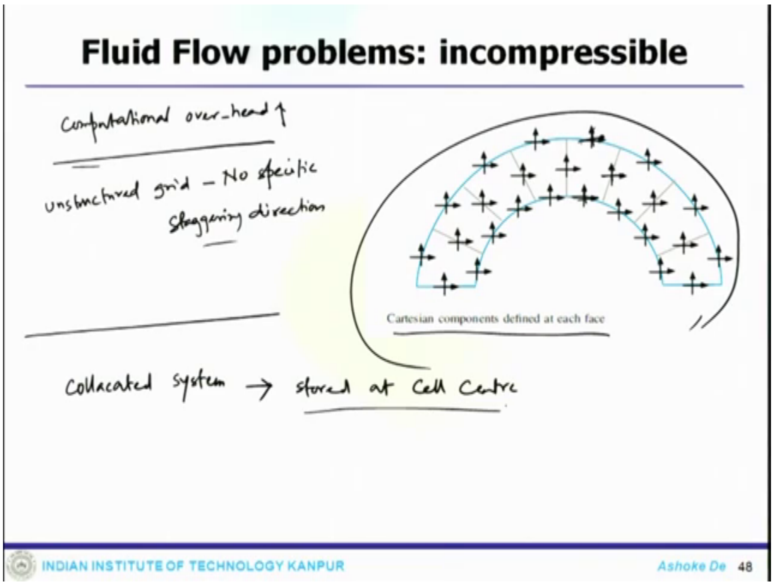

So welcome back to the lecture series of Finite Volume and where will continue our discussion, where we left in the last lecture. So, the computational overhead increases substantially, because all the faces now, actually starts storing all the component of the velocity field. For example, if you look at this face this storing this component also this component. Similarly, this face store both the component, this face also. So, each face stores both the components of the velocity field. Now, the problem is further complicated in this case of an unstructured grid.

So, this is n still a regular grid, but in a (Refer Time: 01:09) system, but when you have an unstructured grid, there you can get more problem. Now, in that case, the unstructured face you do not have any staggering direction. So, unstructured grid no specific staggering direction, which can allow you to do that and a only way for staggering concept, we apply is by changing the size of the cell elements, which was used for pressure and velocity component. Now, finally, what happens that the geometric information is also stored and more than the doubled when you go like this kind of arrangement increases the computational overhead like anything.

Now, which turns out when you see all these advantages, disadvantages or rather pros and cons of all these, it turns out that the collocated grid system is quite attractive collocated system where everything stored at cell centre is found to be quite attractive. And that is what I mean lot of shape decodes, which are on finite volume they prefer to use cell centre collocated grid system. But one can note that while the velocity components are stored at the centers of the elements as in the case for pressure or any other variable, the mass flux or scalar flux scalar value in a collocated grid is stored at the element faces.

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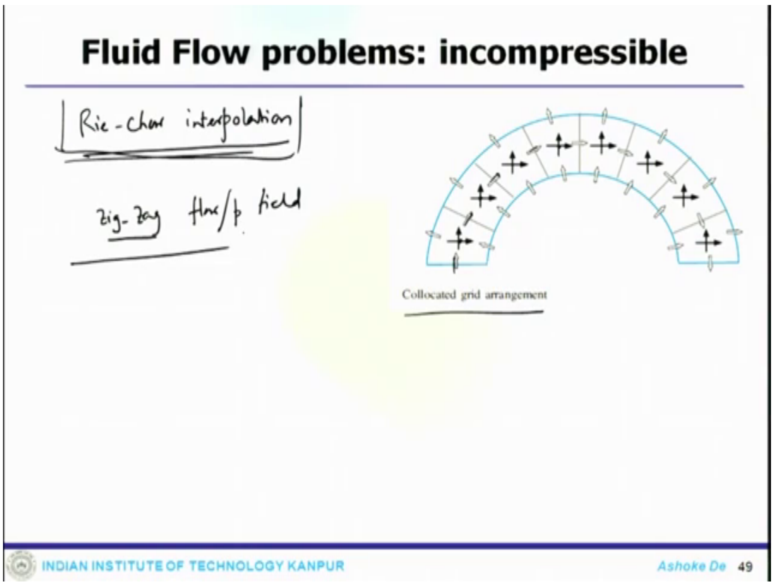

So, the like this the mass flux can actually be viewed as a contravariant component except that is the case, it is computed using a custom interpolation of a discrete momentum equation, which is known as Rie-Chow interpolation, which is one of the famous technique which is used in collocated arrangement. And using these interpolation, you can actually you have everything at the cell centre.

So, its a cell centered collocated arrangement and then you get an using this interpolation scheme, you get the fluxes interpolated at the faces, so that you can avoid the problem of the checker boarding issue. So, this is what the Rie-Chow interpolation does.

Now, the thing is that the deficiency in the original collocated formulation, which is the primarily the checker board or zigzag flow field, zigzag flow and pressure field these are now, can be avoided using this interpolation. Now, this one can see, how one can, it is essentially an equivalent to construct array in this particular arrangement.

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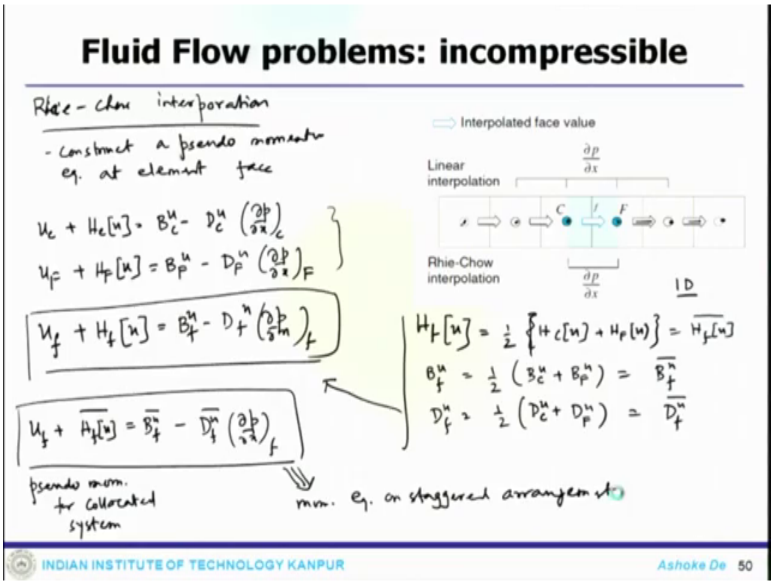

Now, you see we take an 1-dimensional stencil and everything is stored at cell centre; and these are the flux components. And we use this Rhie-Chow interpolation. Now, this Rhie-Chow interpolation are Rhie-Chow interpolation, what it does or in this particular method, what it does that it actually construct a pseudo momentum equation at element face with its coefficient linearly interpolated from the coefficient of the momentum equation, which are based on the cell centre.

Now, how it is mathematically done. So, you start with a cell C and the downstream cell F. and you write U C plus H C equals to B C u minus D C u del p by del x at c. Similarly, for F this is H F u equal to B F u minus D F u del p del x at F. So, now if u F is the velocity equation similar to this equation, then the pressure gradient link to the local neighboring pressure values can be seen here.

And u f can be written as H f u equals to B f u minus D f u del f by del x at f. So, what now since its in the collocated system, so the coefficient of this equation, this particular equation if you look at, it this is at the face. So, the coefficient of this particular equation cannot be computed directly, because its in the collocated arrangement and all the values are stored at cell centre.

So, since they are stored at the cell centre, you cannot do this. So, they are approximated by some interpolation from the coefficients of the neighboring nodes. So, you use some sort of a linear interpolation profile and there you can get H f u equals to half of H C u plus H F u, which is one can think H f u bar. Similarly, B f u, which is half of B C u plus B F u, which is B f u bar D f u, D C u D F u D f u bar.

So, you use these values into these equation and what you get an pseudo momentum equation, which is written as u f plus H f u prime B f u prime minus D f u del p by del x f. So, this is in all practical sense the momentum equation on a staggered grid. So, if you look at this particular equation, what you this is I say its a pseudo momentum for collocated arrangement or system.

But if you look at this is equivalent to an momentum equation on staggered arrangement. So, by doing this kind of interpolation what we essentially do, we actually transform the equation system to a such system, where you get an similar staggered arrangement, I mean similar equations which looks exactly like an staggered arrangement.

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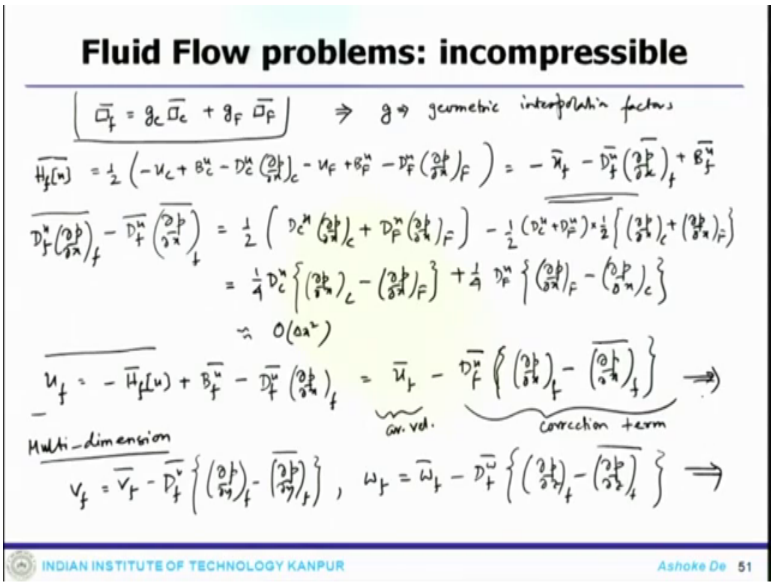

Now, in this one the between the c and f, one has to obtain the linear interpolation. So, any variable which lies there in between f can be calculated as like this plus g F like this. So, you some sort of an where g stands for geometric interpolation factors. So, it related to the position of the face with respect to c and f. Now, we can rewrite H f by u which is half of minus u c plus b c u minus D c u del p by del x c minus u F B F u minus D F u del p by del x F, which is minus u f prime D f prime del p by del x f prime plus B f u.

Now, if you look at the coefficients, this could be of second order. So, one can see that. So, this is my del p by del x at f minus D f u by del p by del x at f, which is half of D C u del p by del x c plus D F u del p by del x F minus half of D C u plus D F u into half of del p by del x C plus del p by del x F, which is equivalent to one-fourth D c u del p by del x C minus del p by del x F which is their plus 1 by 4th D F u del p by del x F minus del p by del x C, which would be second order approximated scheme. So, this shows that this is also a second order approximation.

Now what we do we get this one and then we put it back in the pseudo momentum equation. So, we can have u f equals to minus H f plus B f minus D f u by del p by del x f, which is u f bar minus bar del p by del x f minus del p by del x f bar. So, this is the average velocity and this component is some sort of an correction term. So, one can think of that way that.

Now, if you have a so this is in 1-dimensional, if you have multidimensional. Now, multi dimension, you get v and z component. So, you can write similarly v f equals v f bar minus D f v bar del p by del y at face minus del p by del y face bar, which is like that W f equals to W f bar minus del p by del z f minus del p by del z bar f.

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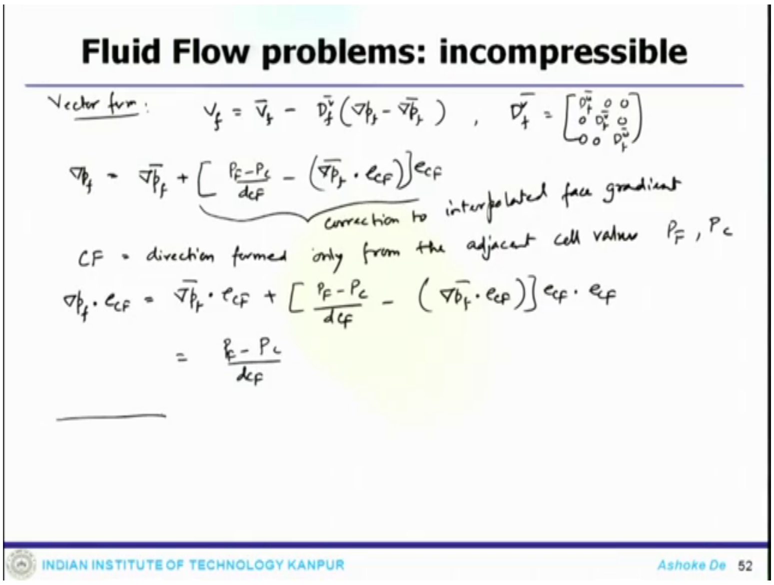

Now, what happens you can see the equation system that you can. Now, all these equation this one and this one for 2 to 3 -dimension one can write in a some sort of an vector form. If you write in a vector form, you can write V f equals to V f bar minus D f v bar delta p f minus delta p f bar, where D f v bar equals to a matrix which is D f u 0 0 0 D f v bar 0 0 0 D f w bar.

And delta p f is calculated as delta p f bar plus PF minus PC by d c F minus delta p f e c F and this is e c F. So, this is the term correction to interpolated face gradient. This already we have seen while dealing with the unstructured grid system and specially in the diffusion system we have done this calculation.

Now, where CF is the direction formed only from the adjacent cell values P F and P C. So, we calculate del pf dot e c F, which is del pf bar dot e c F plus P F minus P C by d c F minus del pf dot e c F e c F dot e c F, which will get you back P F minus P C by d c F, where the face velocity is closely linked to the pressure of the adjacent cells.

So, here you can avoid the checker board problem. Now, before we derive the multidimensional momentum equation, multidimensional pressure corrections equation, so one can look at the some sort of an discretization of the momentum equation or the discretized momentum equation.

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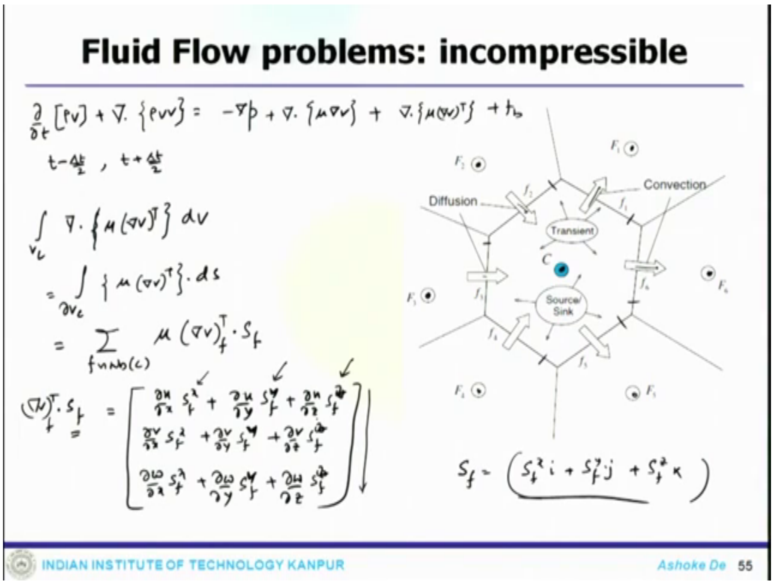

So, let us consider this element, which is very standard element unstructured element and element C, where the cell centre is there, then you have all these 6 surroundings elements, then you have this faces, this face, these, these, this, this, all these faces are there. These are the surface vectors, which are normal to those faces. And the momentum equation, you write in this fashion del del t of rho v plus delta dot rho v v equals to minus del p plus delta dot mu delta v plus delta dot mu del v transpose plus f b source term.

Now, you want to discretize this one within the time interval t minus delta t by 2 to t plus delta t by 2. Now, one can basically take the volume integral and then convert them to a surface integral as we have done. So, if you go about that way, then we look at first individual term by term system. So, first look at this term, which is essentially del dot mu del v transpose d v. If you convert that to surface integral, it is mu del v t dot d s, which will be surface integral of n b c mu del v transpose dot s f.

Now, the expanded form of del v transpose dot s f or s f in a 3-dimensional co-ordinate system, one can think about this would be del u by del x s f x plus del u by del y s f v plus del u by del z s f w. Similarly, del v by del x s f x del v by del y s f v plus del v by del z s f w and del w by del x s f x del w by del y s f v del w by del z s f w. So, this would be x y not v just make a note here this is surface vector. So, this would be s f x, this is s f y, this is s f z. So, along this column more about they are same, it is a surface vector which is a component of s f x i plus s f y j plus s f z k. So, when we take the dot product, it will be all y j k.

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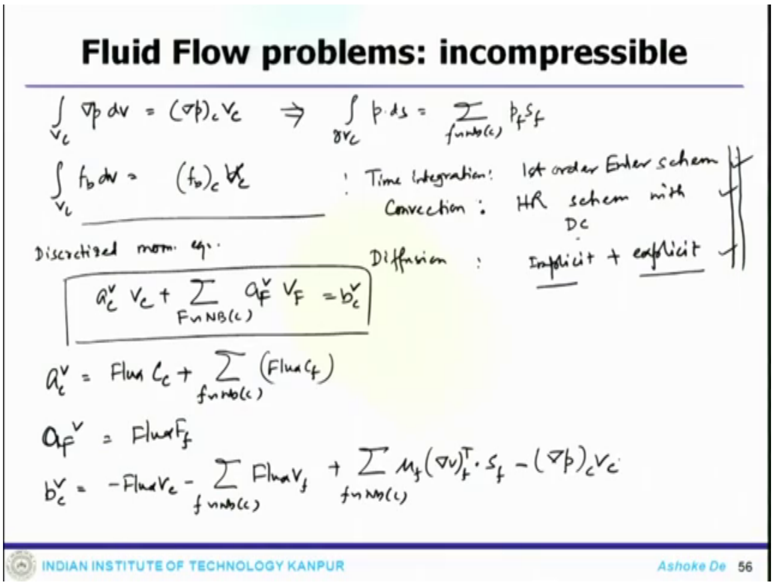

Now, similarly we can look at the pressure gradient term, which will be delta p c v c. And when we transform it to surface integral, this will become essentially del v c p dot d s, which is summation over all the cell faces p f s f and body force term which is also a volume integral, which will be written like that.

Now, one can use for the time integration, if we use the first order Euler scheme, so first order Euler scheme if we use and discretize the unsteady term and for convection term, we use convection, we use HR scheme, which is higher order scheme. And it is implemented HR scheme with default correction, so which can be taken care of the and decomposing the diffusion flux, which is implicit part aligned with the grid and explicit cross diffusion plot. So, diffusion will have implicit component plus explicit component that means this is aligned with the face and this is cross diffusion term. Then the discretized momentum equation would look like discretized.

And by the way you can note or one has to note that all these discretization process, we have done separately and that is why I have been telling when we come down to the fluid flow solved problem individual component integrations and discretization they are taken care of. So, this we all separately discussed. And now we will put together things for the momentum equation and which will look like a c v v c plus summation over NB C a F v v F equals to b c v. Here v superscript stands for the vector. And here the coefficient flux are given by a c v is flux C c plus surface flux or the summation over faces flux C f. And a F, which is flux F f and b c v, which is given as flux V c minus summation f N b c flux v f plus summation of N b c mu f del v f transpose dot s f minus del p c v c.

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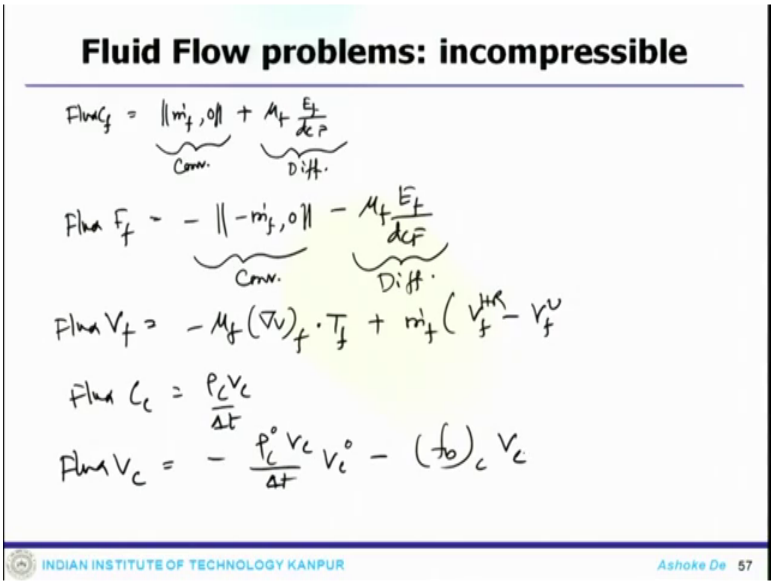

Where you can have the term like flux c f equals to m dot f 0 plus mu f E f by d c F, which is convection contribution, this is your diffusion contribution. And flux F f equals to minus m dot f 0 minus mu f by d c F. This is again conviction contribution; this is again diffusion contribution. And flux V f equals to minus mu f delta v f dot t f plus m dot f v f HR resolution high resolution scheme minus v f upwind scheme. And we can compute other two term which is like C c equals to rho c v c by delta t and flux v c equals to minus rho c naught v c by delta t minus c v c. So, this is how, we calculate individual term and we stop here today and take it up from here in the next class.

Thank you.