Introduction to Finite Volume Methods-II
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Lecture – 35
Fluid Flow Computation: Incompressible Flows-V

So, welcome back to the lecture series of finite volume and we will continue our discussion on the discrete momentum equation. And what we are doing that, we have looked at the staggered arrangement in the previous lecture. And then, we have discussed what is the problem with the staggered arrangement and why the collocated arrangement is a one of the preferred method in a finite volume approach.

Now, while doing the collocated arrangement, then we can actually modify the equation system and then you can get a pseudo momentum equation which will look exactly like an staggered arrangement. So, that allows you to avoid all theses checker boarding problem that you come across in non collocated system. So, this is where we stopped in the last class and now we will continue from there.

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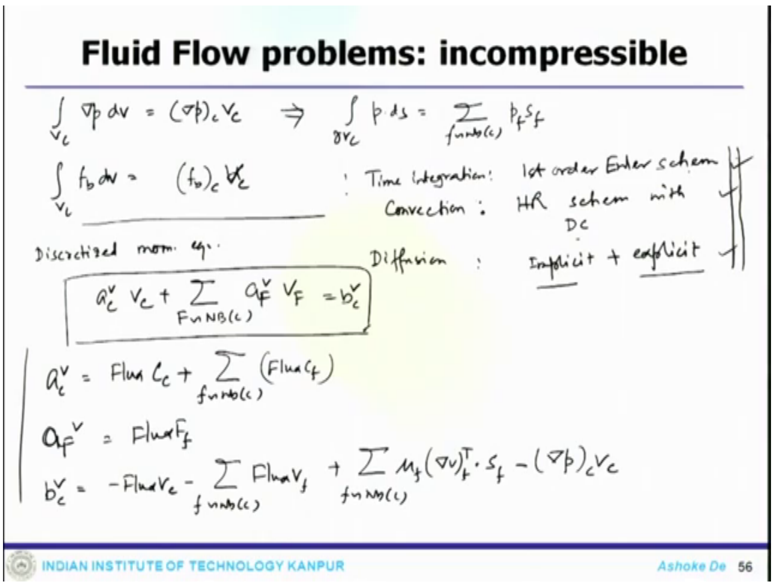

So, if you recall, this is the discretized equation for the velocity system or the pseudo momentum equation and here the fluxes are like this. This is where we stoped. And, all the other term like time integration, convection term and the diffusion term can be obtained which we have done earlier and then the coefficients would be calculated like this.

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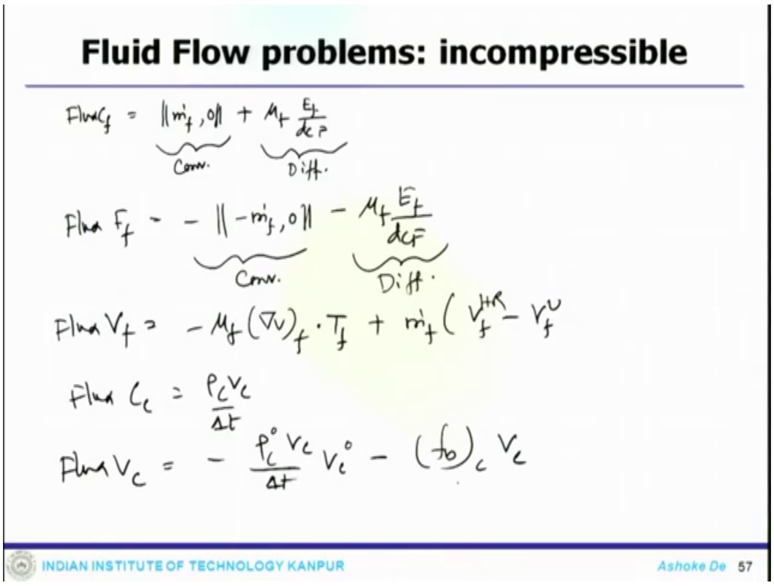

Now, in this particular system, this coefficients are depend on velocity and pressure fields and there is a nonlinearity associated with that. So, the nonlinearities handled by the essentially by your iterative solvers. And now, change in the value of the coefficients which can lead to a change in your flow field.

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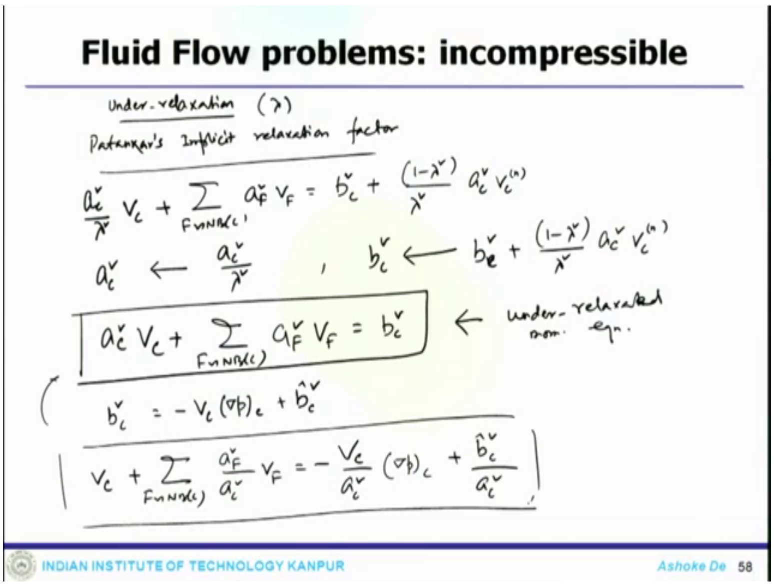

So, that effects the convergence and that is why, we need to do some sort of an under relaxation. So, since the system appears to be stiff, so use some sort of a under relaxation factor and this is adopting the Patankar implicit relaxation factor. So, this is Patankar's implicit relaxation factor. So, by invoking that, one can write that system of equation like a c v by lambda b V c plus F which is summation over cell and a F V V F equals to b c v minus plus 1 minus lambda v divided by lambda v ac v and V c n.

So, now you can redefine your a c v which is essentially a c v divided by lambda v and b c v it is redefined at b c v plus 1 minus lambda v by lambda v, a c v V c n. Now, the under relaxed momentum equation, one can write like a c v V c plus F NB c a F v V F equals to b c v. So, this is what you write as under relaxation or under relaxed momentum equation.

Now, for the derivation of the collocated pressure correction term, the pressure gradient is taken out from the b c or the source term and can be explicitly written as b c v equals to minus V c delta p c plus b c v. And now you substitute this in the this equation, you get V c plus summation of F over NB c a F V by a c v V F V by a c V v f minus v c divided by a c V del p c plus V divided by a c v.

So, what are the coefficients which are associated with this particular arrangement?

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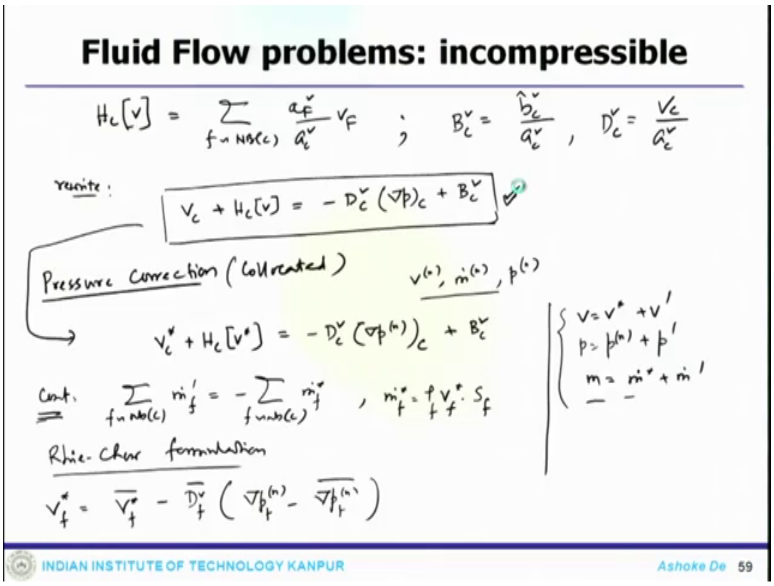

So, we can define the following vector operations which is H c V equals to summation of this V by a c V F and here B c v equals to b hat p by a c v and D c v equals to V c divided by a c v. Now, the equation previous equation this one, you can reformulate as or rewrite in a vector notation like V c plus H c v equals to minus D c v delta p c plus B c v. So, this is how you can write and this should be very useful later on when we will do the discussion of the other terms.

Now, one has to find out the pressure correction term. So, the pressure correction in collocated arrangement, so how do we find out that? Now, you can use our previous iteration values like v n m dot n p n. So, these are the previous iteration values that we can use and this vector form of the momentum equation if we put this things and the intermediate velocity field which we will be calculated as V c star plus H c v star equals to minus D c v delta p n c plus B c v.

Now, the final solution should satisfy the exact solution. Now, now the difference between these two equation is going to give us the correction equation. And now, when you look at the velocity field, they must satisfy the conservative equation or the continuity equation and then if there is no mass flux from there, we get the pressure correction equation.

Now, if you say the correction in a standard form which we have been using v equals to v star plus v prime, p equals to p n plus p prime, m equals to m star plus m dot prime, so all with the guess value and the corrections which should satisfy the conservations equation. Now, substituting this into the equation, will get the conservation equation for the. So, we will use these things first in the continuity equation. So, the continuity equation essentially give us back that all the faces m dot prime f equals to minus faces where m dot f star with m dot f star equals to rho f V star f dot S f.

Now, the face velocity is computed using the Rhie Chow formulation. So, once we use this Rhie Chow formulation, you get V f star equals to V f star average minus D f v delta p f n minus delta p f n average. Now, in the computed mass flow rate field is conservative, the right hand side of this equation should this continuity conservation equation should go to 0; from this side should go to 0. Now, if you have a incorrect velocity field, then there will be a source term and from there, the correction has to be imposed. Now, the mass flow rate corrections can be written in terms of velocity corrections and which one can derived from this vector equation.

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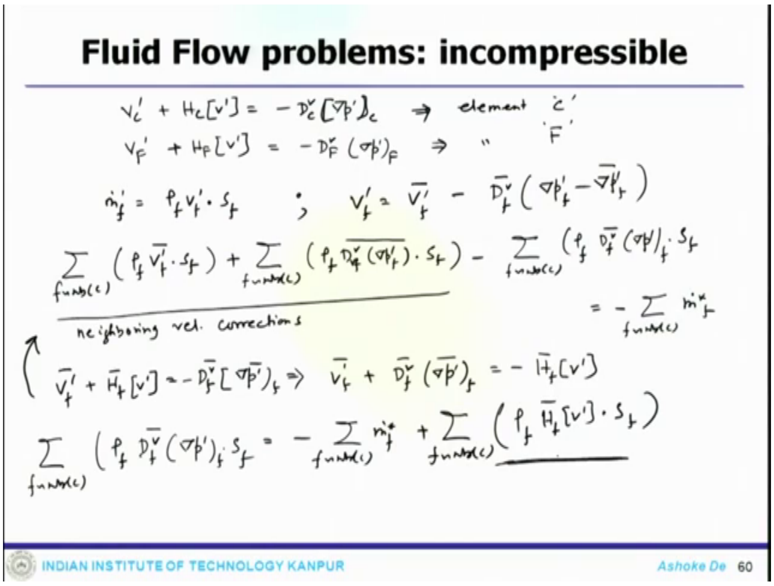

So, the corrections equation for the velocity would become V c prime plus H c v prime minus D c v delta p prime c. So, that is the correction for the velocity field. So, this is for element C.

Now, the neighboring element F, the similar equation will be there which is H F v prime minus D F v delta p prime F which also hold good for element F. Now, the mass flow rate corrections at cell face is estimated as rho f V f prime dot S f where the correction velocity field is obtained as minus D f v delta prime f minus delta prime f bar. Now, if we put together all these equations, so, finally, we will get the pressure corrections equation in this format which is faces over rho f V f dot S f plus all the faces rho f D f v delta p prime, this dot S f, then minus f rho f D f v delta p prime. So, dot S f equals to minus, this is going to be m dot star f.

Now, this portion actually represents the neighboring velocity corrections on the velocity of the element. Now, this influence will become much more visible when we interpolate this things to the face and get an equivalent expression like V f prime plus H f v prime minus D f v delta p prime which will lead to v prime plus D f v delta p prime and equals to minus H f v prime. Now, once we substitute this one here, you can get summation of f

NB c rho f D f v delta p prime f dot S f equals to minus f which is m dot f plus f goes over the faces, rho f H f dot S f.

So, this term one can find it out more explicitly in the following form.

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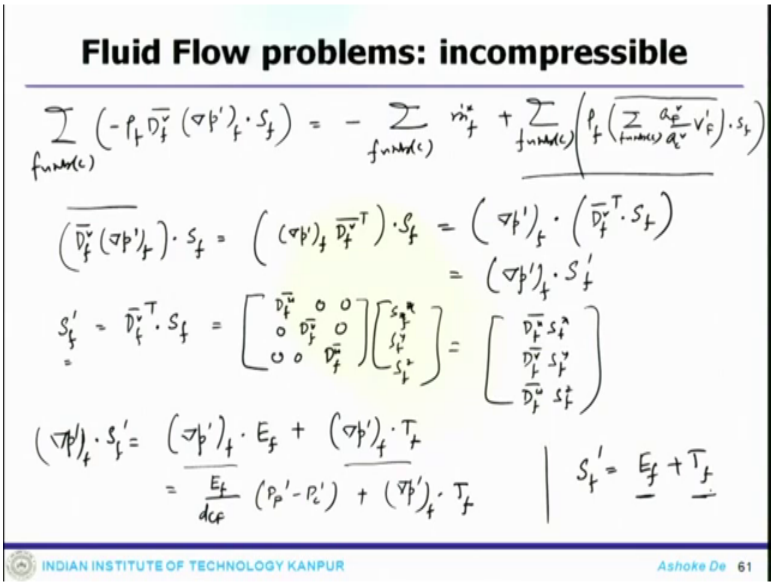

So, you can find out this is f which is minus rho f D f v p f S f which is minus n f c m dot f plus f goes from their rho f summation of over n B c a b by a c v V prime F. So, this is bar dot S f. So, this is done and the previous equation this underline term. So, this term is critical for solution of this equations. Now, in the original simple equation or algorithm, this was actually neglected for the coupling of pressure and velocity.

But because this is a correction equation, the modification or the dropping of these turn will not affect the final solution since at convergence, the correction becomes 0; however, it may affect the convergence rate in the larger. Convergence rate in that the larger is neglected term, the higher will be the error present in the approximate at each iteration.

Now, the remaining term can be easily treated and the coefficient of the pressure correction terms can be obtained in a simplified form. For example, we take these term which is present there and del p prime f which is dot S f equals to del p prime f D f v transpose dot S f which is del p prime f dot this transpose dot S f which is del p prime f dot S f prime.

Now, the expanded expression is giving that S f prime equals to D f prime transpose dot S f which will be D f u 0 0, 0 D f v 0 0, 0 0 D f w into f x S f y S f z which will get you D f u S f x, D f v S f y, D f w S f z. So, now, you can work with this prime component the surface vector component and the pressure correction gradient can be calculated like delta p prime f dot S f prime equals to now delta p prime dot E f plus delta p prime dot T f. So, which is like an implicit component, this is the cross diffusion component. This we have seen while looking at the discretization of the diffusion equation.

So, this will become E f by d c F and p F minus p c prime plus del p prime f dot T f where the S f prime will have two component E f plus T f. So, one is along the line which is connecting between the cell centre and this is normal to that. So, these are already discussed while discussing. Now, if you drop the non orthogonal contribution and linearize the term of the pressure correction equation, then this guy will become del p prime f dot S prime f equals to E f by d c F where p F minus p c prime which will become D f minus p F prime minus p c prime.

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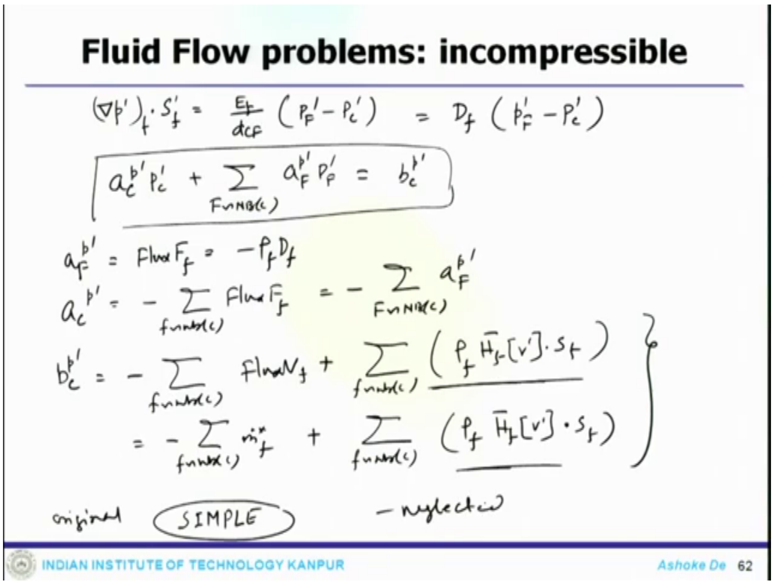

Now, once we substitute that in that this in the pressure corrections equation, so that we will look a simple correction equation like plus F go over all the cells a F p prime p F prime equals to b c p prime. So, and the coefficients are a F p prime equals to flux F f which is minus rho f D f, a c p prime equals to minus F which is flux F f minus summation over all the cell a F p prime and b c p prime equals to summation flux V f plus summation over rho f H f V prime dot S f which one can write summation over this plus summation over rho f H f dot S f. So, you can actually different approximation can be used for these terms, I mean either of these or this. And once you use different approximation for this term, that will lead to a different sort of algorithm but when we will look that the original simple algorithm, the original simple algorithm, these terms are essentially neglected, these are neglected.

Now, one can think about once you neglect that, what can happen to the solution. The solution finally, still you can achieve the solution because it is a iterative process, but it may slow down or some sort of errors will accumulate over the iteration.

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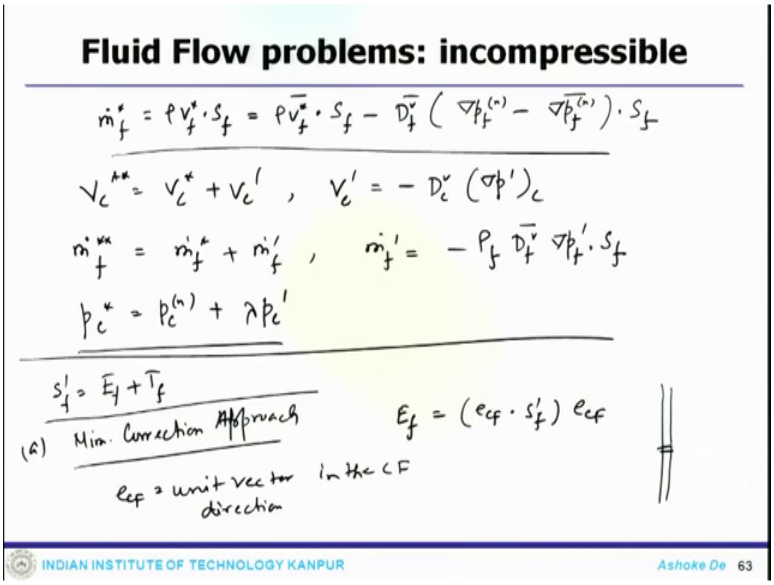

Now, after getting all these corrections and the Rhie Chow interpolation, finally, the mass flow rate can be calculated as rho V f, now rho V star f dot S f minus D f v minus delta p f n minus delta p f n bar dot S f.

Now, the pressure and velocity field which are corrected and calculated at the cell centre, they can be used to correct the mass flow rate. And now one thing is there, once you get those things, then the velocity field is corrected as V c star plus V c prime where V c prime is corrected as del p prime by c and m dot star star f which is corrected as plus m dot f prime and m dot f prime equals to minus rho f D f v delta p f prime dot S f and p c star is p c n plus lambda pc prime. So, some sort of an under relaxation which is used to update the pressure.

Now, the kind of decompositions which can be used as we have looked at in our previous diffusion calculation for this component where your S f is E f plus T f, so here one can different multiple approaches can be adopted one can do minimum correction approach which we can refer to our diffusion system where this can be calculated as E f e c F dot S f e c F and e c F is the unit vector in the c F direction.

Now, once we combined that, so the readers are or all of your strongly encourage to go back and look this calculations in the context of diffusion equation because these are discussed in details there.

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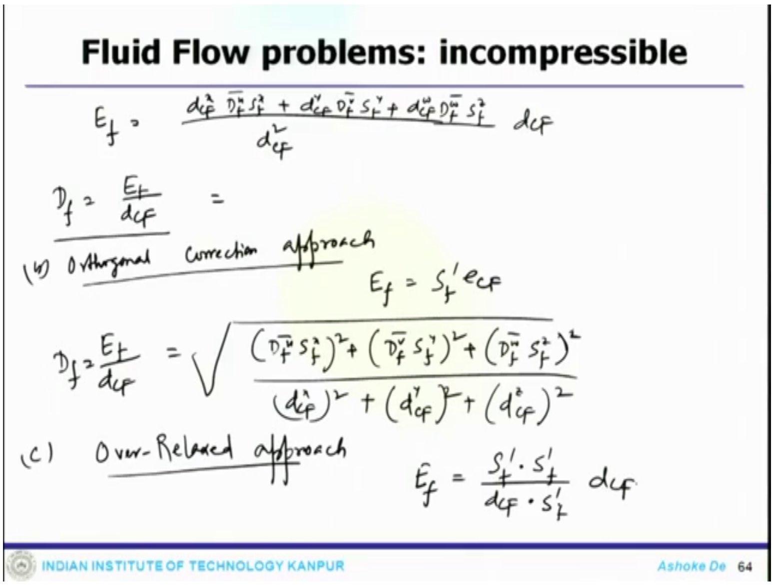

So, E f would become d c F x D f u S f x plus y D f v S f y plus d f w D f w S f z divided by d square c f and d c F which following the expression one can find out E f by d c F. So, one can put those expression back here.

Now, other way round, one can do the orthogonal correction approach where you find E f as S f prime e c F and then you calculate E f by d c f which is nothing but D f u S f x square D f v S f y square D f w S f z square divided by d c F x square d y c F square d z c F square. So, and third option could be over relaxed approach over relaxed approach where you get E f S f dot S prime d c F dot d c F. These are again; these are all discussed in details in the context of diffusion system.

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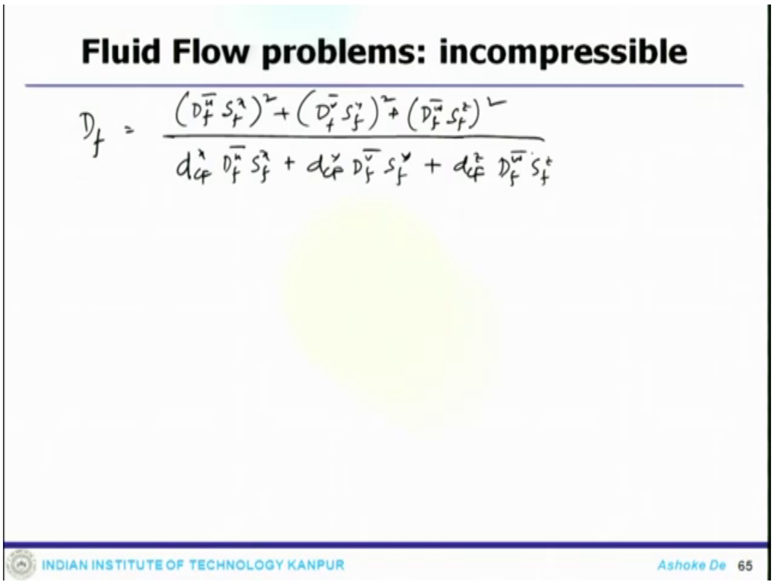

So, the reader can go back and look at those derivation in details there. Then, you can correlate the thing here easily, D f w S f z square divided by d c F x D f u S f x plus d c F y D f v S f y plus d c F z D f w S f z. So, one can be done. So, we will stop here today and we will take from here in the fallow up lectures.

Thank you.