Introduction to Finite Volume Methods-II
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Lecture – 08
Linear solvers-VIII

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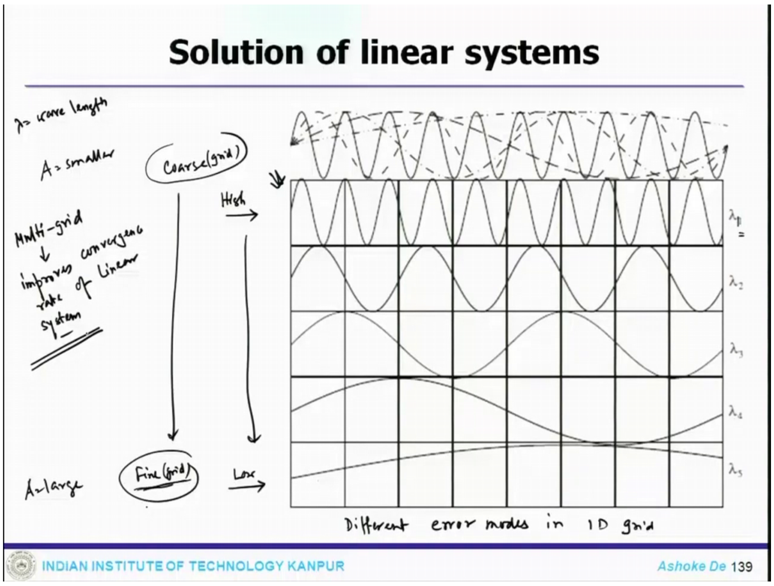

Welcome back. So, the whole idea behind that because you see this kind of error which appears and this has to do with grid and the solution procedure. So, what it does that one basic approach which is commonly known as is called the algebraic AMG, which is Algebraic Multi Grid method.

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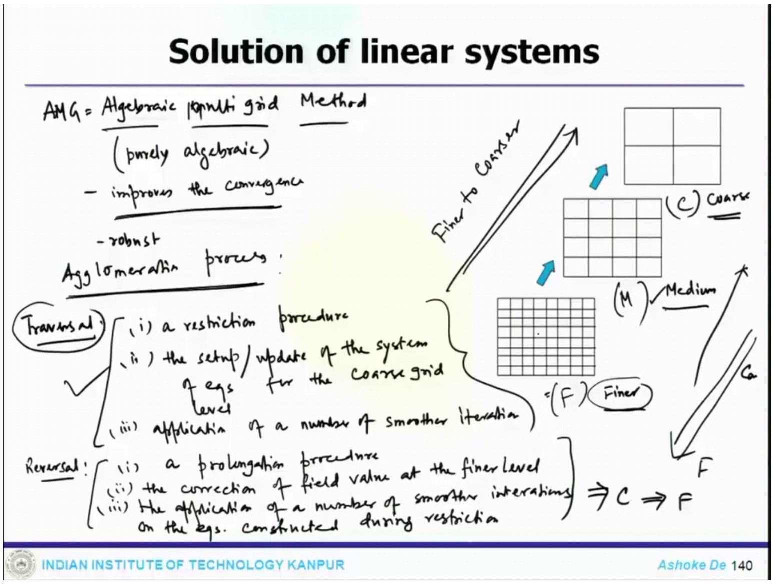

So, that is what it stands for AMG algebraic multi grid method, now this process is purely algebraic that is why it is called the algebraic multi grid method. And what it does is actually improve the convergence of your iterative solver. So, this improves the convergence of your iterative solver by ensuring that the resulting low frequency error that arise from the application of a smoother at any one grid level are transformed into higher frequency error at a coarser grid level.

So, let me re iterate this things, it only improve the convergence and what it does that what you achieve at the finer grid level this is the finer grid level one can see, this is the finer, medium and coarse. So, the low frequency oscillation that arises at the final grid level it is transformed to the high frequency oscillation at the coarser grid level and that is why it ensures. So, it is a hierarchical approach from finer medium to coarser and able to, multi grid methods are able to overcome this convergence problem that you see when we go down to a finer grid level with a large system. So, this is how it used and also this multi grid method is quite robust and process is algebraic in sense.

So, the weight is done through some agglomeration process. So, this can be used to build highly efficient and lower robust linear solver. So, in either of this process a multi grid cycling procedure essentially is used to guide the traversal of various grid hierarchies. So, this is finer level grid then the medium level grid then the coarser level grid. So, and it could be you can one can have another level of grid refinement if they want. But typically three layer of grid refinement works quite nicely for multi grid system. And how it is done that traversal process? The traversal process it goes through different procedures of the process.

It involves a restriction procedure first, then the setup or update of the system of equation for the coarse grid level then third is the application of a number of smoother iteration. So, you operate or do carry out some sort of an iteration to smooth out those issue. So, that is essentially the traversal process and this process is only happening when you go from this direction. That means, this is applicable, this guy this process or the steps are applicable when you move from finer to coarser. Now the reversal process which is a one can say it is a reversal process. So, there you need to do a prolongation procedure.

That is number 1, number 2 the correction of field value at the finer level. So, that is the correction of field value at the finer level. And third the application of a number of smoother iteration on the equation constructed during restriction, constructed during restriction ok. So, what one has to do this is the reversal process and this is applicable when you come down from this direction. That means, coarser to finer. So, this is applicable C 2 if this is C this is m this is f. So, one case you follow this step to go from this direction to that direction; that means, you have a fine level of grid from here you move to the medium level medium to coarser level.

So, these are the process one has to follow and when you do the reversal; that means, you get a solution at the coarser level now translate back to the finer level. So, what it does if you one can think about by looking at this picture and the procedure is that you are getting a solution at different grid level. Maybe this F actually is your finer grid level where you would like to obtain your solution. But then your iterative process will end up giving you some sort of an error, convergence is low so in order to improve that you refine it to two different grid level get your iterative solver, solve their reduce your error due to iterative solver translate back to the original level.

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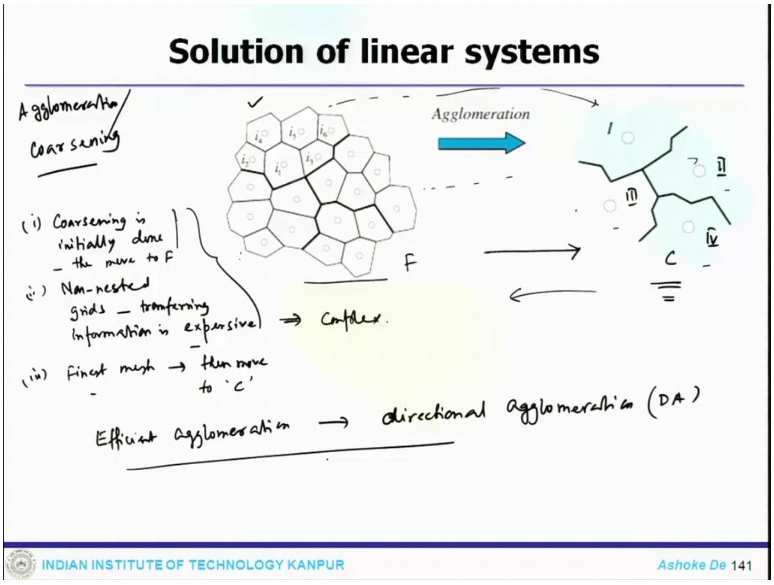

Now, what one needs to do it is essentially one has to do some agglomeration. So, this is called agglomeration or coarsening of the grid so; that means, this is my finer level of grid F which stands for finer. So, I am doing the coarsening level. So, F to C. So, here if you can see all this cells i 1 2 3 4 5 6 they are merged to one particular cell from there to here. Then in between all these they move to that site this side they move to here. So, essentially 4 to 6 elements in the finer grid levels are sort of merged and here you get 1 2 3 4. So, you get an coarser level of grid. So, what essentially one needs to do is that first your coarsening is done.

So, coarsening is initially done or carried out; that means from the finer grid level to the course grid level is generated. Then the fine level refinement is done so also one can use non nested grids which can be also used so, but that case there is a problem that transferring or transferring information from one level to another level is quite expensive. That is why this kind of process has some limitations.

Then you can go to finest mesh and get your solution done and from there you do the coarsening and through this agglomeration process. But what happens is that these two coarsening process, I mean this is a process where you can first do coarsening and then move to F. Or you can have non nested grid where the information transfer is quite expensive, in both these two cases or the approaches they do not allow good resolution of complex domain.

So, when you first essentially these two step say that you can have first this coarser mesh. And then you move to the finer mesh or you can have non nested grid. But in that case the transfer between these different level is quite expensive, in both these cases are not having very good resolution or accuracy of the solution when it goes to complex domain, but the other case the third approach where actually you first have a solution on the finer mesh and then you move to coarsening the process; that means, you first get a solution here which will have.

So, many elements which solution must be quite accurate, but then you do the coarsening through agglomeration process one level of coarsening two level of coarsening reduce all this error get a solution towards the convergent. Then you come back to get the final solution. So, and the coarsening done is that I mean it is done inside the coat through some sort of a fusing of the multiple elements as we have seen here multiple elements are kind of merge together to get one element here this 5 got to here, this 5 got here, these 4 here.

So, this is how the process is done and in that case and one more thing that any efficient agglomeration process. So, any efficient agglomeration process is the directional agglomerations is known as called directional agglomeration which was initially developed by marvelius so this is what it says.

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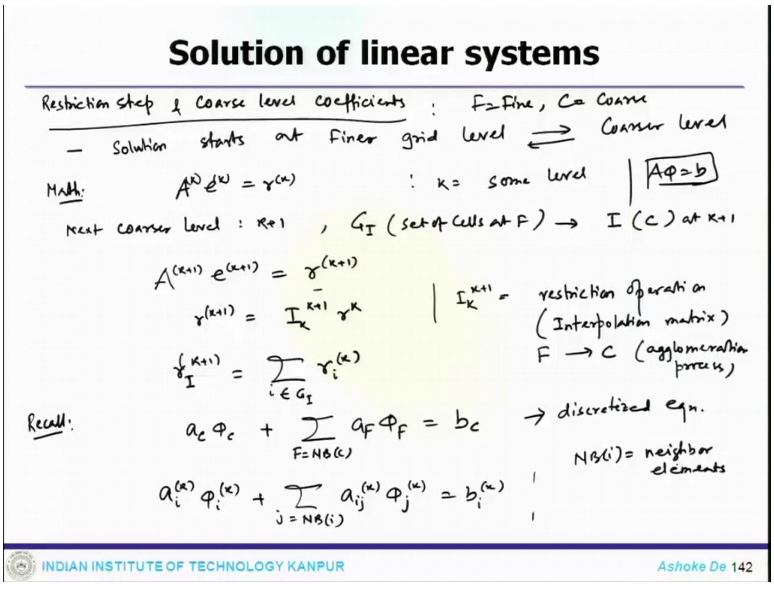

Now, one can see the restriction step and the coarse level coefficients. So, how one can obtain these things? So where the important point here the solution starts at finer grid level so that is the important point. So, after performing few iteration the error is transferred or restricted to a coarser grid level and the solution is found at that level. Then after performing few iteration at that level the error is restricted again to a higher level and the sequence of events is repeated. So, solution starts at finer level then you carry out some number of iteration, move to the coarser level then carry out some level of iteration and then go back and forth so this is how this done.

So, if one has to put mathematically then one can write A k e k equals to r k. Where k stands for some level, which denotes some level. So, where and the actual system that we are doing is if a equals to b that is our standard system. Now the next coarser level which is at k plus 1 and the error will be restricted. So, let us say G I represents the set of cells I on the finer grid level and that are agglomerated to form cell I of the coarser grid level at k plus 1. So, essentially these are set of cells at F level. So, F stands for fine C stand for coarse immediate level.

So, which are agglomerated to form a cell I at the coarser grid level which is at k plus 1. Then one can write that A k plus 1 e k plus 1 is the residual k plus 1. Now the residuals here on the right hand side which can be computed as r k plus 1 equals to I k k plus 1 r k. So, this guy this I k k plus 1 is essentially the restriction operator or the other way one can think about it is an interpolation matrix ok. An interpolation matrix for from fine grid level to coarser grid level through agglomeration process; agglomeration process through that you do the transformation.

Now the AMG will solve for this residual and one can get r I at k plus 1 equals to summation of I belongs to G I r i k then. Now the coefficient of the coarse element are constructed by adding the appropriate coefficients of the constituting fine elements then if you recall the linear system that we are solving in the discretize system is a C phi C plus summation of F N b C a F phi F equals to b C. Now we can write a different grid level this is our discretized equation discretized equation. Now we can write at different grid level, one can write a i k phi i k plus j goes to n b i a i j k phi j k equals to b i k and here N b I refers to the neighbour elements ok.

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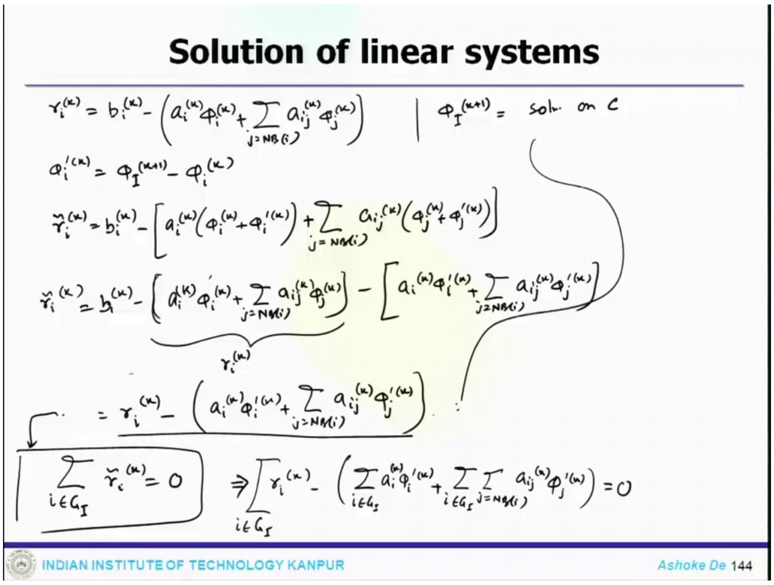

And then the residual can be computed and so the residual one can compute like r i k equals to b i k minus a i k phi i k plus summation of j equals to NB i a i j k phi j k. So, at k level one calculate this step. Now phi i where phi i at k plus one level denotes the solution on coarse mesh element i. So, then one can write phi i from k equals to phi I k plus 1 minus phi i k. Now what it is expected this correction to result must be 0 residual over the coarse mesh element i.

So, the due residuals one can write as an r i tilde k equals to b i k minus a i k phi i k plus phi i prime k plus summation of j equals to NB i a i j k multiplied by phi j k plus phi j prime k. Or equivalently one can write this one r i tilde k equals to b i k minus a i k phi i k plus j equals to NB i goes to a i j k phi j k, this is one component. And other component one can write a i k phi i k plus j goes to this a i j k phi j prime k. So, you take this out. So, this is nothing, but your r i k.

So, one can write this one as r i k minus a i k phi i prime k plus summation of j equals to NB i a i j k phi prime j k. So, this is what one can get the residual in this way. So, if you put them together. So, if you enforce the residual sum i to be 0 then summation of i belongs to G I which is r i k equals to 0. So, that enforcing the sum of the residuals 0.

Now, if you substitute this one this guy into here then this essentially get you that this particular term the r i k summation over i belongs to G I minus summation of i belongs to G I a i k phi i prime k plus. Summation of i belongs to G I summation of j belongs to NB i a i j k phi j prime k which is essentially 0. So, just put this particular expression back to this one and you get this one 0. So, what happens is that you can rewrite this equation.

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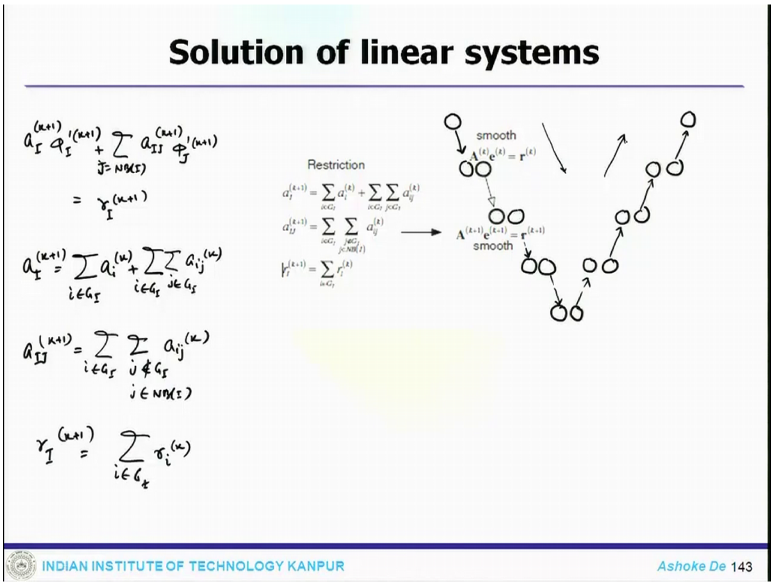

And one can see if you rewrite that equation then one can write a I. So, these are How a I k plus 1 phi I prime k plus 1 plus summation of J NB I equals to a I j k plus 1 phi prime J k plus 1 equals to r I k plus 1. So, here what is that a I k plus 1 is nothing, but I belongs to G I a i small k plus summation of I belongs to G I summation of small j belongs to G I a i j k, a capital I j k plus 1 is I belongs to G I small j does not belongs to G I a I j k. And j belongs to NB I and r I k plus 1 is nothing, but I belongs to G I r I k.

So, this one can see in the figure this is one element and this is what the smoothening is done. So, this goes from here, then this is next level of this things and this goes to here this is the third level. Then you go back you get this back, then you get this then you move back move back to the system. So, one hand you come down this way then you go back in this way. So, that is how you get the solution for this system.

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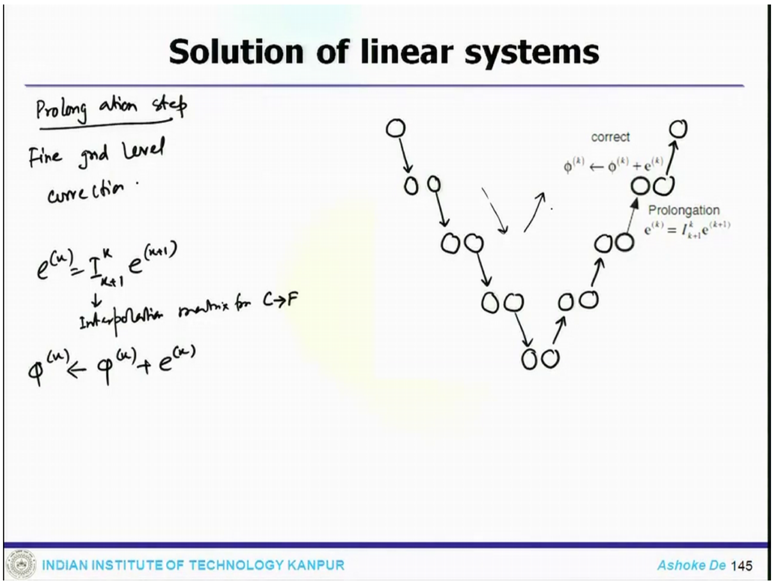

Now, the other step which could be used as the prolongation step and fine grid level corrections. So, the prolongation step or fine grid level correction. What you do that? It is the operator that is used to transfer correction from a coarser to a fine grid level. So, what you do the error vector is I k k plus 1 error k plus 1 and where this guy is the interpolation matrix matrix from C to F. So, that is the interpolation matrix. And finally, the phi k can be obtained as phi k plus e k. So, the correction is basically obtained from the coarser level grid to the finer level.

Now how it is obtained? You can see that graphically this is your level from there you move here and then from there you move to this, then again you move back here this will get you to this from there you get you get the corrections done and finally, so its goes like that. So, this is what the coarser grid level to finer grid level it is done.

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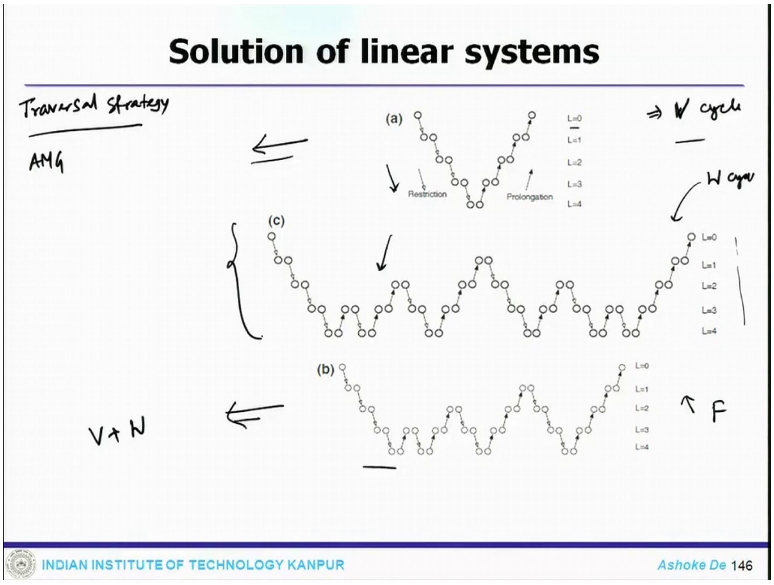

Now, the finally, one thing which is called the traversal strategy and algebraic multi grid system. There are three different ways one can obtain this; the first one which shows the, it shows essentially the b cycle and the second one is called the w cycle, third one is called the F or full multi grid cycle.

So, one case is the b cycle which is shown in this particular I can you can see this is my restriction step and this is how my prolongation step. So, at L 0 to L 2, L 3, L 4 similarly these are L 0, L 2, L 4. So, what happens the usual practice is to perform few iteration, at restriction phase and then to inject the residual to a coarser grid until you reach some coarser level of convergence.

So, for very steep system this b cycle is not at all, I mean not very good or not sufficient for accelerating the convergence of a iterative solver. So, to get a good convergence you need more iterations at this v cycle. So, that is why the improved cycle is w cycle; which actually if you look at it, it applies multiple some sort of a b cycle of nested coarse and fine grid level sweeps through some complicity. Because their complicity level is increasing as you move this and the F cycle is essentially this is a some variant of b plus w cycle. So, it is split into two different segment, some split as per the v cycle some split is as per the w cycle.

So, this has some very typical restriction that very steep system this cannot be obtained very good convergence at the w v cycle cannot get you good convergence with less number of iteration, there you prefer a w cycle. And the F cycle or full multi grid is the combination of both this two. So, that talks about all the iterative solver and we will stop here today.

Thank you.