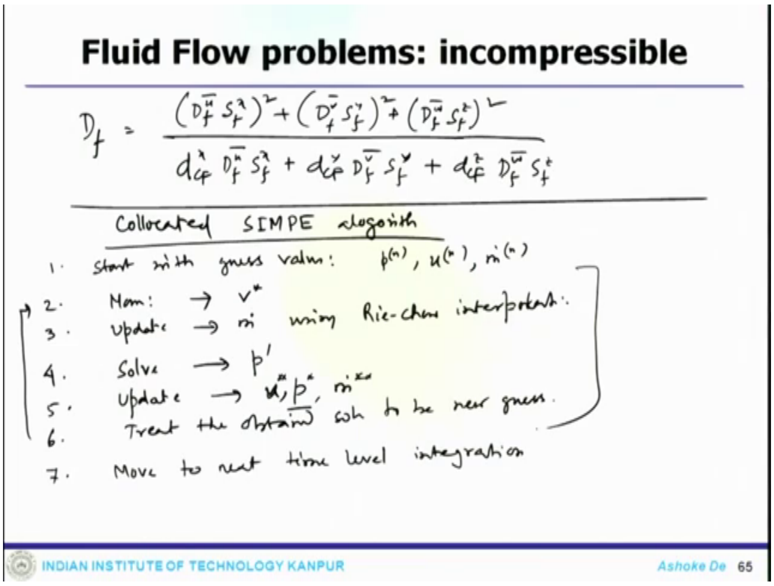
Introduction to Finite Volume Methods – II

Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture – 36
Fluid Flow Computation: Incompressible Flows – VI

So, welcome back to the lecture series of Finite Volume.

(Refer Slide Time: 00:19)

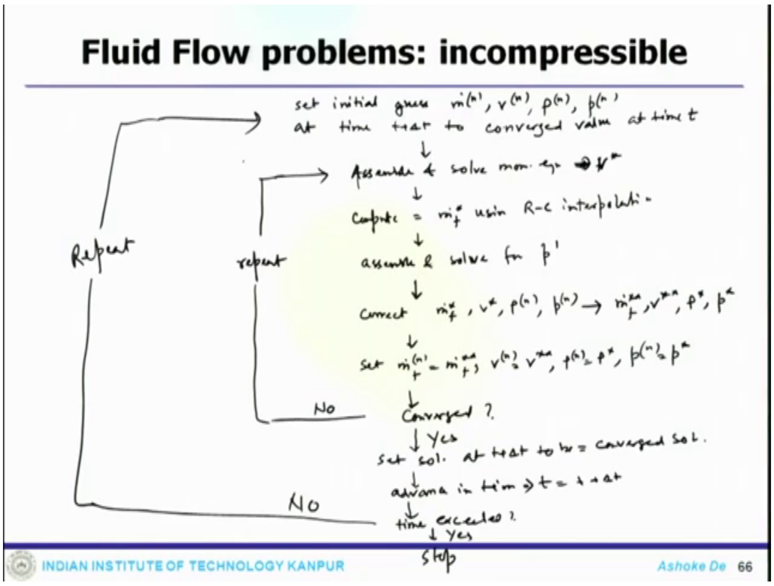


And, where we will continue our discussion where we left in the last lecture. So, if you put the collocated algorithm together. So, collocated simple algorithm so, what it says that so, again you start with guess values which is p n, u n, m dot n. Then, solve the momentum equation to get V star then update the m dot using Rhie-Chow interpolation and compute the momentum satisfying mass field. Fourth; now, you solve the pressure correction equation to get p prime.

Now, with the pressure correction field so now, you update velocity and pressure field, u and pressure field and also the mass flow rate which is u double star p star m dot double star and check. So, now you can treat this new solution to be a the updated solution to be the new guess. If it is mass conserving so, treat the obtained solution to be new guess. If it satisfy everything stop here otherwise you can go back to step here and repeated once you get a converge solution then move for move to next time level integration.

So, in every time level you carry out this process to get a converge solution and then you keep doing this. So, you can someone put the flow chart which will look quite nice.

(Refer Slide Time: 03:20)



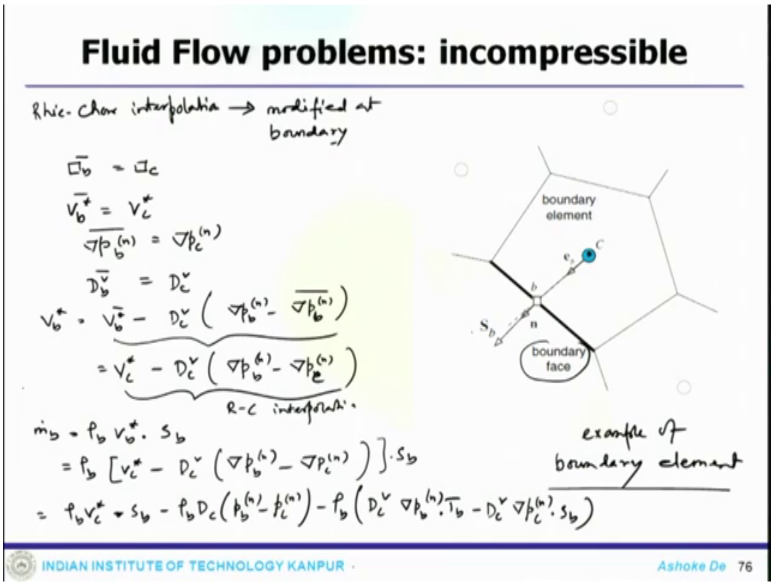
Let say set initial guess which is m dot n, v n, rho n, p n at time t plus delta t to converged value converged value at time t. Then, from there you assemble and solve momentum equation for to get V star from their one can compute m dot f using Rhie-Chow interpolation m dot f star. Then, assemble and solve for p point pressure correction from their you can correct m dot star f v star rho n and p n to get m dot star star f v star star f rho star star and p star star.

Now, you can set m sot n f equals to m dot star star f dot v n equals to v star star rho n equals to rho star and p n equals to p star. So, converged if no, then from here you go back to this process and repeat. So, if this is no, if it is not converged then go back and repeat this process. Now, if it is yes, then set solution at t plus delta t to be equal to the converged solution now you advance in time and set t equals to t plus delta t.

Now, if time exceeded, yes, to stop it will get you stop if no from here you go back and repeat. So, this is no. So, this goes in physical iteration and within that this is a iterative process and this is how you get solution for the simple algorithm get working, so, for this collocated arrangement.

Now, moving ahead once you get a system ready then it is important to also see how one can implement the boundary condition.

(Refer Slide Time: 07:49)



So, to do that let us start working on the boundary condition and this is a this is an example of an of boundary element. So, this is an example of an boundary element where you can see this is the boundary face and this is the cell center and this is the direction of that and this is the surface vector.

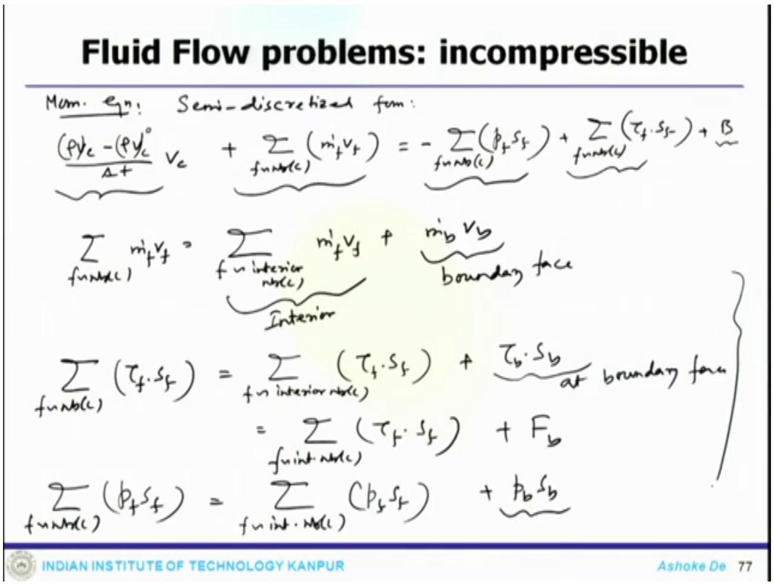
Now, one important thing is that when the face located at a boundary this is the boundary face has to provide the boundary condition now the first thing to be of interest is the expression of the Rhie-Chow interpolation at the boundary face. So, which now the Rhie-Chow interpolation at boundary face needs to be modified it needs to be modified at boundary.

So, what one can write that any variable which is at boundary is variable of the c. So, b refers to the boundary face and in that way the adopting this practice error is the Rhie-Chow interpolation so, at the boundary values can be written as b c star delta p b at n equals to delta p c at n and D b v equals to D c v. So, this one can do and your V b star equals to V b star bar minus D c v delta p b n minus delta p b m bar and this is your standard Rhie-Chow interpolation which one can write minus D c v delta p b n minus delta p b p b c n bar which is at the boundary face Rhie-Chow interpolation.

And, the mass flux which can be written as m dot b equals to rho b V b star dot S b which is rho b V c star minus D c v delta p b n minus delta p c n and dot S b. So, if you expand this would be V star dot S b minus rho b D c multiplied by p b n minus p c n minus rho b D c V del p b n dot t b minus. So, this an component which comes along this line and the tangential of the default correction along the normal line. So, that has this component. So, this is a implementation of the boundary condition which presented for the momentum equation.

Now, secondly, we can look at the boundary condition implementation for the pressure equation or the pressure correction equation and for the cases when the boundary condition for the momentum and pressure corrections equation are codependent then you need the full treatment of the pressure correction equation.

(Refer Slide Time: 12:22)

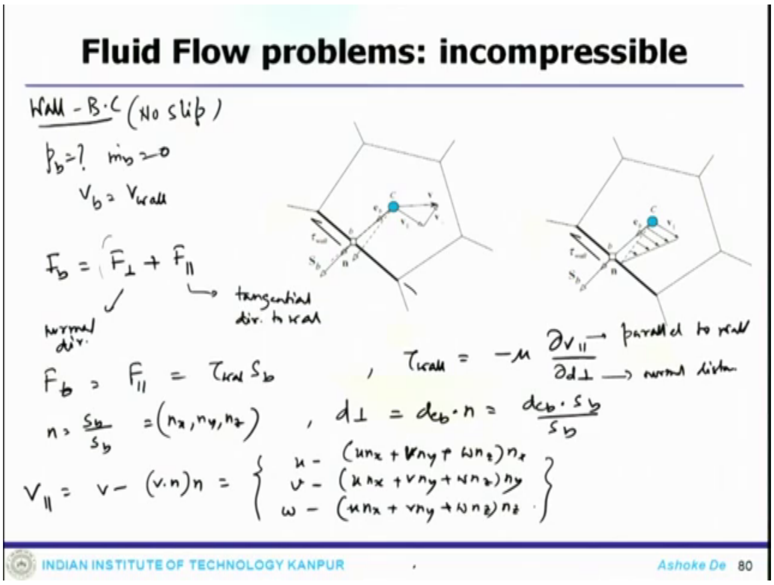


Now, momentum equations once we look at it. So, for momentum equation the semi-discretized semi-discretize form looks like rho V c minus rho V c divided by delta t V c which is previous time step, so, plus summation f n b c m dot f V f.

So, this is elemental discretisation face integration equals to summation of n b c p f S f this is again face discretisation plus summation of tau f dot S f this is also face plus B, which is again elemental discretisation. Now, these are already taken care of individually. So, straight away what we can write for this face which is m dot f V f one can write that interior n b m dot f V f plus m dot b V b which is at boundary face and these are all interior face. So, you can decompose into two component and similarly one can decompose the component of the stress component tensor which is again f at the interior n b c tau f dot S f plus tau b dot S b.

So, this is at boundary face which one can write summation of f interior n b c tau f dot S f plus as a source term and similarly the pressure discretisation of the boundary face one can write f interior faces p f S f plus p b S b. So, this is at boundary face. So, one has to look at individual component and then treat the boundary values.

(Refer Slide Time: 15:34)



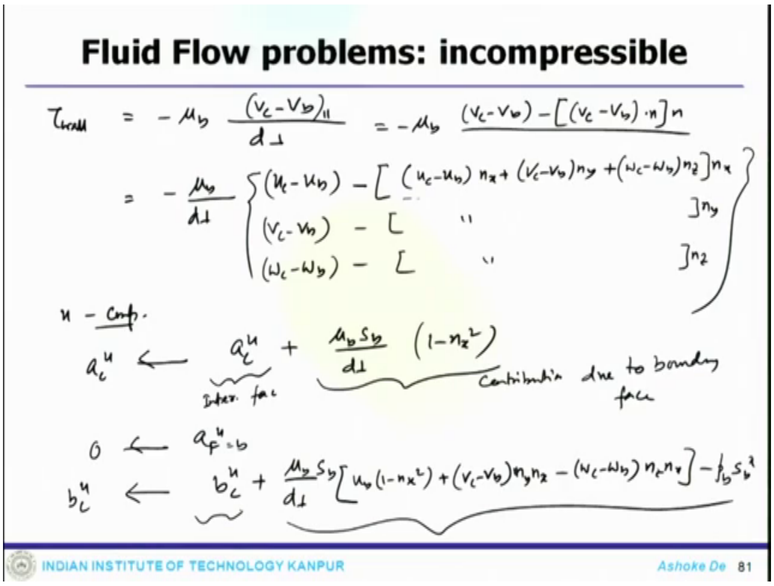
So, for example, now if you come to wall boundary condition; now this is wall boundary condition what you have? You have no slip wall. So, you can see this is my face or the boundary face and the no slip wall condition means what you do not know p b, but you know mass flux is 0 and you know the velocity of the boundary face is wall. So, this is the boundary face.

Now, a no slip boundary condition means you have a velocity, but typically this would be 0. So, now, what one can do this the F b that can be expanded as a plus a parallel component. So, the F b can be extended on the fluid can be written as this where this guy represents the tangential direction to the wall tangential direction to wall and this is the normal direction. So, which means this would be along this direction normal and this is along the tangential direction.

So, the F b would be F this. So, in the normal direction which is supposed to be actually 0. So, the parallel direction would be tau wall S b. Now, tau wall is the shear state all wall which can be calculated as del V tangential to the directions by del d perpendicular direction. So, V is the velocity vector parallel to the wall. So, this is a parallel to the wall and d perpendicular is the normal distance.

So, this is how so, this is the parallel to wall and this is the normal distance. So, unit vector would be S b by S b where n x, n y, n z would be the component then d perpendicular would be d cb dot n which is d cb dot S b by S b and V parallel would be V minus V dot n n which will let you have u minus u n x plus v n y plus w n z and n x v minus u n x, v n y, w n z n y and w minus u n x, v n y, w n z n z. So, this is how you compute all this component.

(Refer Slide Time: 18:54)

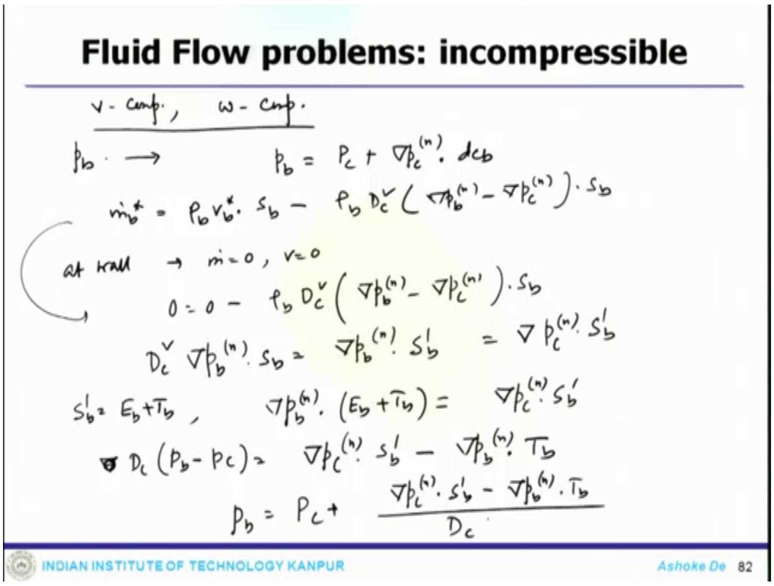


And, you can calculate the tau wall which would be minus mu b V c minus V b which is parallel d perpendicular minus mu b which can be written as V c minus V b minus sum V c minus V b dot n n and this is d perpendicular and you get all this component like u c minus u b minus u c minus u b n x V c minus V b n y plus w c minus w b n z multiplied with n x. So, this term would be common.

So, one can write similarly V c minus V b minus this is the same term multiplied with n y and w c minus w b minus same term multiplied with n z. So, that is what one get.

Now, for laminar flow; now this is what one can obtain. Now, another the thing is that u component direction the coefficients of the boundary elements for the momentum equation. For u-component in that direction your a c u would be written as a c u plus mu b S b by d perpendicular one minus n x square. So, this is the contribution due to boundary face and this is interior face similarly a F u equals to b 0 and b c u would be b c u plus mu b by d perpendicular S b u b 1 minus n x square plus V c minus V b n y n x minus w c minus w b n z n x minus p b S b x. So, this is the contribution come from the boundary and this is from the interior face.

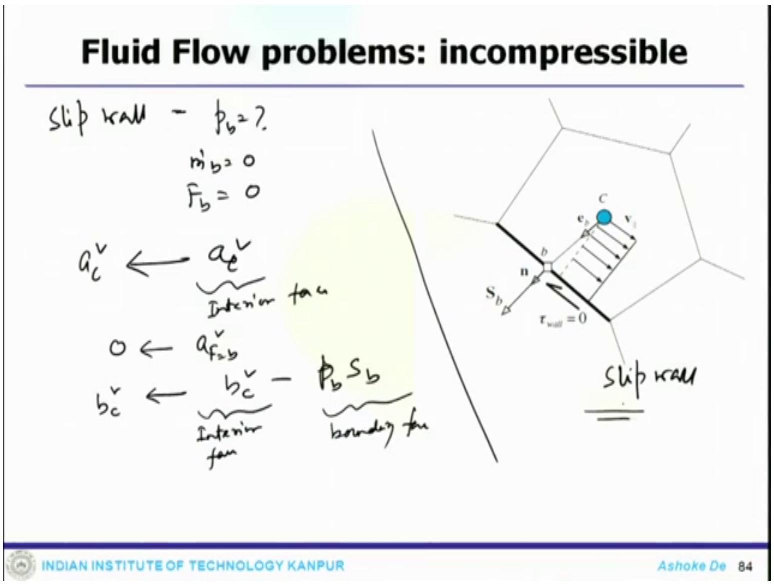
(Refer Slide Time: 22:25)



Similarly, one can actually get the V component and w component. So, it can be obtained similarly. All the matrixes are already been written. Now, the important point which comes with that at the boundary the pressure p b which is unknown and it needs to be extrapolated from the interior solution. So, one can use some sort of a Taylor series expansion and write that p b equals to P c plus delta p c dot d c b. Now, the mass flow rate which is expressed as Rhie-Chow interpolation which will have the contribution like this delta p b n minus delta p c n dot S b.

Now, the mass flow rate and the velocity at wall boundary at wall the mass flow rate is 0 velocity is also 0 then this guy walls down to 0 equals to 0 minus rho b D c v minus delta p b n minus delta p c n dot S b. So, which one can modify it and like right that n dot S b equals to delta p b n dot like this which is dealt p c n dot S b. Now, S b has two component like E b plus T b. So, one can write the term delta p b n dot E b plus T b which will get you the delta p c n S b prime. So, one can write D c p b minus p c equals to delta p c n dot S b prime minus delta p b n dot T b. So, all these one you put together you get p b equals to P c plus delta p c at previous time iteration dot minus p b n dot t b by D c.

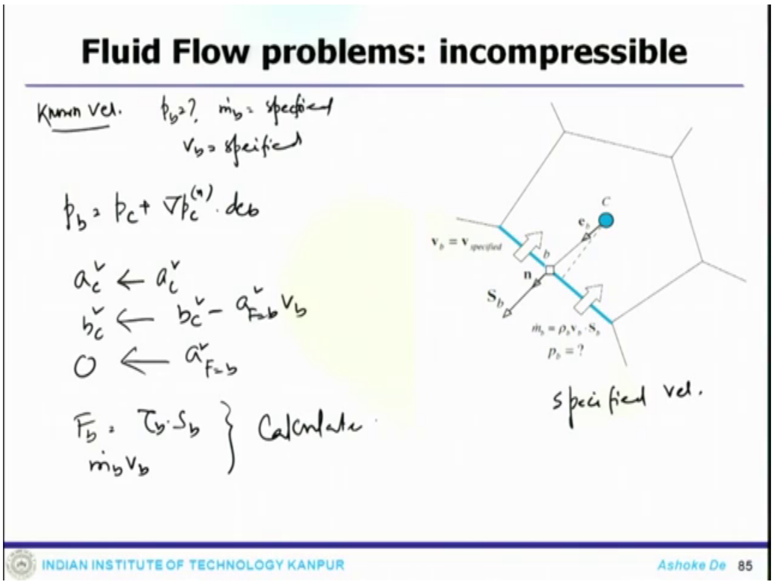
(Refer Slide Time: 25:12)



Now, similarly one can have slip wall. So, this is an example of slip wall and at slip wall what happens your p b is still unknown, mass flow rate is 0, force is 0. So, what one can modify that coefficients would be like this a c. So, this is only contribution come from interior faces and a F equals to b which 0 and b c is b minus b which is again interior faces and boundary faces.

So, one can see once you write everything mathematically in this fashion then when you come to an particular application of the boundary condition then it becomes quite easier.

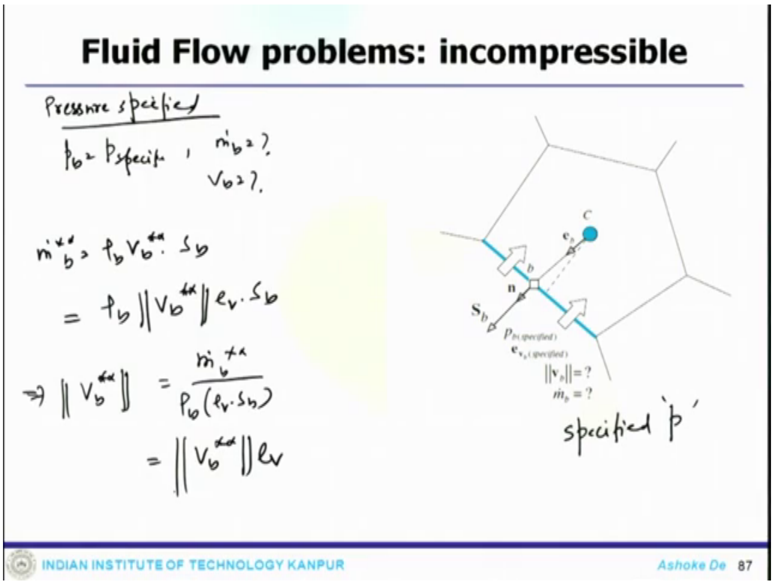
(Refer Slide Time: 26:21)



Now, this is a case where you have a specified velocity. So, when you have a known velocity which means p b still unknown m dot b can be specified and V b is specified. So, one that is the case then you can have this calculation like p b equals to p c plus delta p c n dot d c b and your other things will get modified a c v is a c v and b c v which will modified like a F equals to b V b and a f equals to b V is 0 and the conviction mass flow rate m dot V b and diffusion term of the boundary face can be calculated.

So, like this F b equals to tau b dot S b and m dot b V b this can be calculated as the given condition is known. Now, similarly one can have specified pressure.

(Refer Slide Time: 27:51)



So, this case is a specified p. So, the pressure specified at the boundary which means my p b is specified and m dot b is not known. So, velocity b is also not known. So, one has to modify the equation of the mass flow rate correction mass flow rate would be rho b b V correction dot S b which will be rho b V b star star e V dot S b, which get you back the V b star star equals to m dot b star star divided by rho b e V S b which is V b star star and e V.

So, we can actually see the other kind of boundary conditions also in the next lecture. So, will stop here and take it up from here in the next lecture.

Thank you.