Introduction to Finite Volume Methods-II
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Lecture - 37
Fluid Flow Computation: Incompressible-VII

So welcome back to the lecture series of finite volume and we are almost towards the end of our discussion on the fluid flow problem. So, what we are doing or rather in the middle of doing the discussion is the navier stokes solver and the discretization and in the last lecture if you recall, we have looked at the pressure velocity coupling the simple algorithm and then we started looking at the implementation of the boundary condition and what we have discussed, we discuss the inlet boundary condition and now today we are going to look at the outlet boundary conditions and different kind of outlet boundary conditions, how the discretized equation needs to be modified and what are the information that is available. So, let us start with the outlet boundary condition.

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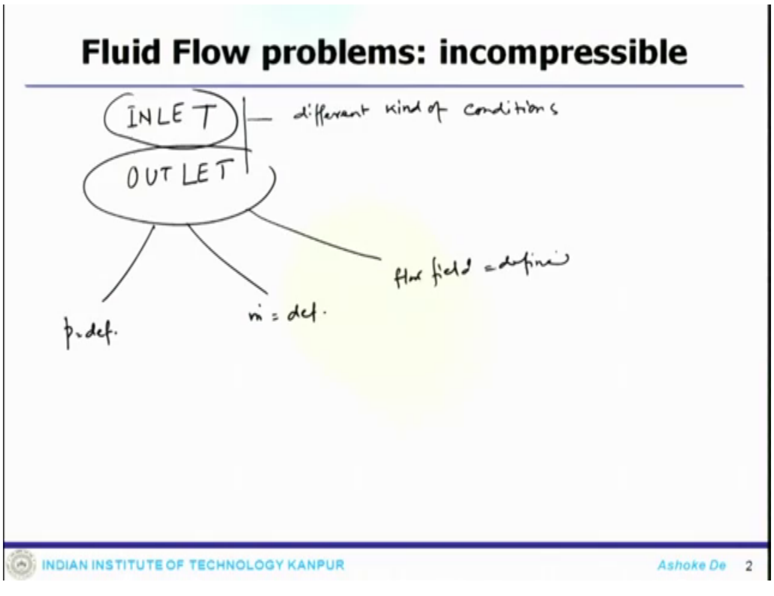

So, what we have done we have done the inlet boundary condition and under that umbrella different kind of conditions and now today, we are going to look at the outlet boundary condition. So, the outlet boundary condition could be also different type, different type in the sense could be static pressure defined, it could be mass flow rate defined, it could be a flow field defined so there are different ways one actually define the outlet boundary condition.

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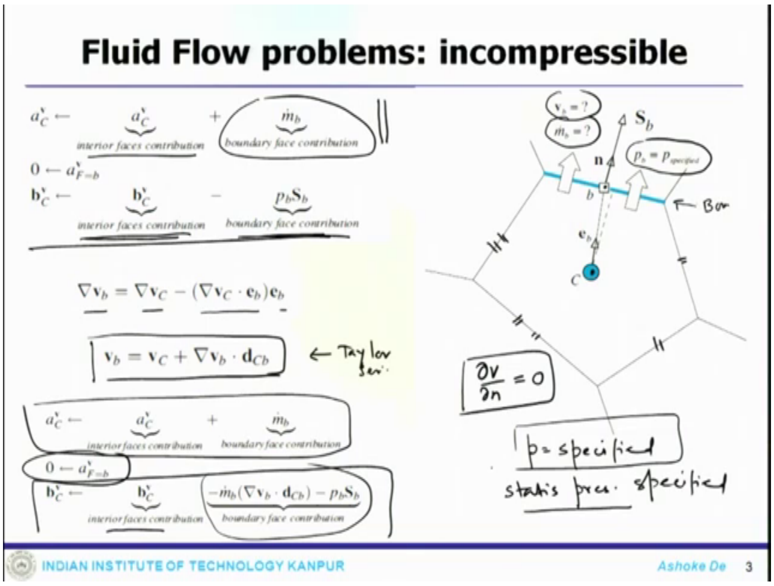

So, let us look at this schematic of a boundary condition where your this is the case where static pressure p is specified or static pressure specified. So, once you define the static pressure and you can see how the so once you define the static pressure; that means, at this, this is the boundary surface so this surface is the boundary surface and your p b is p specified, which is the static pressure please note that do not get confused with the static and dynamic pressure or static or total pressure this is purely the static pressure which is defined here and which actually appears in your discritize equation.

So, when you define the static pressure, then the velocity condition is neither known neither the mass flow rate is known, but what it implies that when you apply the static pressure this essentially implies that the velocity gradient, which is going to be there del v by del n that is going to be 0 or the gradient 0 condition will be there at that condition and also the which is assuming the velocity at the outlet would be equal to the that of the boundary element.

So, assuming that if you modify then the coefficients of your discritize equations would get modified so a C v which will come from the interior faces and then there will be a contribution which will come from the boundary face contribution. So, that will modified your coefficient a C a F b would be 0 at the boundary and the source term. So, the source term there will be a contributions from the interior faces and there will be a contribution from the boundary face contribution.

So, these are the 2 contribution so a C v plus m dot b and the b C the source term the contribution come from all the interior faces, these are the interior faces and the boundary face contribution. So, that is how it turns out, but one has to also ensure once you get this coefficient modified, one has to also ensure that the flux is zeroed in the outflow surface vector direction only.

So, the velocity is usually extra polated to the outlet by using the boundary flux and it can be computed as a delta b v equals to delta v C in delta v C e b, e b e b is the along this perpendicular line along this point which is at the face boundary face and connected with the cell centre. So, these typically ensure that the gradient along the boundary surface vector is 0.

So, then one can use the Taylor series expansion and expand this v b equals to v c. So, you write the velocity at that face using the velocity plus delta v b d C b. So, this is purely from Taylor series. So, with the taking the only the first term into the consideration and then once you get this then the additional correction is added to the source term.

So, the a C coefficient of a C will not get change the F b will not get changed, but the change comes here in the source term where your inter interior face contribution which will come from all these faces they will not change then, but the boundary face contribution is modified like minus m dot b delta v b dot d C b minus p b S b. So, this is the change one can add to that when you have a pressure boundary condition which is specified. So, at the outlet if you apply the pressure or the pressure is specified then your rest of the coefficient gets change in this fashion.

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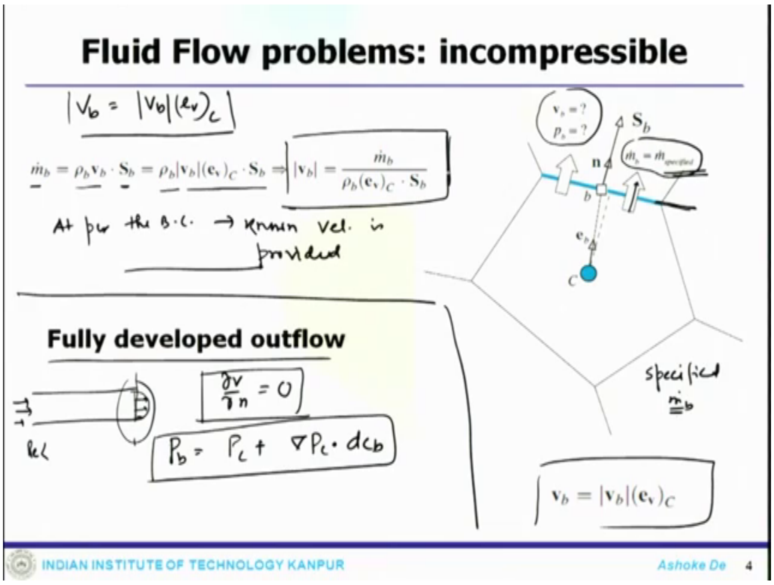

Similarly, now if you use a so this is schematic for the specified m dot or mass flux at the boundary. So, you can see at this face the m dot b is specified. So, this is known, but what is not known in the velocity and the pressure at that face. So, that is also not known. So, now, so this when you actually apply a mass flow rate or mass flux is specified in incompressible flow which is essentially equivalent to the specifying the normal component of the velocity. So, the velocity which is calculated by assuring its normal a direction can be like this. So, when you which is V b equals to V b e v c. So, the normal component of the velocity is known. As soon as at this face I specify the mass flow rate one can calculate the normal component of the velocity. Now V b can be obtained from m dot b rho v v b dot S b rho b magnitude of v b S b so v b you get like this.

So, once you get this which will means so even then this case you have a mass flow rate which is specified you can get back your velocity which in terms of that mass flow rate and once you get this velocity you can actually this is now the coefficients will be at per the boundary condition where known velocity; known velocity is provided. So, now, rest of the calculation would be similar where we have specified a velocity. Now apart from that one can also provide a fully developed outflow.

So, fully developed outflow means the at the outlet face if this is the outlet face, there you assume the flow field is fully developed for very simple example one can think about when there is a flow through a channel and if it is laminar very low Reynolds number so its laminar then there is a inflow and at the channel outlet it will take an parabolic profile. So, that is a fully developed parabolic profile if the flow field is turbulent then at the end of the channel, this could be fully developed turbulent profile. So, this is what the fully develop profile means.

So, which means so velocity gradient at the normal to the outlet face is going to be again 0 so del v by del n is going to be at this particular face, this is the face if it is outlet del v by del n is going to be 0. So, velocity at the outlet essentially going to be or assume to be known and computed from this 0 normal gradient and the pressure at that boundary when we assume so one can calculate P b equals to P C plus delta P C dot d C b. So, the pressure at that boundary can be calculated like that.

So, velocity will be treated as known and the coefficient the momentum will be modified according to the known velocity condition. So, all these conditions when you provide mass flow rate or fully developed condition, these are some our other connected with the previous boundary conditions when you have already specified a velocity condition, they are the coefficients are going to be modified accordingly. Here, what matters here when you specify a mass flow rate, from mass flow rate first you calculate the velocity then the coefficients or the discritize equation can be used as we have developed for a known velocity condition.

So, one can think about that these conditions like specified mass flow rate or fully developed conditions these are some sort of and derived boundary conditions from the standard this let or normal kind of boundary conditions or it is sort of combination of all those fundamental boundary condition.

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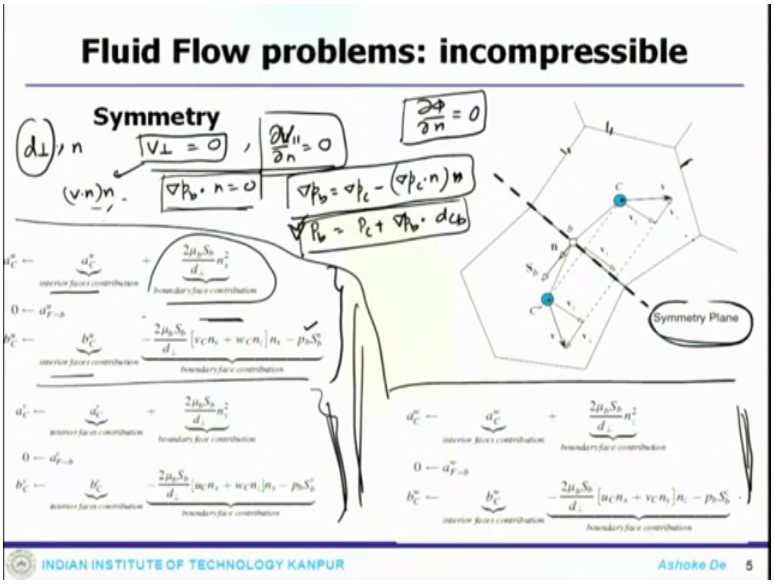

Now one more conditions which could be also interesting to look at is the symmetry boundary condition. So, here is the plane of symmetry now you can see this side is could be a interior cell and this is an cell which is imaginary replicating the same interior cell and the conditions at this particular face where the symmetry plane is there the condition supposed to be symmetry.

So, in a physical understanding, before we go into the mathematics the physical understanding is that the whatever value you have this will be the exactly same that is how the symmetry is maintain and if the symmetry is maintain, then along this line the gradient is going to be essentially 0.

So, for any variable here if it is maintained to be symmetry, the del phi by del n has to be 0 that is the gradient which is not going to contribute anything. Now when you have this and you relate this all this information with the normal component becoming 0, so the results is the 0 also for. Now when you apply to the velocity boundary conditions that can actually I mean turns out to be a 0 shear stress condition along the symmetry boundary. So, when you apply this gradient 0 and calculate that.

Now the unit vector in the direction normal to the boundary is n and the normal distance let us say d, d this is the normal distance and the unit normal vector is n then, the velocity components normal and parallel to the boundary can be given that v normal to that is 0 and del V which is parallel to that del n equals to 0 that is what the symmetry condition does and then you can now find out the this one from the v dot n dot n and you can expand this and calculate and then from there one can also calculate the stress term or the source term like F b and from the parallel conditions also you can actually get the other gradient.

Now also since it is a symmetry condition pressure also it will satisfy the del p b dot n equals to 0 so the pressure gradient is also going to be 0 and the pressure at the symmetry boundary can be extrapolated using the condition like delta p b equals to delta p c minus delta p c dot n n. So, like this you can actually extrapolate the pressure at the symmetry by boundary. Now then the pressure could be obtained as p b equals to p C plus delta p b dot d C b which we have done earlier in the previous case.

So, once you get all this information, now you get pressure velocity and everything so their momentum equation this is your x momentum equation. So, the x momentum equation the coefficient a c this will come so the superscript u stands for the u velocity component and then the v stands for the v velocity component, w stands for the w velocity component and now this will a contribution there would be from the interior faces so these are the interior faces and then there is a boundary face contribution which will be 2 mu S b n x square by d normal distance. So, this normal distance one needs to put place.

Similarly, the v component will also get modify the source term also will have interior component and the component from the boundary face where you get a slightly involved expression which will also in work pressure term and the diffusion term. Similarly you can see v direction you need to change; that means the y momentum equation and if you have a 3 dimensional z momentum equals are will also get change.

So, the symmetry conditions to apply which is a primarily at that is plane of symmetry, you need these two condition to be satisfied V perpendicular to be 0 and thenV parallel gradient is 0, then from there the pressure gradient in the normal direction 0 you get the pressure, from here also you get the velocity then you modify the source term like the stress term and accordingly you modified all the coefficients which are sitting here in your momentum discritization.

So, that is actually talks about the boundary conditions which one can use for the velocity equations or momentum equation.

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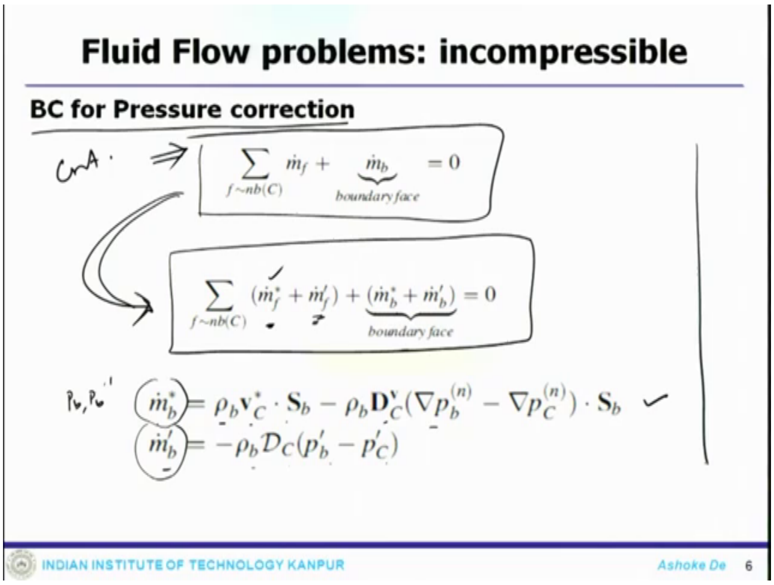

Now, we need to look at the boundary condition for pressure correction equation because in the simple algorithm we have a equation for pressure correction equation and when you are trying to solve the pressure correction equation, one has to provide proper boundary condition for pressure correction. So the what one can share the boundary cell, the mass flow rate from the interior and the boundary face this has to be conserve this will be coming from my continuity or one can write that at the boundary face this is the intermediate value m dot f star or m dot f prime which is the corrections and the boundary face that is going to be 0. So, this is what it actually get you.

Now once you have this condition, then you know this mass flow rate the star condition is calculated intermediately during the process of the iteration and this prime is the correction. When the solution converges this correction become 0 so the stars become the solution.

Now one can also note that, since at the boundary face only the boundary cell contributes to the average quantities so the equations can be slightly modified like m dot b star equals to rho b v C star dot S b minus rho b D C v this coefficient delta p b minus delta p c n dot S b. So, this equation we have already seen and the prime equation would be minus rho b and D C p b prime minus p c prime. Now for implementation of the boundary condition these values like m dot b star m dot b prime p b p b prime all this informations are required.

So, like p b p b prime these are required or needs to be calculated. So, so far whatever discussion we have than the first type which is designated by a specified mass flow rate where we can apply that m dot v to be 0. Now this is how for the pressure corrections equation one has to do.

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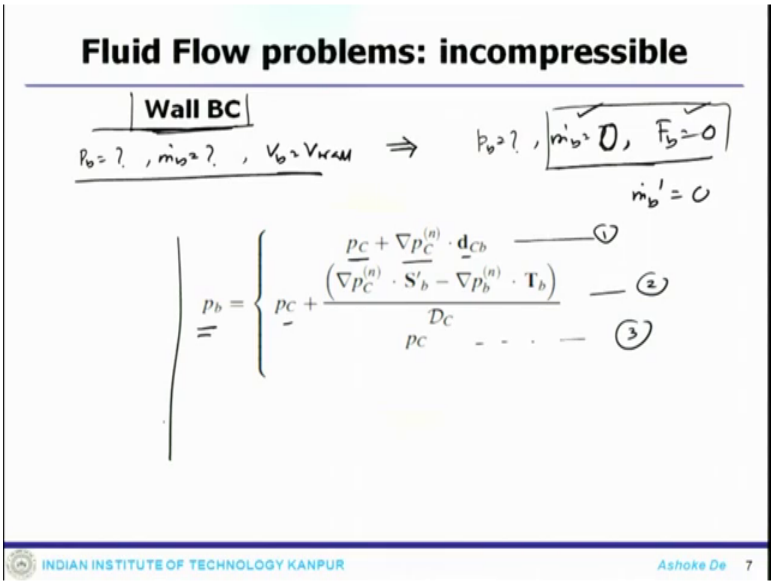

Now will go and look at a different kind of pressure corrections equations I mean boundary conditions like when you have your wall boundary. So, these boundary conditions or this type of specific boundary conditions that we have already done a discussion while talking about the velocity field discussion. So, this already been done.

So, now again same kind of boundary condition if you are dealing with a physical geometry, it might have an actually wall and now once you put the wall, the boundary condition needs to be provided for velocity it is much easier because the velocity at mostly boundary condition can be applied, but when you come down to pressure this is little tricky because pressure corrections equation what it requires we do not know the p b, we do not know the mass flow rate we only know v at wall which is essentially equivalent to p b still not known m dot b equals to 0 and force also 0 why this is happening?

This is happening because the no slip boundary condition provide the velocity component to be 0 from there we get this, but please note that these boundary conditions that we are right now discussing this is required for pressure correction equation. So, having a condition on velocity owned help that much.

So, what you require that the wall boundary condition where the mass flow rate is 0 so also it will lead to m dot b prime 0; that means, the correction would be 0 when there will be no modification which is needed for the pressure correction equation; however, the pressure at the wall is required as is computed using an or a lower order kind of profile so p b can be calculated as the interior cell p C plus delta p C previous iteration dot the distance or you can actually this is one approach one can do that, second approach one can use some sort of an p C delta p C from the previous iteration dot S b prime minus this is also we have seen or third some lower order extrapolation could be done which will just assign the t b to be p c.

So, these are 3 ways one can actually calculate the pressure at that boundary where you need to provide the wall boundary conditions and one boundary condition means you do not know the pressure, only condition which is known that the velocity because of the no slip condition and that gets to the mass flux 0, source term 0 from there you can calculate the pressure.

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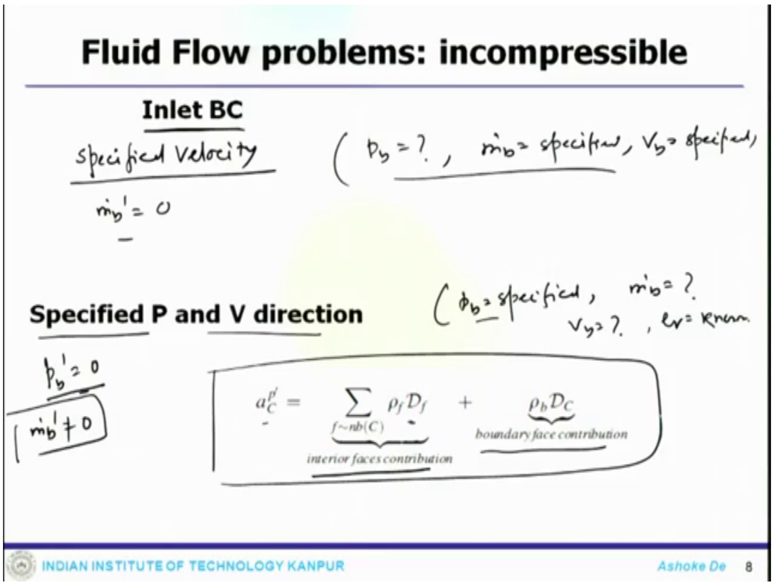

Now the second approach could be inlet boundary conditions. So, also in your domain there is a inlet condition, then the inlet boundary condition one can have let us say specified velocity. So, that could be one easy condition where specified velocity which means my p b is not known m dot b is some sort of a specified and v b is also specified.

So, once you have this thing specified, then for a specific velocity at the inlet the mass flux is known and its corrections would be m dot b prime would be 0. So, this is similar to the wall kind of boundary conditions, but the term is simply dropped from the pressure corrections equation. So, the pressure at the boundary can be extrapolated from the internal pressure field like what we have already done using similar kind of things.

Now another way to do that you can specified the pressure and the velocity directions in these case you have p b which is specified and then you have m dot b which is not known and then you know v b which is also not known, only thing the direction is provided e v is known that is the direction of the velocity. So, what one can do seen p b pressure is known and p b prime would be 0 for corrections.

So, one can do that, but this does not mean m dot b prime would be 0 mass flux would be not 0 so note that. So, then the inlet can be treated as a dirichlet boundary condition for the pressure correction equation and the p prime the coefficient of the p prime now would become like this a C p prime rho f D f this is interior face contribution and then the boundary face contribution. So, with this you can actually solve the specific boundary condition which provides the pressure condition.

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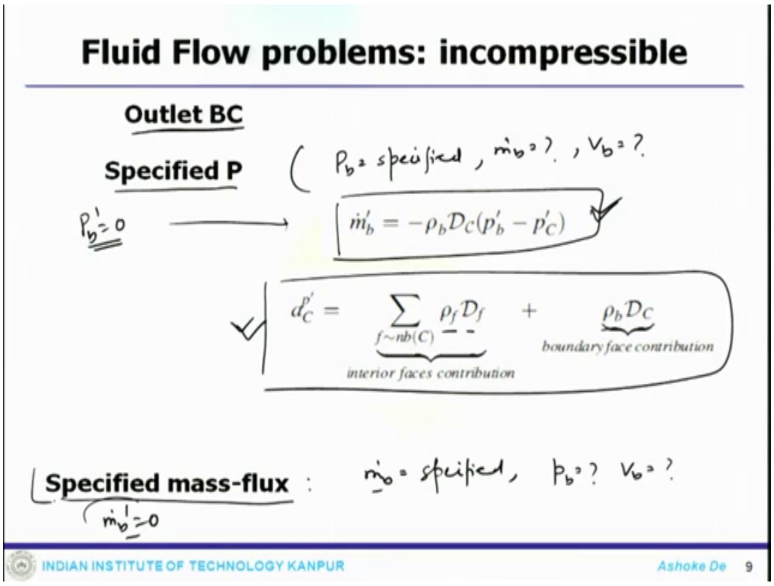

Now, similarly we need to look at or tackle the outlet boundary conditions. Now, outlet boundary conditions one option could be specified pressure; that means, p b is specified and what is not known m b not known v b not known. So, for the specified pressure the p b prime is going to be 0 for no corrections or from there one can calculate m dot b prime equals to rho b D C into p b prime minus p C prime. So, using that you can calculate m dot b and secondly, the velocity direction could be needed and then to take the direction of the v b to that with the append case of the v C and then the corrections coefficients will be modified like that.

So, you get summation over all the interior faces rho f d f and then the boundary contribution rho b D C that coefficients. So, this corrections would take care of that. Now alternatively outlet case it could be specified mass flow rate where m dot f b specified and p b is not known v b is not known.

Since m dot b specified then m dot b prime would be 0 for all the corrections I mean not required corrections and simply drop the pressure corrections equation with no modification required for the coefficients of the boundary element by setting this 0 one can do that and then the pressure correction the boundary set equal to the pressure corrections at the boundary cell centroid. So, using that, one can solve this specified mass flux conditions.

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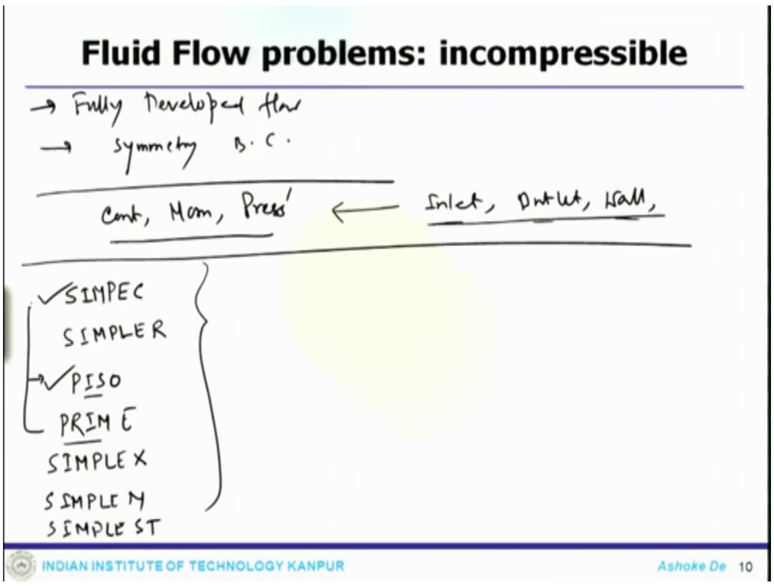

So, a apart from that there are few more which are also derive kind of conditions like also at the outlet one can have fully developed flow that could be one condition, then one can also have symmetry boundary condition. Now apart from that these are the some derived conditions.

Now what we have done now we have a simple algorithm that we have discussed and which actually solve for both continuity, momentum and pressure corrections. So, while solving that you need boundary conditions. So, we have talked about inlet boundary condition for all the equations, outlet boundary conditions, wall boundary condition and then other boundary conditions which come under inlet, outlet and wall. So, different kind of these conditions we have talked.

Now, there is a different class of simple family which are slightly advanced. Simple is one of the basic algorithm that was proposed and lot of commercial C F d code that uses even other C F d codes also which are based on finite volume technique they use that, but there are different class of algorithm which are available like simple C, then simple R, then you have PISO prime, simple X, simple M, simple ST. So, these are different different variant of simple algorithm, but the underline algorithm remains the simple algorithm and some sort of an modification is done while doing those calculations of the iterative process.

Now we will discuss couple of more like let us say we will discuss simple C and PISO obviously and to discuss PISO we may need to discuss about prime because PISO is some sort of a combination of these two is PISO. So, we will stop here today and will take from here in the follow up lectures.

Thank you.