Introduction to Finite Volume Methods-II
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Lecture -38
Fluid Flow Computation: Incompressible Flows-VIII

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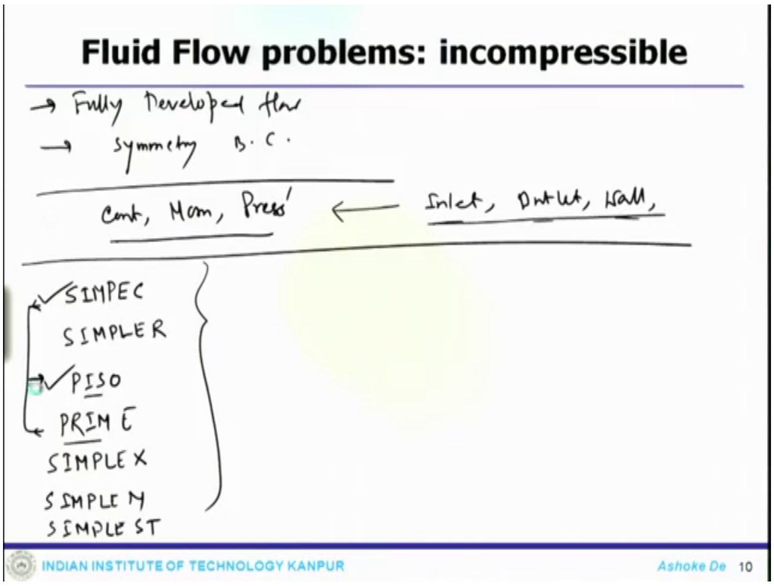

So, welcome back to the lecture series of Finite Volume and where will continue our discussion where we left in the last lecture. So, PISO is kind of a combination of this to solver.

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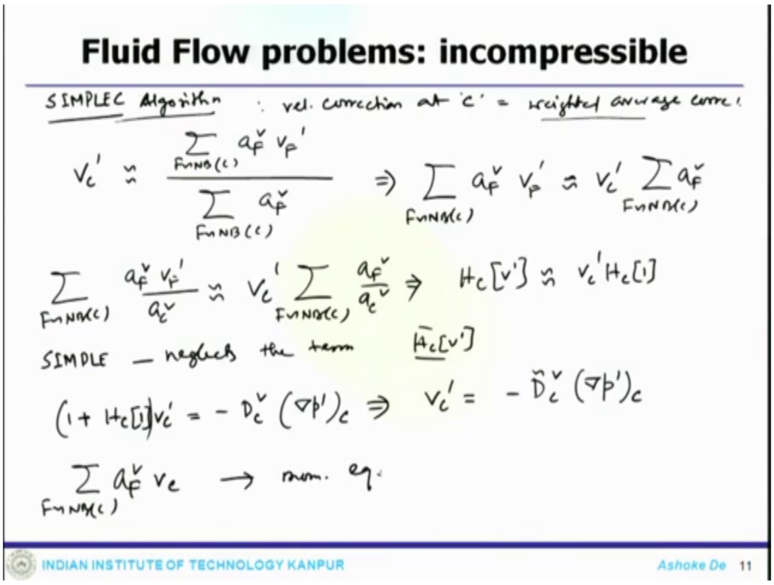

So, what happened to simple c algorithm, now again as I said this is a modified version of simple algorithm and it is derived by assuming the velocity correction at point c is some sort of an, so the velocity correction at c is some sort of an weighted average correction.

So, I mean taking care of that some weighted average corrections of the neighboring elements. So, one mathematics becomes then V c prime which was calculated as overall the cell a F V V H prime over F NB c a F V which will get you summation of F over all the cell a F V V F which will be V c prime F NB c a F V.

Now we use our H operator that we have done, so, this guy can be written as like F NB c a F V V F prime by a c V equals to V c prime summation of F over c a F V by a c V equals to H c V prime equals to V c prime H c 1 and then instead of neglecting the a c bar V prime term. So, in simple what we have done that it neglects the term is H c bar V prime

So, instead of now in this case we do not neglect this term rather it is replaced. So, in this particular case this term is replaced by an equation , which can be corrected as 1 plus H c 1 V c prime equals to minus D c V delta p prime c which will lead to V c prime equals to minus D c V delta p prime c. So, now what it does? So, from here one can used to derive the pressure correction equation and the same result can be achieved by adding and subtracting the term of F NB c a F V v c from the momentum equation.

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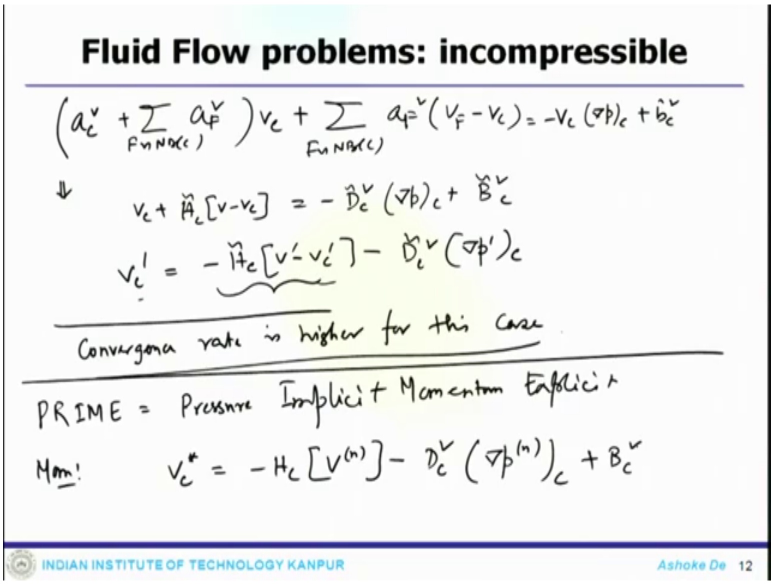

So, use this one in the momentum equation and then you once you do that the combining everything it will lead to the a c V plus summation of F NB c a F V v c plus F NB c a F V v F minus V c equals to minus V c delta p c plus b c V which in turn in a compact form one can write V c plus H c V minus V c equals to minus d c delta p c plus V c.

So, this you can be used for the velocity collections as V c prime equals to minus H c V prime minus V c prime minus D c V delta p prime c. So, the term this term is dropped which is equivalent to the approximation of the previous equilibrium in a modified velocity correction is used driving the pressure correction; so this is dropped and only this written this term.

So, this is a better estimate in simple c the relaxation of the pressure becomes unnecessary and as compare to simple. So, in this case we may not required to be the relapse the pressure and the result in velocity corrections will satisfy better the momentum equation but consequently at higher convergence rate is obtained. So, the convergence rate is higher for this case so that is important.

Now the second another algorithm which sits in between is called the prime algorithm, prime is stands for pressure implicit pressure implicit momentum explicit. So, the momentum equation solve explicitly and the explicit treatment is justified by the mall contribution to the convergence on the other hand the pressure corrections or pressure equation is solve implicitly.

So, since the momentum equation is solved explicitly one can get the new or intermediate velocity corrections like minus H c V n minus D c V delta p n then c plus B c V. And the velocity correction is applied to direct the pressure correction equation.

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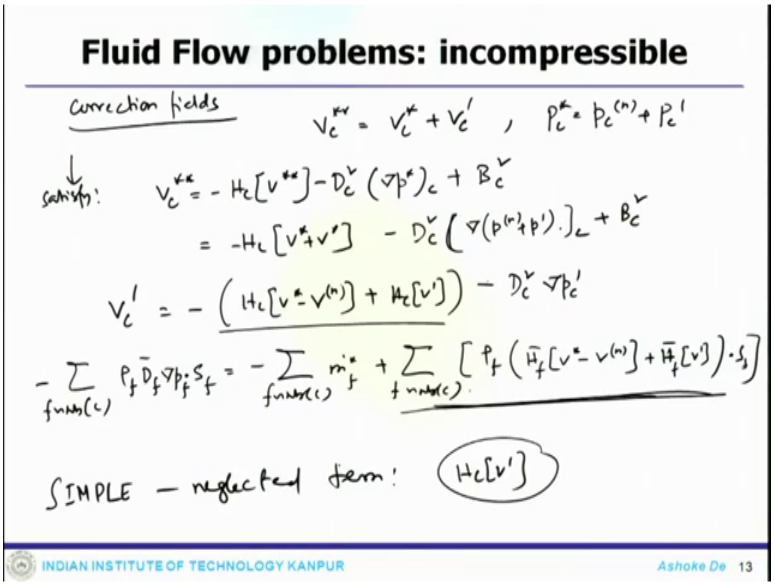

So, the correction fields are now the correction fields one can obtained as V c double star equals to V c star plus c prime and P c star equals to p c n plus P c prime and the corrected field. This will satisfy the equation V c double star equals to minus H c V double star minus D c V delta p star c plus B c V which is minus H c V star plus V prime minus D c V delta p n plus p prime which is c plus b c V.

Now, which will lead to the expression relating to the pressure and velocity felid like V c prime equals to minus H c V star minus V n plus H c V prime minus D c V delta p c prime. So, this particular term I mean if you substitute in the continuity equation which will get you back minus F nb c rho f D f delta p f dot S f equals to minus summation F nb c m dot star f plus summation n b c rho f H f V star minus V n plus H bar V prime dot S f.

Now the underline term which is here this can be neglected or typically this is neglected. So, the term which is neglected in the prime this underline term can become smaller and the, but then the term neglected in the simple. The simple algorithm, the neglected term was H c V prime. So, this was not that smaller and can lead to the I mean sure convergence rate, but in this prime algorithm this term is quite smaller compared to that and the neglection of that term does not have too much of difference and it actually leads to better convergence.

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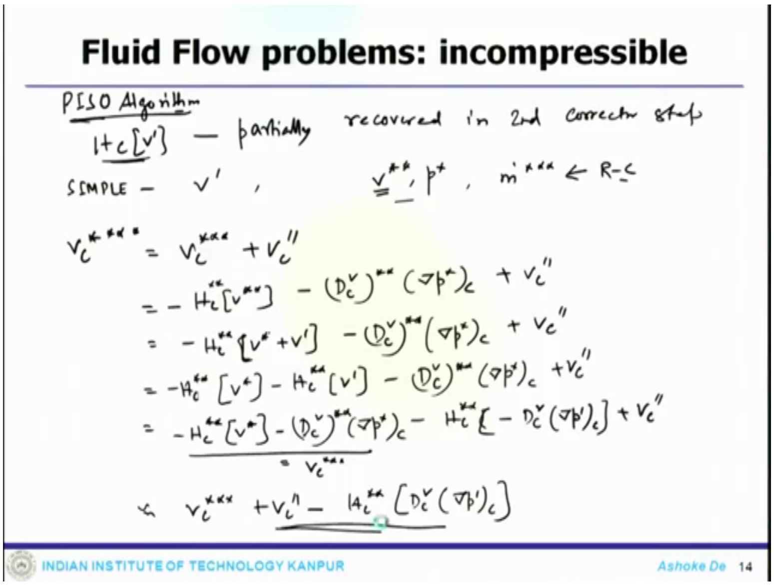

So, now using this simple c and prime the algorithm which actually works is a PISO algorithm. So, PISO algorithm is an interesting it stands for again the in between of this simple and these things and what happens in the PISO algorithm that the which one neglected term in the simple algorithm like H c V prime it is partially recovered in second corrector step.

So, the PISO is essentially based on some sort of an predictor corrector kind of approach and so what one can look at it that because we have in the simple we are computed V prime. and then neglected this guy the continuity satisfying the V star star the quantity V star star and pressure p star.

So, which were used to recalculate the coefficients of the momentum equation and then to solve it explicitly the new velocity field V star star is also used to calculate the m star star star at the element faces using Rhie Chow interpolation and this guy a c V prime is also partially recovered. And the equation that does is that equals to V c triple star plus V c double prime which is minus H c V double star this is double star minus D c V double star delta p star c plus V c double prime which will be minus h c double star V V star plus V prime minus D c V double star delta p star c plus V c double prime which is minus H c double star V star minus H c double star V prime minus D c V double star delta p star c plus V c double prime and this if you write H c double star V star minus D c V double star delta p star c minus H c double star H c double star minus D c V delta p prime c plus V c double prime.

So, this equivalent to your V c triple star, so this will become V c one triple star plus V c double prime minus H c double star D c V delta p prime c. So, this term represent the portion of the H c V prime which is neglected and it is kind of in this algorithm partially recovered and the second velocity correction.

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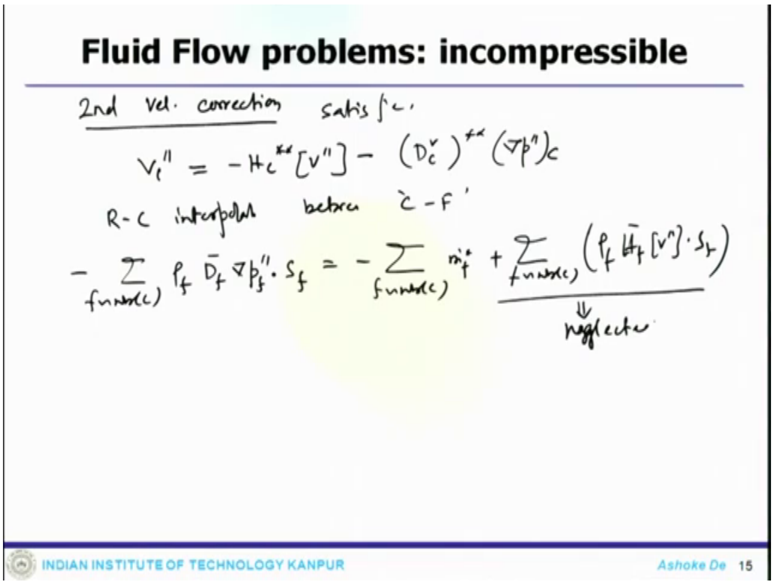

The second velocity correction that satisfies that V c double prime equals to minus H c double star V double prime minus D c V double star delta p prime c. Now, using the Rhie Chow interpolation between points so interpolation between C and F these are the cell neighbors one can obtain the new pressure corrections equation which is n b c rho f D f delta p f double prime dot S f equals to a minus F nb c m dot f star plus summation of F nb c rho f H f bar v double prime dot S f.

So, this is again a term which are again this is neglected in the PISO also and this corrector step may be replaced as many as time desire so that you can lead to better convergence.

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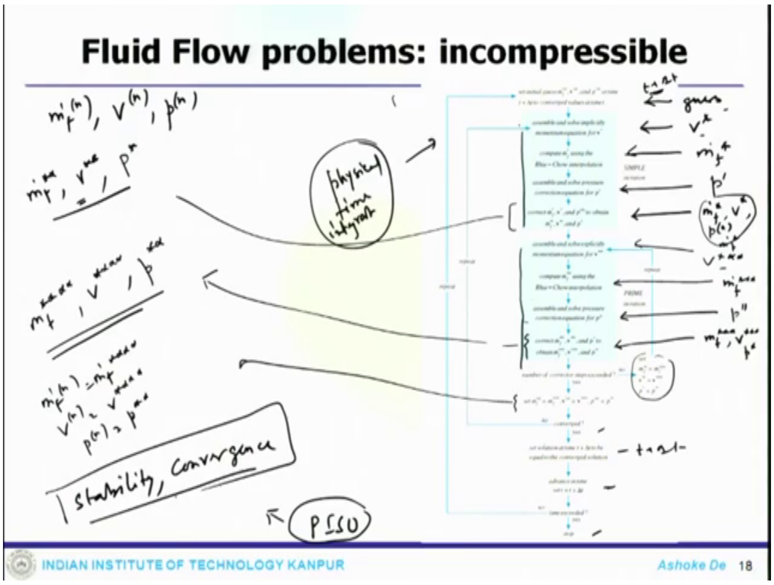

So, if you look at how it works so this actually kind of giving you the complete PISO algorithm and one can also look at it any finite volume books those are mentioned is a very standard algorithm. So, what happens you first this is at that p plus delta t time iteration, so this is a physical iteration. So, outer loop as we have done in the simple this is physical time integration.

So, what you do at the time level t plus delta t? So, you have all m dot f n so here m dot f n V n p n so these are all guess. So, these steps is the guessing step so you guess everything, then you assemble and solve for momentum equation for all the star quantity; that means, the first level of intermediate step where you calculate all the star quantity then you compute here the m dot f star using Rhie Chow interpolation, you then after that at this step you collect everything and solve for pressure corrections. Once you are done then; obviously, using the pressure correction the m dot f star V star and p n is obtained.

So, these are use this correct this things to get m dot f double star. So, at this step you finally, get m dot f double star and V double star and p star which is calculated based on these values which are corrected values using the p prime equations. Now this portion of the algorithm particularly this is exactly what we do in the simple algorithm. Now here we again assemble everything and explicitly solve for the momentum and here you solve for V triple star because already at the second level of correction, so what happens is that it goes in multilevel iteration.

So, the first level you do simple kind of algorithm and initially you started with the guess value. Once you get a corrected value you use that for the next level of algorithm where you obtain V star V double star. Then again you calculate the m dot f triple star using Rhie Chow interpolation then you get pressure corrections equation here you corrections for p double prime and then after that at this step what you do this step you actually get the corrections field m dot f triple star and V triple star and p star. And this correction fields are use to calculate m dot f four star V four star and p double star so, these are used.

Now, if the number of corrector step exceed it if it is yes then you move, if it is no then you actually assign those value here to the double star value and repeat this process. So, this is the part of prime algorithm and this prime algorithm is repeated, but if that is exceeded that you do at this step you get m dot f n equals to m dot f four star V n equals to V four star and p n equals to p double star.

So, you assign that if it is converged to move ahead if it is not you go back and repeat this step. So, again you compile simple at PISO algorithm, if it is converge you update this thing for the next t plus delta t iteration at once in time again the time limit exceeded stop or you go back and do the physical iteration.

So, it does twice kind of or two times of the iteration in the process and now that is why PISO is provide better stability, better convergence and if you have a slightly non orthogonal skewed grid. And those kind of cases actually PISO was better than simple algorithm or simple c algorithm. Now that is why it is special preferred and lot of the c f d course actually is based on PISO, they provide a better stability and better convergence.

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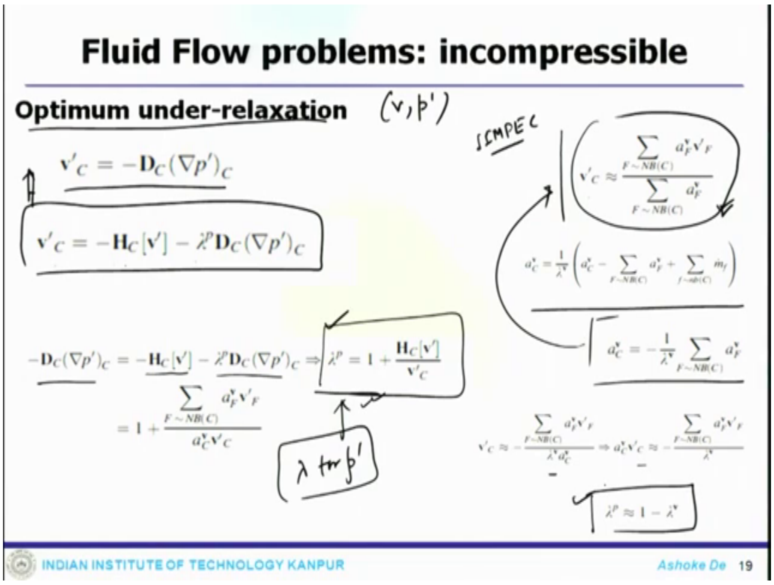

Now, moving ahead we have used under relaxation factor, so now, one can look at the optimum under relaxation factor for V and pressure corrections. So, what one can see if you recall the velocity corrections how we do that? We do that minus D c into del p c and once I am calculating the pressure field which use some sort of and under relaxation in the velocity corrections and we wrote in this fashion where v c prime equals to minus H c v prime D c delta p prime c.

So, that is how we written and now if you equate these two guys or these two equations one can write minus D c delta p prime equals to minus H c v prime minus lambda which will provide lambda p equals to this or so that gives an, so the simple c algorithm eliminated the need to under relaxation pressure corrections and result in an optimum acceleration rate.

So, therefore, using an approximation introduce with the simples will the velocity corrections at c can be written and weighted average like this. So, this is done in simple c and this is the corrections for lambda for p prime; so the pressure correction equation this kind of correction factor. Now when you move ahead from the simple to simple c the kind of corrections was eliminated and the velocity corrections can be estimated like this already we have seen, so there is nothing new here that we are writing.

So, if you go back to your expression for the simple c or simple algorithm you will find this expression and then the coefficients can be written in this fashion invoking the is a under relaxation factor. So, if you simplify that you will get this and so the role of under relaxation would limit to this.

Now if you substitute this equation in the velocity corrections approximation here if you use this one here then you get v c prime equals to a F v by lambda from where you get lambda p equals to 1 minus lambda v. So, what people have seen that, that simple algorithm which is under relaxation factor satisfying this equation provides to similar to that kind of simple c kind of algorithm.

So, that is an optimum lambda calculation for simple algorithm which can behave like an simple c algorithm.

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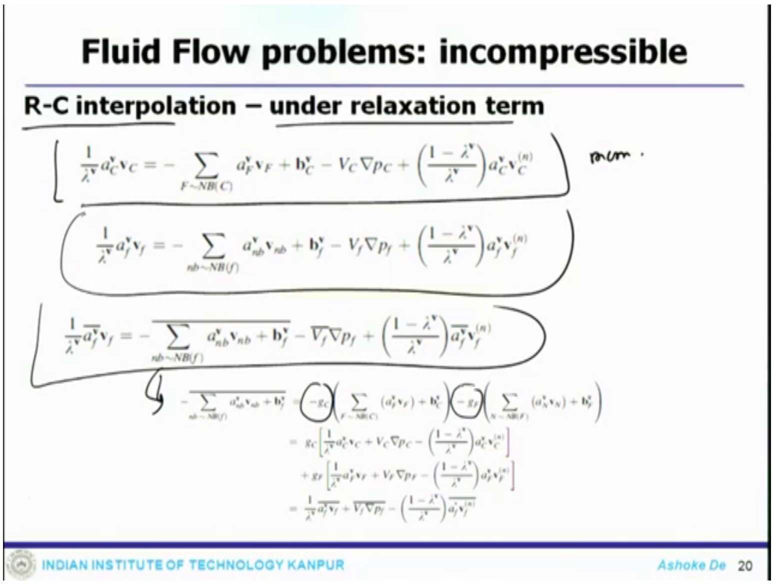

Now, one more important thing which one can discuss is the Rhie Chow interpolation and the treatment of the various terms in the Rhie Chow interpolation. So, first one can look at the under relaxation term, now the under relaxation term in the momentum equation this is how one has written the momentum equation and the under relaxation term written like that.

So, if you do some sort of an algebra this is for momentum equation and velocity corrections you get back this equation which is again we have looked at while looking at the momentum corrections equation and then the Rhie Chow interpolation should be written in this fashion and the this particular term one can express that contribution from the elements c and the contribution from element f. So, finally, this actually leads to a some sort of an calculations of the face variable which is written as v f.

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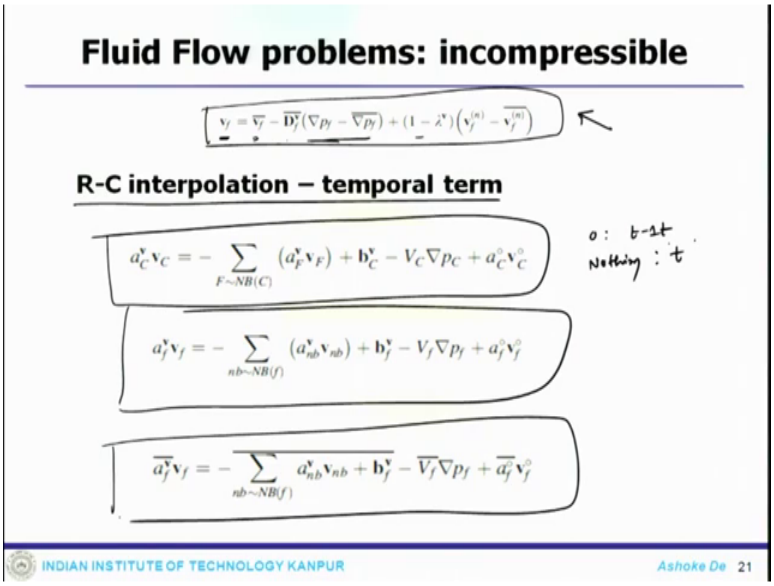

So, all this algebra you can carry out because this starts with the momentum equation, the important point here is that you get back the surface value which is a average surface value, then there is a delta p f and delta p gradient and then 1 minus lambda. So, this is how when we use under relaxation the Rhie Chow interpolation actually gets modified.

Similarly same equations can get modified for the temporal discretisation, so when you have written the temporal here these stands for t minus delta t or previous time steps and nothing stands for t or the present time step. So, this is my momentum equation with all the discretize form then you write at the face this is the modified and the face value can be average like that.

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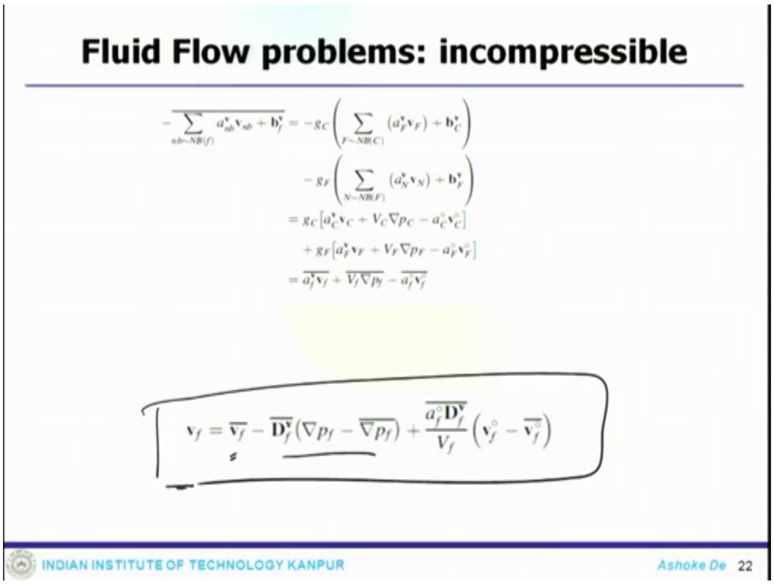

And once you expand this finally, you get this is an important calculation for the transient term how the face value one can treat and that case you get face value minus this and you use this things. So, also one can do the treatment of the under body force term and body force term if you do the treatment.

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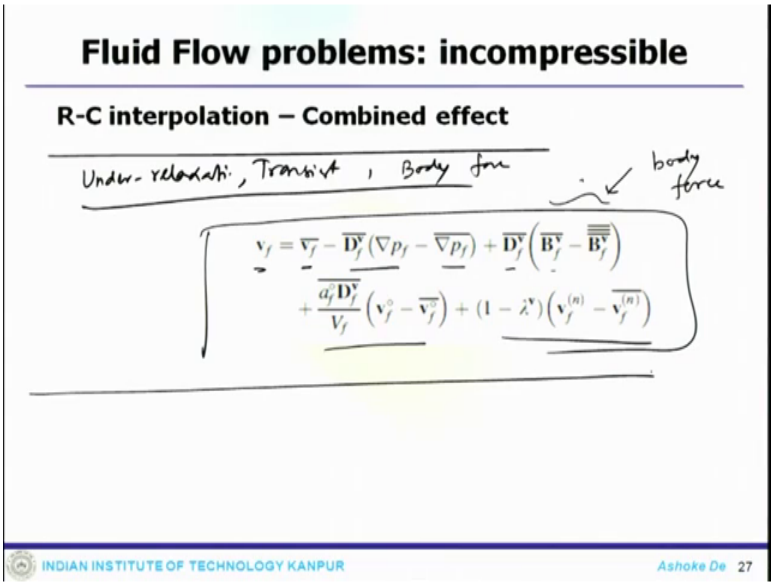

So, you can actually get, so and then just let us look at one important thing for the interpolations for all combined effects which takes care of the under relaxation term; under relaxation term, transient term and it also takes care of the body force term.

So, then the surface vector v f is a average D f v delta p f minus delta p f bar D f v this will the term which will come due to body force term and then this contributions are there. So, this is how one can look at the interpolation scheme in this kind of system. Now have been said that that pretty much actually takes care of the discussion on the incompressible flow, now will extend the discussion similar discussion where you can also obtain the pressure corrections equation or all this things for compressible cases.

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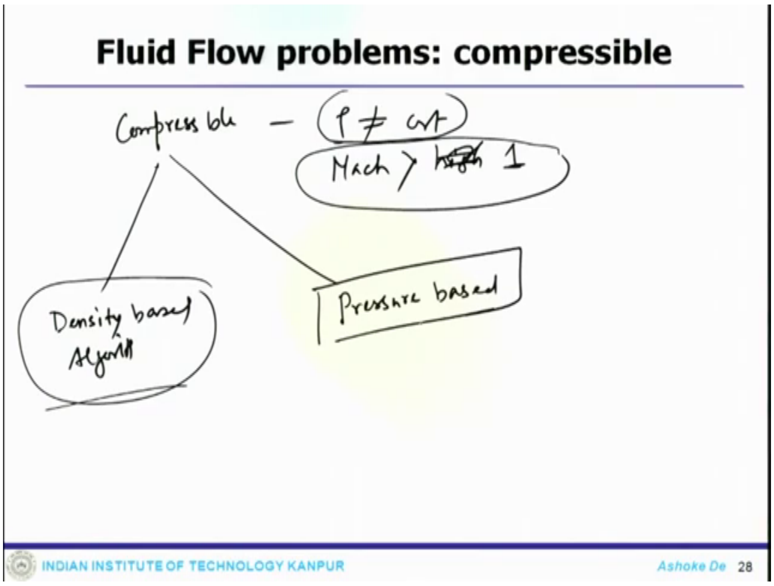

Now, there is a important difference between compressible case where it is like that is density is no more constant and mach number is quite high or greater than 1. So, that is where the compressibility if it starts acting and once that is happening you cannot actually assume the density to be constant rather density to be approximated using ideal gas law.

So, that is why the, now in the compressible flow solver there are two types of approach one can have density based algorithm and one can have pressure based algorithm. And what we are discussing so far all are pressure based algorithm and when deriving the equation for the incompressible case we did not bother about density too much, though we written the density in the term.

So, similar algorithm we can directly extend for the compressible cases. Now, the other thing is that density based algorithm is not the I mean scope of this particular lecture. So, one can talk about that in details in a separate lecture because it has different kind of issues all together. So, right now we will concentrate on pressure based algorithm and can see how we can achieve.

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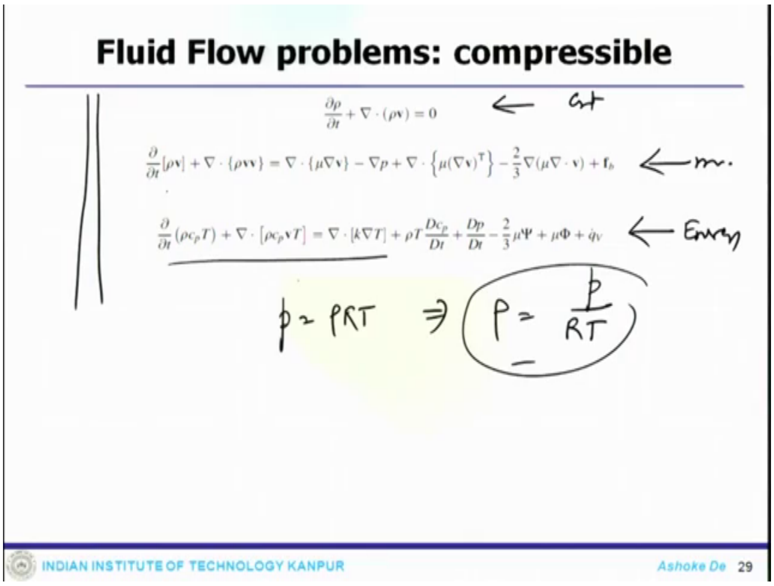

Now, these are the set of equation this is your continuity equation, this your momentum equation, this is your energy equation. Now, when you and one more equation which will be required is the ideal gas law p equals to rho RT which will get you calculate the rho by p by RT. So, top of this all you need this equation where the pressure and density can be calculated and connected with the temperature.

So, in compressible case when we are talking about now if you see the difference between in compressible case we did not really bother about the energy equation, but when you come down to compressible case so these are very standard equation and any textbook on fluid mechanics or compressible flow you can find this equation. So, there is no point going into the details of these discussion. So, and rather we have done detailed discussion in our initial lectures quite we derive the our governing equations, now these are connected.

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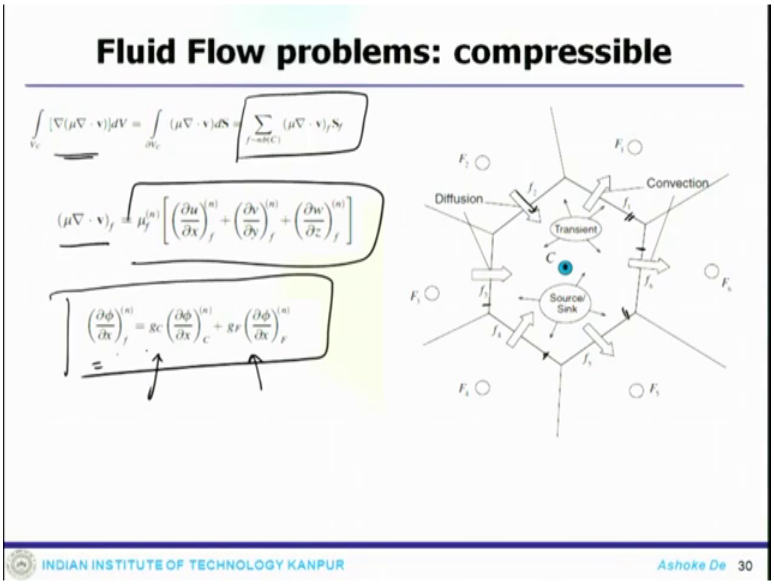

So, let us look at a element c which is our standard element you have all this six faces and these are fluxes or flux vector. And then if you actually take a volume integral of the diffusion term which will convert to a surface integral like this which we have already seen and delta dot mu f will be mu f into this and any flux gradient calculation can we contribute one is the gc and gf gc and gf are the geometric coefficients which we have used earlier. Now how would you derive actually pressure corrections equation.

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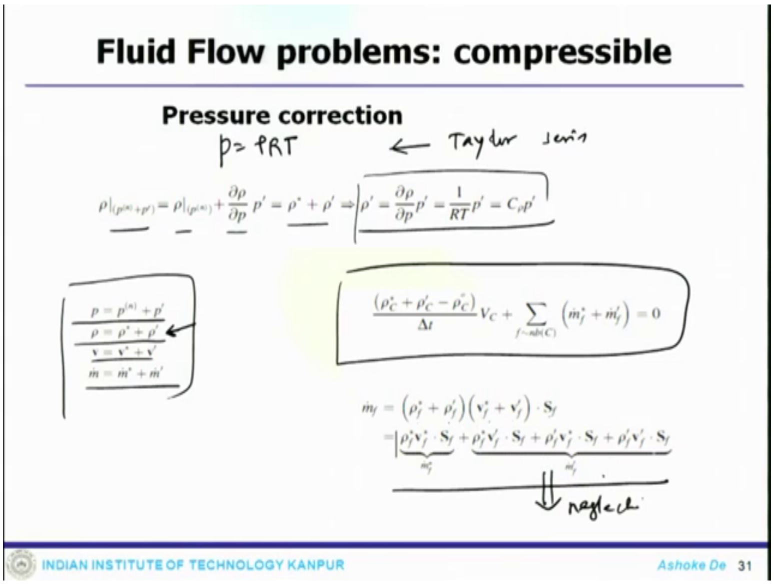

So, first equation that will start with p equals to rho RT and if you expand rho using some sort of an use Taylor's series to expand rho then you write a rho equals to rho plus del rho by del p so it is rho star plus rho prime, so rho prime become c p like this. So, similarly you have the corrections of the velocity and pressure field. So, the pressure previous pressure plus correction density velocity mass flow rate, when we looked at the incompressible case we did not bother to look at the corrections of the density.

Now, we need both corrections of all the density and all these then if you put this in the semi discretized equation this is how it looks like where mass flow rate would be rho f star plus rho f prime v f star plus v f prime and which will boils down to two quantity. So, this is the second correction term is usually this is neglected and then we obtain the detail correction term. Now we will stop here today and will take from here in the follow up lectures.

Thank you.