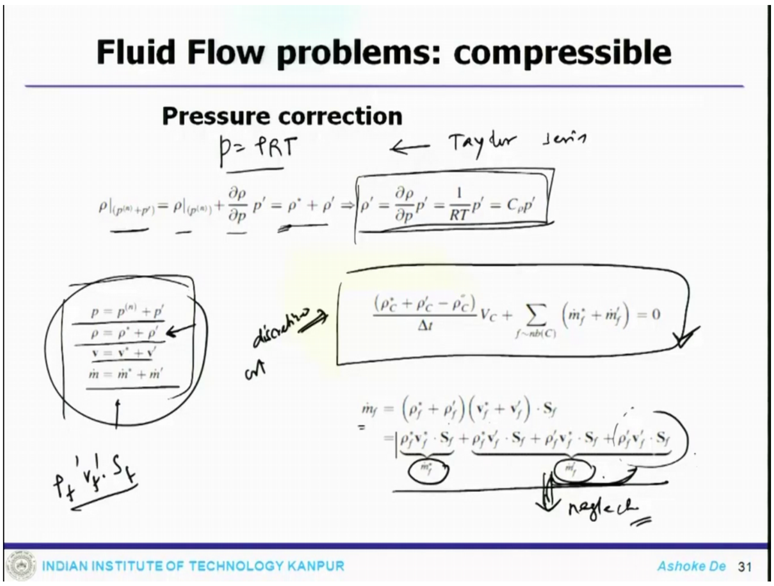
Introduction to Finite Volume Methods-II
Prof. Ashoke De
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 39
Fluid Flow Computation: Compressible Flows-I

So welcome back to the lecture series of Finite Volume and what we have discussing is now after finishing the incompressible fluid flow problem, we are now in the middle of the compressible fluid flow problem and once we do that, then we will discuss some of the special topics. And there I mean essentially we will just patch up on those special topics how to discretize and implement in the finite volume context.

So, let us go back where you left in the last lecture is in the compressible pressure based corrections.

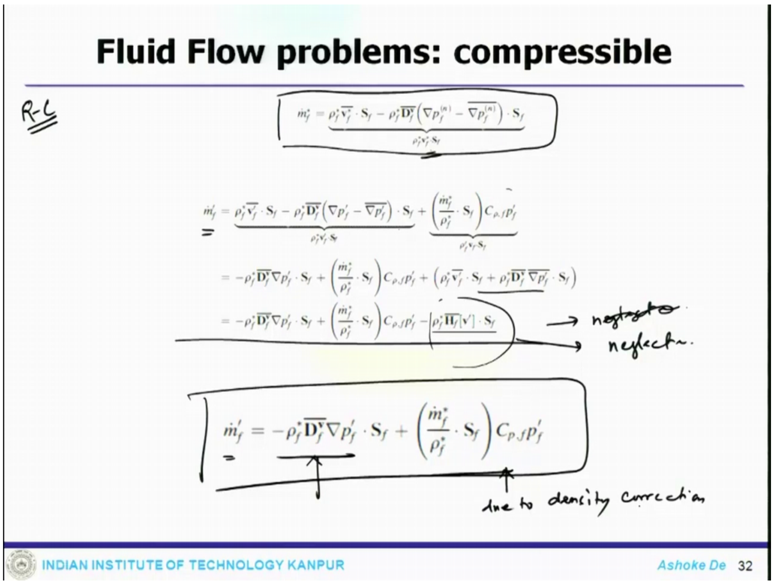
(Refer Slide Time: 00:48)



And where once you try to look at the pressure correction equation, this is where we started with the pressure density relationship, then we used the Taylor Series expansion to get the density where one component is the star component. Again this case like our incompressible case you have a correction component which is prime and the intermediate component which will be calculated like n star, then the density prime was obtained like this and once you put it back in the discretize continuity equation, this is your discretized continuity equation, you get this equation and where m dot f is m dot like this and these term here which is can be treated as intermediate m dot f star and this is m dot f prime the second order correction term like rho f prime v f prime dot s f prime s f.

So, which is these term the second order correction terms these are neglected. So, then because these term is typically smaller; so once you neglect that the approximation doesnt influence too much the iterative process or the solution process which you now, use the Rhie Chew interpolation

(Refer Slide Time: 02:12)

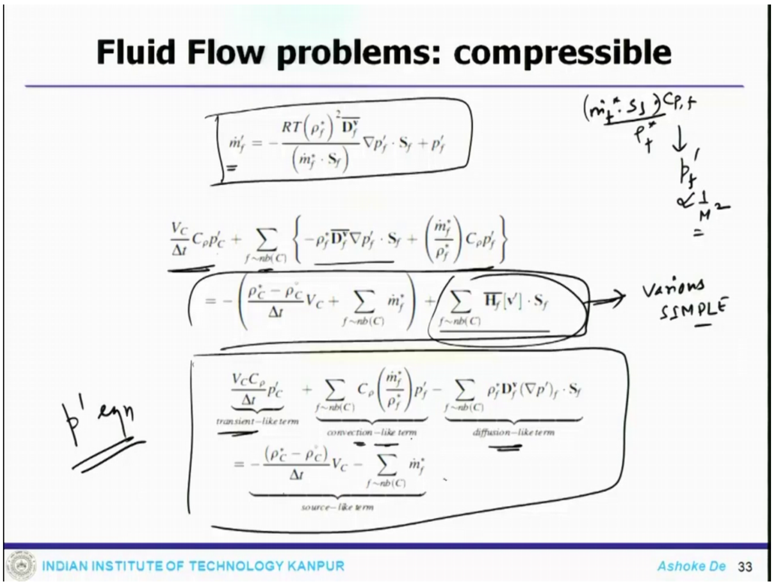


So, similarly Rhie Chew interpolation is applied and of the flux the mass flux at the cell face can be obtained like rho star f v star f bar dot s bar and then. So, this is equivalent your rho f star v f star dot s f and here m f prime is rho f star v f prime bar dot S f minus rho f star d f v minus pressure corrections equation and then, one contribution come from the CP and if you do the algebra, finally you get this expression and again the second order term which is shown here the under line term these are actually neglected in the iterative process.

So, now I mean the second order term here it is neglected and the under lined term which is here that actually present some difficulties as was done in the incompressible algorithm. So, typically this one also sort of dropped or neglected and if you neglect this term, the correction to the mass flux or m dot f prime begins rho f star dot S f and this much. So, now your corrected mass flux is looks like that in the first term here is similar to what arising at the incompressible case. So, if you look at the first term this is exactly with the kind of term that you have seen in the incompressible case and second term is due to, so this is due to due to density correction. So, this is due to density correction. Since it is a compressible case, the density corrections do appear in the system.

Now, in the compressible system the second term is quite important because it transforms the pressure corrections equation to an elliptical system than 2 and 1 also in a other format. Now at the same time we can devise our simple algorithm or compressible simple algorithm based on this.

(Refer Slide Time: 04:34)



Now, one can do some sort of a normalization of these term like m dot f star dot S f into CP f by rho f star. If this kind of normalization is done of weighting factor one for the for P f prime term which will be proportional to now becomes an weighting factor P f prime term which will be proportional to 1 by mach number square where m is the mach number now and that case the m dot f prime will be r t rho f star by this and like this expression.

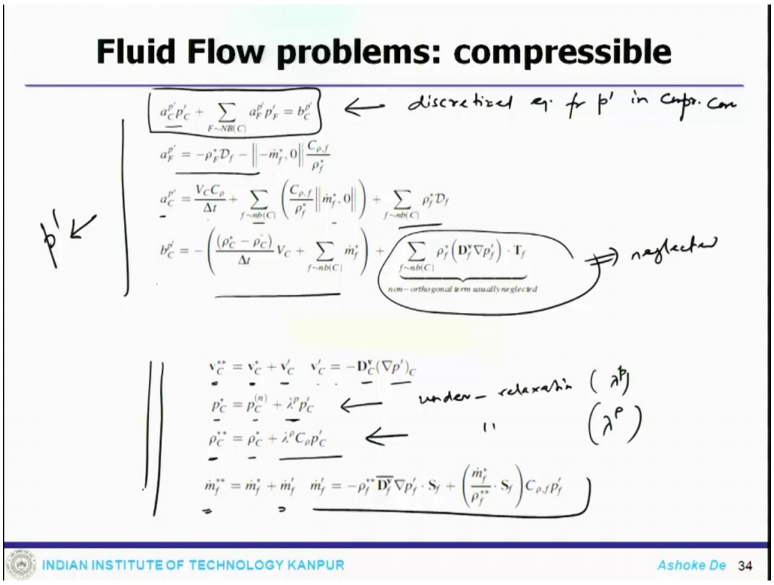
So, when you have a low mach number value, the delta P correction term dominants and that returning equation is actually elliptically nature, but on the other hand when your mach number is too high, so the P prime correction and can no longer be neglected which will given rise to a hyperbolic character to the correction equation. So, these combined behavior actually allows the prediction of the fluid flow of all the speed. That means, there is a strong correlation between mach number and pressure. So, and that changes the system behavior accordingly when mach number is low or the lower mach number cases rather incompressible cases your delta P prime equation behave like an elliptical system, but when you go down to high mach number cases, they behave like an hyperbolic system.

So, now once you substitute everything back in the continuity and compressible flow of the pressure correction equation, so this will be the transient term V c by delta t c P P c prime summation over cell faces and then, you density correction. So, you can put them together nicely in a compact form and same thing once you put the another treatment of the underlined term yields, the variant of simple.

So, how you actually treat this term? This gives you a various simple algorithm which we have already seen in the case of incompressible flow where simple simple c prime or piso it depends how we take care of this correction term. Now, dropping the underline term the pressure correction equation for the simple algorithm can be modified like Vcc rho by delta P and so this is a transient term. This is conviction like term summation over all the faces due to density diffusion like term and then which will be a right hand side you have a source and corrections. So, that is a source term.

So, this is your P prime equation for compressible case. Now one can note that because that convergence the correction fields always leads to 0. So, the order of scheme used to discretize this convection like term is of no consequence on the accuracy of the final results. So, however there is not the case for m dot f star. The use of higher order schemes in its evaluation can improve the capture of socks in the algorithm. So, to enhance the robot robust nest and its helpful to use the append scheme for the discretization of this convection like term one can actually neglect the non-orthogonal contribution for the diffusion like term and then, the pressure correction coefficients would become like this.

(Refer Slide Time: 08:38)

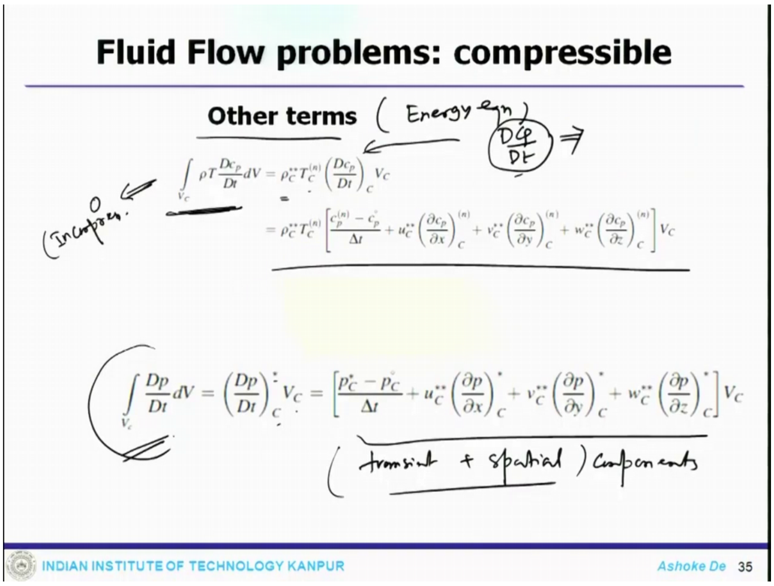


So, a c P c prime summation over all this would give rise to this source equation, that is my discretized equation for p prime. In compressible case a p prime would be minus rho F d f minus mass flow rate m dot f c f rho f a C P prime would be Vcc rho by delta t summation overall interior faces c p m dot f and then correction and rho this will be behaving like an; so you have one along the face; another one the tangential. So, this is like non orthogonal corrections and this can be neglected, but this will only once you neglect that your convergence should be slower, but eventually since its iterative process you will end up getting a converge solution.

Now, once you do that your corrections components can be estimated. Once you get the p prime from this, once you get p prime, then your v c star equals to v c star plus v c prime where v c prime equals to minus d c v delta p prime c and p c star is p c m and lambda.

So, here you use some sort of a under relaxation and the factor is lambda because lambda p, then rho c start start equals to rho c star plus lambda p c. I mean this also for some sort of a under relaxation for lambda rho and then finally calculate m dot f double star equals to m dot f star plus m dot prime and m dot prime is estimated like this. So, you have different under relaxation term and you get the corrections term. Now, top of the it one has to also discretize the energy equation. Now, before doing that why I mean to get the discretize energy equation we can look at the different terms which are part of the energy equation. For example, first thing that we can look at the specific heat term.

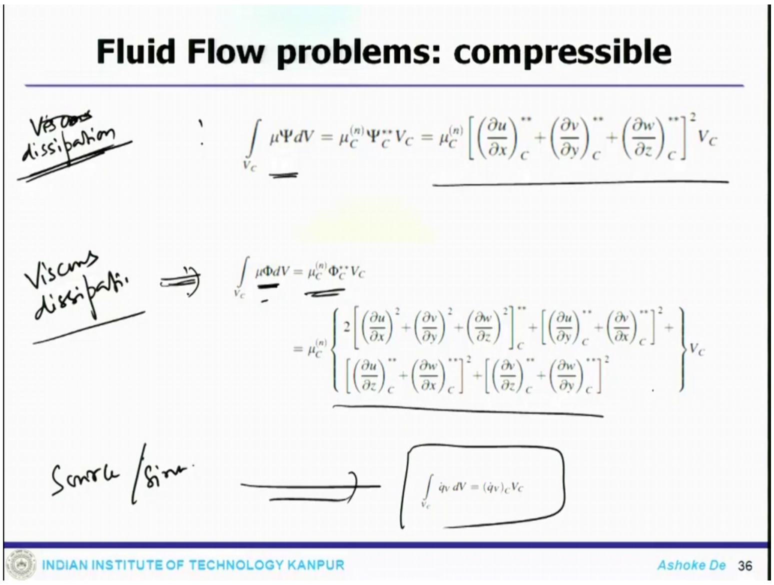
(Refer Slide Time: 10:56)



So, the other terms in energy equation, so the specific heat term that if you look at the volume integral of that, it will get corrections and the previous iteration value and you can write that term like this where if you see that is a d c p by d t setting there which is a material derivative or substantial derivative of the specific heat. Now, as long as your flow is incompressible, you can actually neglect these terms or the contribution due to this become essentially zero in incompressible crisis, but since we are discussing the compressible case we cannot neglect this term and this leads to some sort of and transient and special derivative of that.

Similarly, the pressure term or the substantial derivative term which for the pressure if you do that, this will be d p by d t star at c v c. Again this will be written some transient plus spatial component. So, you need to take that also into account.

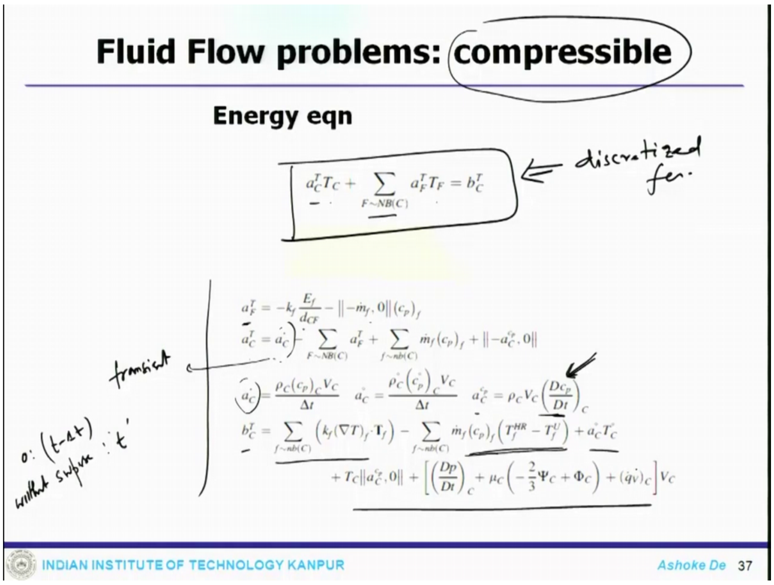
(Refer Slide Time: 12:22)



Now, you have this is your viscous dissipation term. So, the viscous dissipation term if you do the volume integral, this will only get you back a volume integral with the corrections and the other term is the if you have any source or sink like term, then you get a volume integral of that which will written like that now this would be source or sink term. So, this guy is the dissipation term and this is your viscous dissipation term.

So, correct that thing. So, what first term is the only dissipation term, this is the viscous dissipation term where mu is setting there and this will be the source term which will be integrated. This we have already seen. The only thing which is the new term arising even then if you have a viscous dissipation term all the it only actually gives rise to the velocity gradient. Apart from that the integration over a cell element is not that difficult , but the difficulties arises from this d p by d t term and d c p by d theta.

(Refer Slide Time: 13:47)

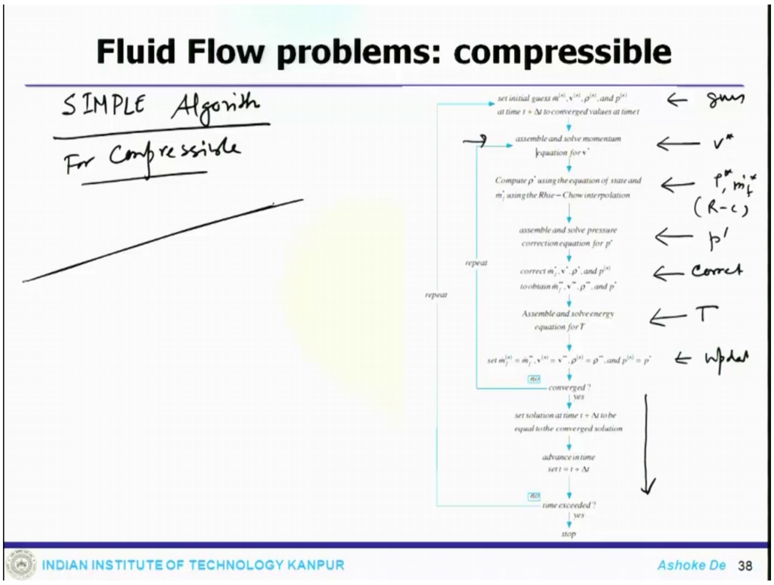


Now, if you took the energy equation and this will be your discretized form of the energy equations. Discretized form again it looks like similar to our momentum or anything else. So, a c t c summation over cell a f t f, but the superscript p stands for the temperature coefficients a f where minus k f e f by d c f minus mass flow rate c p f a c t is a c dot minus here again dot. This guy is the transient contribution.

So, this is a transient contribution minus this and the transient contribution is rho c c p c v c by delta t and a c not and the subscript from t minus delta t and without superscript is t level which can be calculated like d c p by d t and the source term we have some over the faces then c p temperature deferred correction. This is a deferred corrections where you take the high resolution and the append corrections previous time integration and this.

So, now if you look at the coefficients here they are bit involved and why they are bit involved, the reason is because of the compressibility since it is a compressible flow. The pressure density and the other quantities like c p they are connected and one cannot neglect from one other. Top of that you come across like this kind of spatial I mean the material derivative or substantial derivative term in your discretized equation.

(Refer Slide Time: 15:47)

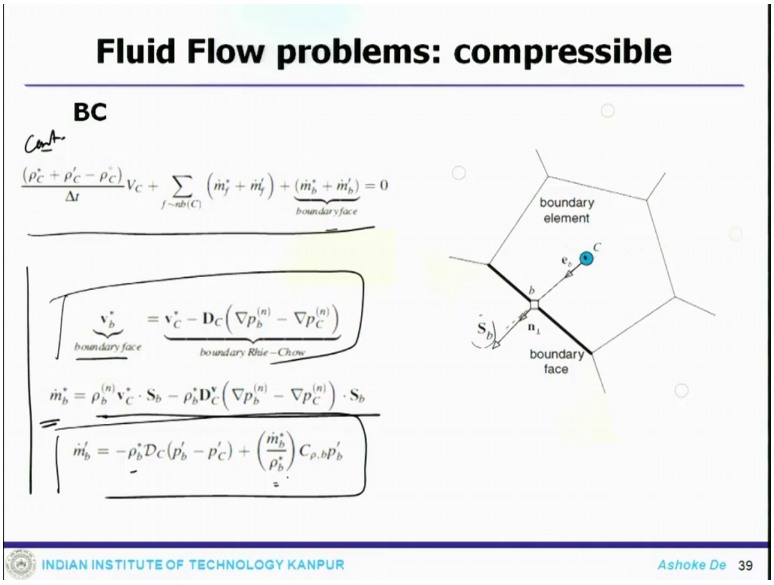


Now, having said that we can actually this is our simple algorithm for compressible case. So, we can see. So, initially you do the guess value as we have done. You solve for the momentum for v star, then do the rho star calculation and then m dot f star where you use Rhie Chow interpolation solve for p prime, then you correct then again you solve for temperature, then you update everything is just like a simple algorithm.

Only thing is the density is now taken into consideration and you take that into consideration if it is converge move for the next time iteration. If not, you go back and this place and repeat. So, the algorithm wise it is exactly similar that we have discussed in the incompressible case. The difference comes here density c p these are the term which are no more can be neglected and top of that energy equation becomes a part of your solver or the system you cannot about the energy equation.

Now, having said that one can understand when you talk about this algorithm. You need to talk about the boundary conditions. We have done some detailed discussion for the incompressible case. Now look at the same in the context of compressibility.

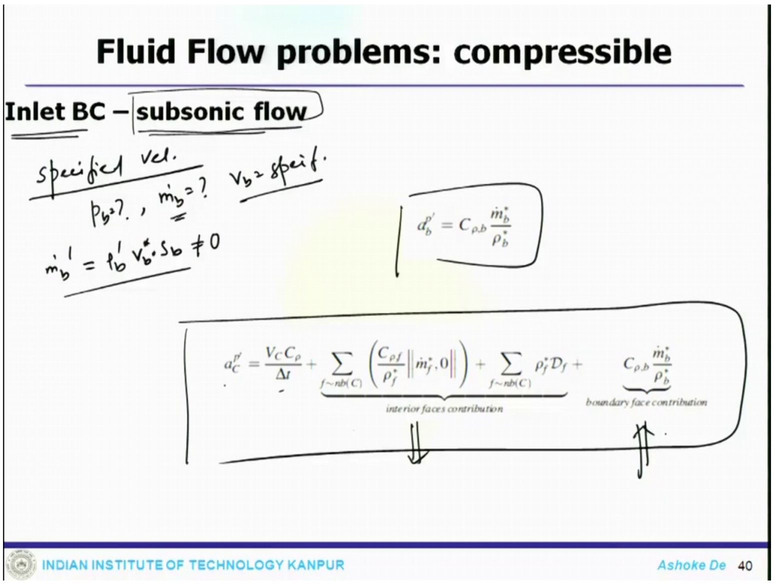
(Refer Slide Time: 17:27)



So, this is a typical boundary element and here is the s b is the circus vector, this is the c centroid and now in the boundary cell this is my continuity equation rho c star. So, this is my continuity rho c star plus rho c prime minus rho c from the previous time iteration mass flux correction and boundary face. So, the boundary face velocity would be corrector can Rhie Chow interpellation using the pressure and this and the corrected mass flow rate of the calculated mass flow rate would be rho b with the previous iteration and this is the corrections.

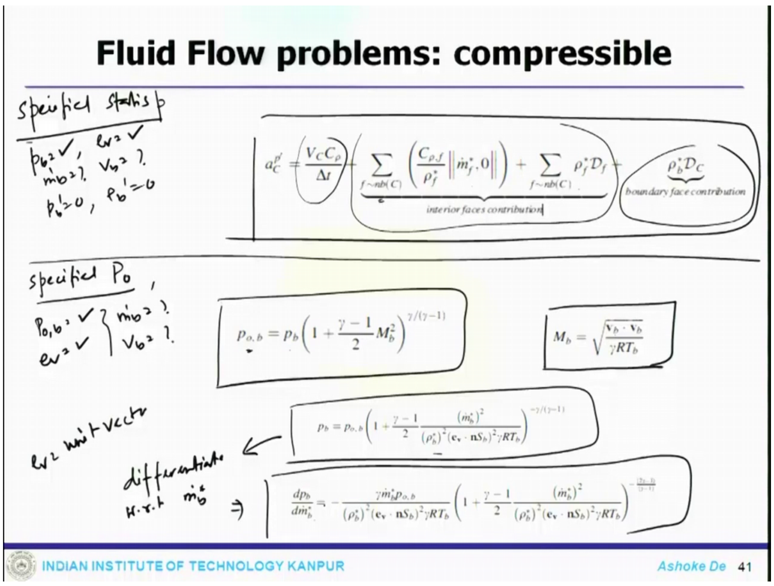
So, when actually now if you have a different kind of boundary condition, this term of the system of equation they get modified and they look quite a bit of similarity accept the term where your density these are coming in for the mass flux calculation. Other than that they are I mean they are looking quite similar.

(Refer Slide Time: 18:33)



Now, the first thing is that inlet boundary condition, now inlet boundary condition if you have a subsonic flow, that means now while you talking about the compressible case or high speed case you can encounter two different kind of boundary condition. One could be subsonic; one could be supersonic. So, let us first start with the inlet, but subsonic flow if it is subsonic flow and then it could be specified velocity if that is the case, your p b is not known m dot b is not known, but v b is specified. Now like incompressible flow or compressible flow the density depends on the pressure.

(Refer Slide Time: 20:24)



So, the mass flux remains unknown even with the specified velocity. See that the big difference when you talked about the incompressible case as soon as you know the velocity you could calculate this, but in this case without knowing the density you cannot do that.

So, that means m dot b prime is equals to rho b prime v b star dot s b which is not 0. So, at the inlet boundary the coefficient is multiplied with the p b prime and it could be written as a b p b prime c p b by like this. Now for the implementation of the pressure correction p b prime is expressed in term of some interior note and the coefficients like this. So, a c p prime would be b c c rho by delta t by c p f by rho f star m dot. So, these are for coming from the interior faces. This is some boundary face contribution. So, this gets modified for the pressure. Now, another could be you can specified static pressure static pressure, that means p b is known and also you this is known velocity direction. What we do not know this and this we do not know now in this case for static pressure known the p b is known.

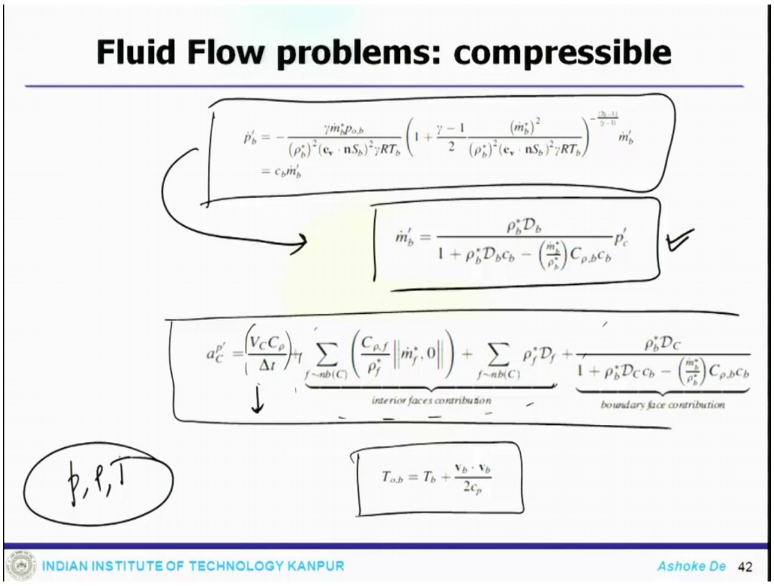
So, p b prime can be set to 0 and consequently your rho b prime is also 0. Now the implementation would be similar to incompressible case like a dislet condition and the correction term coefficients for pressure gets modified like this. So, one would be the transient component, then you get summation over all the faces c p f rho f star m dot f plus rho f d f and rho b d c. So, this is a boundary face contribution, this is coming from all interior faces.

Now another kind of boundary condition that we did not talk about in the context of incompressible flow which is quite important here when you define the specified p naught for the total pressure and velocity direction. So, that means p naught b is known, e v is known. So, which means m dot b is not known, v b is not known. For this case to calculate the magnitude of the velocity, one has to use the stagnation conditions where mach number at the face can be defined at v b dot v b by gamma r t b and the pressure which is unknown at the face can be correlated with this standard isentropic law which is stagnated condition is known. So, the pressure can be correlated with gamma.

Now, here b refers to the boundary. Now p b 1 can rearrange with gamma by m dot f by rho b like this. So, which you can get rid of mach number and velocity. So, it can be terms of total pressure one can actually rearrange this since e v is a unit vector in the direction of the velocity.

So, once you differentiate this equation, so differentiate this equation differentiate this equation with respect to m dot star b which will get you d p by this. So, that is an algebra. One can carry out that and now you substitute this guy into the equation of the pressure corrections.

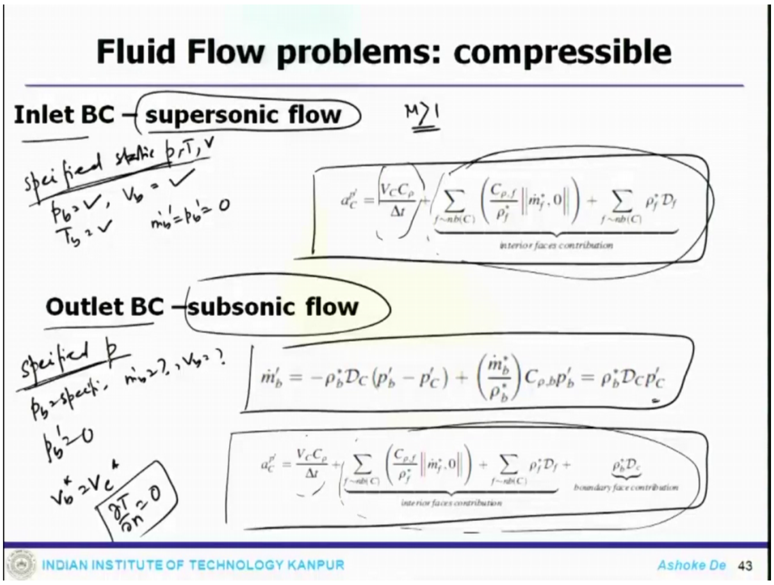
(Refer Slide Time: 23:23)



Then what you get p b prime equals to c b some constant like this. So, that will get you back the mass flux prime like this. So, these are the condition which you get for the pressure correction.

Now, the modified cell coefficient which is obtained for the m dot b from this equation, you can apply that and the pressure corrections equation get modified like this where this will be transient term rho c p by delta t, then you have a term which is interior face term and you get a term on boundary and your stagnation temperature would be estimated like this. So, in a compressible case your pressure density temperature they are linked and wherever you do an condition with the pressure, then you can have you need to take care the density and temperature are the same time; now that talks about different kind of boundary conditions are subsonic case.

(Refer Slide Time: 24:31)



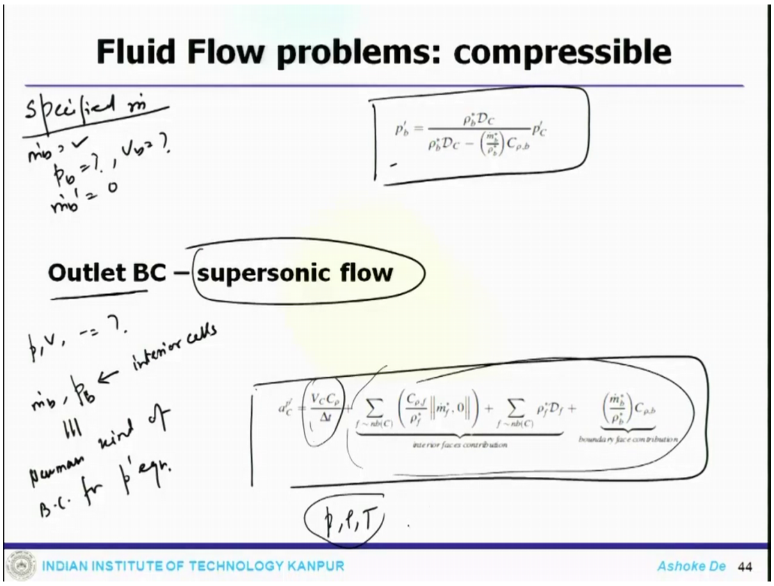
Now, you could also have inlet boundary condition which is supersonic in condition. That means, mach number is greater than 1. So, in that case you can have again a situation where specified static pressure; that means p b is known, v b is also known and t b is also known. So, it is specified p t and v.

So, all three are defined then which is implied condition like m dot p prime p b prime, they would be 0 and the coefficient of the pressure corrections equations gets modified like that. So, it will be written the transient term and this is the term which is the transient term and there will be an integration over all the interior faces; so there we two contribution which will come from this.

So, one can note here your inlet conditions in compressible cases could be of two different types. One could be subsonic; one could be supersonic. So, now when you have a supersonic we do not have too much of variation. You can specified them as a dislet condition. Now similarly you should have outlet boundary condition and outlet boundary condition could be also subsonic flow where again you can have specified pressure which means p b is specified, m dot b is not known, v b is not known.

So, once specified pressure your p b corrections would be 0 and the mass flow corrections can be estimated like this where rho b star d c p c prime p c prime is computed and since the v b star is not known, it is necessary to assume that v b star is equals to b c star and the expression for the pressure correction coefficient a c could be modified like v c c p by delta t and then you have all this contribution from the interior note and also one has to note that energy equation a 0 gradient or dell t by dell n kind of 0 at the energy flux zero or neumann kind of boundary condition needs to be specified because this is a supersonic face. You cannot leave the energy equation and connected. So, it has to be connected with the system.

(Refer Slide Time: 27:12)

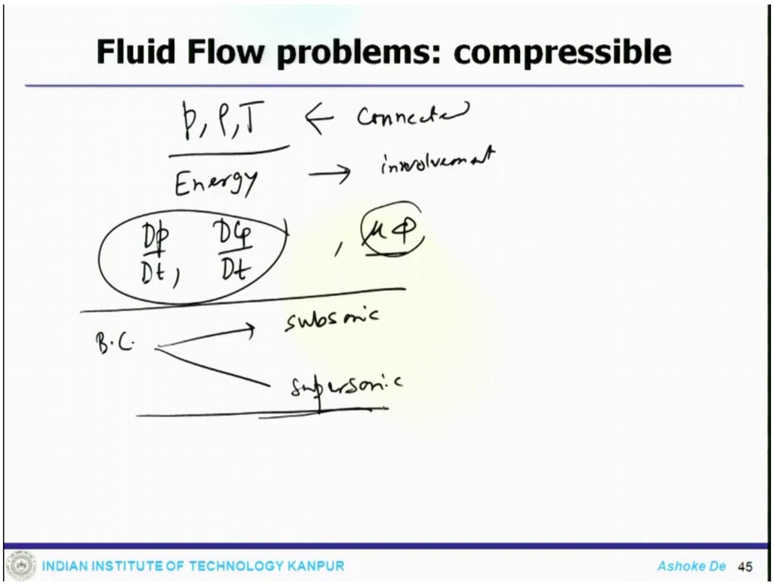


Now, similarly one can have specified m dot. So, that means m dot f b is known, but p b not known, v b not known. So, specified mass flux mean m dot prime would be 0. So, you can simply drop the pressure correction equation with no modification and then, if setting m dot b prime equals to 0, the coefficient of the pressure correction equation or p b prime would be like this.

So, you can actually calculate the pressure and density corrections at the boundary and the last one which I would like to just touch with the outlet boundary conditions and which could be of the supersonic type. So, in that case here the none of the variables should be specified. So, p v and all these are not known. Pv density temperature and they should be extrapolated from the interior value. So, m dot b and p b they are interpolated or extrapolated from interior interior cells and this is equivalent to applying a neumann kind of boundary conditions on pressure corrections which will lead to this. So, which is equivalent to neumann kind of boundary condition for p prime equation which will lead to this pressure correction coefficient to be modified like that.

So, a c prime would be v c c rho delta t. This is transient term and these are all summation over interior faces and. If you notice here again pressure density temperature p rho t these are all connected. So, when you compare with the subsonic case with the supersonic case, the one important difference which appear is that the not only the formulation.

(Refer Slide Time: 29:39)



Because formulation only takes care of the determine takes the energy and density into system also there this pressure density temperature, they are connected that is number 1. Then you have to involve the energy equation that is another involvement or inclusion and also in the discretization you get some sort of a term like d p by d t d c p by d t. These are the term which appears and then, viscous dissipation term these are the term or extra term which appear in this kind of situation.

Now, the other thing which is possible is that the when you come to compressible case, your solution algorithm has to have to different kind of definition of the boundary inlet condition and outlet condition they could be of solve your boundary condition can be of inlet or outlet, could be of subsonic type and supersonic type. So, one has to take care of that. Now that essentially concludes the portion of our fluid flow problem where we have discussed both incompressible and compressible.

Now, what I would like to patch up on some of the advance things like how you generate bit, but this should not be done in a detailed discussion, but one can always look at the text book and then look at some of the turbulent modeling issues and how you discretize that. So, we will stop here today and will take from here in the follow up lectures.

Thank you.