Introduction to Finite Volume Methods – II

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Lecture – 09
Convection term discretisation – I

So, welcome to the lecture of this Finite Volume Method and now, we will start with the next level of Discretization process. So, just to recall where we stopped in the last lecture, we finish the discussion on our iterative solvers. So, essentially the process that we have gone through, we looked at the diffusion equation, we looked at all the numerics related to that like how the discretization is done on a orthogonal, non-orthogonal, Cartesian, unstructured system then finally, we derived our discretize system and once you discretize the system you eventually get the linear system which is in terms of ax equals to b.

And then we had our discussion or we other had detailed discussion on different kind of linear solvers. Along with the properties of the linear system which are essential for the discussion of the linear solver. So, we started with that and then we did discussion on some direct solvers or their other direct approaches and those direct solvers met the platform for the iterative solvers. And in the iterative process we started with the simplest one and then finally, we added the increasing complexity to have better and better iterative solvers. And then finally, we looked at the multi grade system how one can improve the performance of an iterative solver. So, now, we will move to the discretization process of our convection diffusion system.

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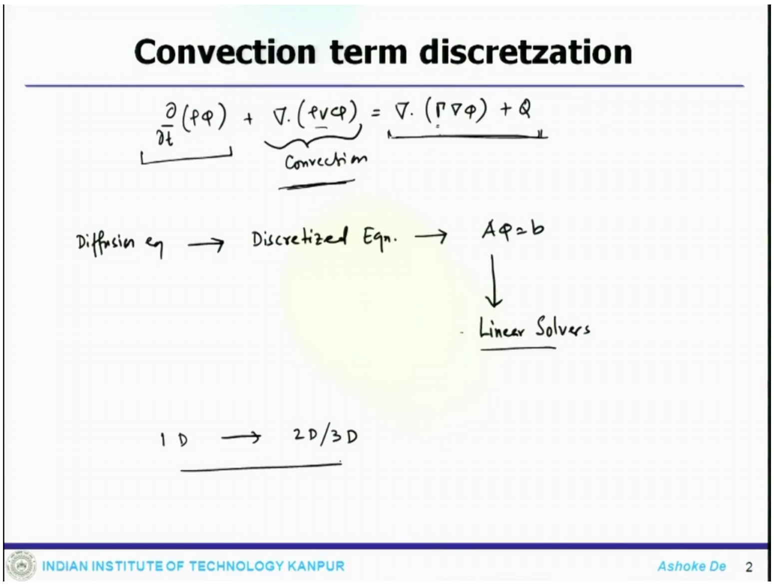

So, if you look at our equation, so, the complete equation was of this kind. So, for any variable phi we had the complete equation on rho v phi equals to delta dot gamma v phi plus some source term. So, what we have looked at it the discretization technique and everything we have looked at it this portion. So, we did not consider the transient term, we did not consider the convection term, we just looked at this and we had our discussion on the diffusion equation which lead to a discretized equation and from there you got a linear system. Once we get that we had our discussion on the linear solvers and from linear solvers we got different kind of linear solvers and the solution pattern which can get you the solution.

Now, we will look at the term the convection term. So, this is the convection term. So, essentially we will look at the discretization of the convection term. So, though the pattern look similar, but there are certain difficulties which are associated with the convection term discretization. It is not going to be straight forward like the diffusion term because in the diffusion term you had a diffusivity coefficients, where if you look at the convection term the gradient retain the velocity vector. Since the flow field or underlined flow field is associated with the convection term the difficulties arises and how you want to proceed? We will look at the 1D system first and then discuss some of this concept like how you discretize this then once one we are done with the 1D system we will move to multi dimensional like 2D or 3D.

And, also here we would like to discuss the orthogonal and non-orthogonal system and the corrections which requires due to non-orthogonality.

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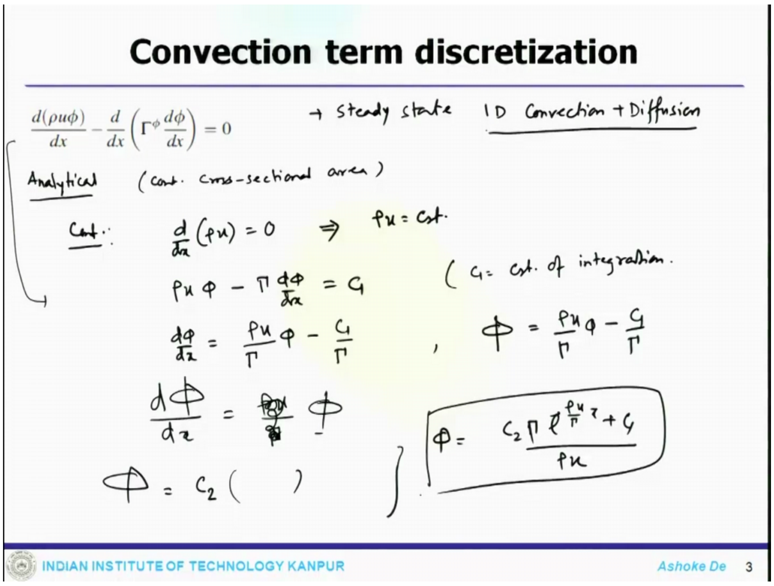

So, to begin with one can start with the 1D convection diffusion system. So, this is steady state 1D convection diffusion system without source term. So, we will start with this. So, once we look at the convection diffusion system then we can move forward. Now, this particular system if you look at this convection diffusion system one can find out since in the one direction, one can find out the solution quite easily. In the sense one can obtain the analytical solution of the same and how do you get that the. So, if you have a constant cross sectional area, then the continuity equation the continuity equation or mass conservation equation can be written at rho u equals to 0, which implies rho u is constant.

Now, using this you look at that 1D system then one can write from here is that rho u phi minus gamma d phi by dx equals to constant C 1. So, C 1 is the constant of integration. So, this equation if you integrate now you rearrange this term. So, one can write d phi by dx or d phi dx equals to rho u gamma phi minus C 1 divided by gamma. Now, you do the change of variable and this guy will so that you say capital phi equals rho u by gamma small phi minus C 1 by gamma, then this will become d capital phi by dx equals to rho u by gamma capital phi. So, or rather this term will not be there you will get this.

Then, if you find out the solution the solution would be in terms of finally, that phi is C 2 some constant and what you get and finally, when you write down the final expression this one can on the small phi it will be C 2 gamma e to the power rho u by gamma x plus C 1 by rho u. So, that is the analytical solution.

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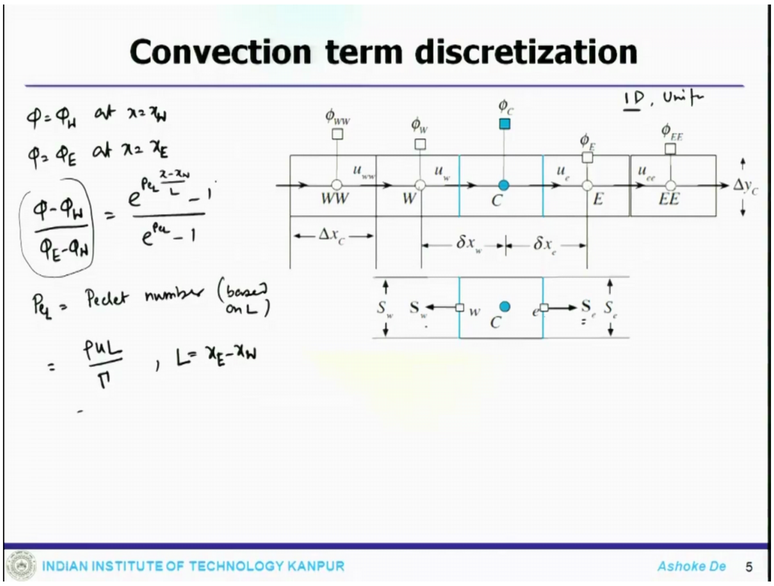

Now, once you get the analytical solution you can show in the particular figure how the solution would look like this is in your 1D system and if you look at the elements we still follow. So, the analytical solution we follow the similar pattern like it is again uniform. You have centre C of the element C ahead of that E, then EE, W, WW; that means, you follow in one directions delta x c would be delta x w all these are uniform. Then you have the face fluxes e or the flux vector S e is the surface vector, S w is the west side surface victor. So, all these are and the centre values are C, W, WW, EE, W. So, this is how we have followed exactly the similar notation in the diffusion system.

So, one said that now the analytical solution between the points W and E one can write phi equals to phi W at x equals to x W phi equals to phi E at x equals to x E. So, one can find out the solution like analytical solution phi W by phi E minus phi W equals to e to the power P e L into x minus x W by L minus 1 divide by e to the power P e L minus 1, where P e L is called the Peclet number. So, this is based on L based on L and which is defined as rho uL by gamma and L is the x E minus x W. So, that is the distance between these two.

Now, one can evaluate the Peclet number and then see how that varies. So, and once you change the in this particular equation if you see this solution on the left hand side it is primarily dependent on the Peclet number and if you vary the Peclet number then one can find out how exactly the solution is varying.

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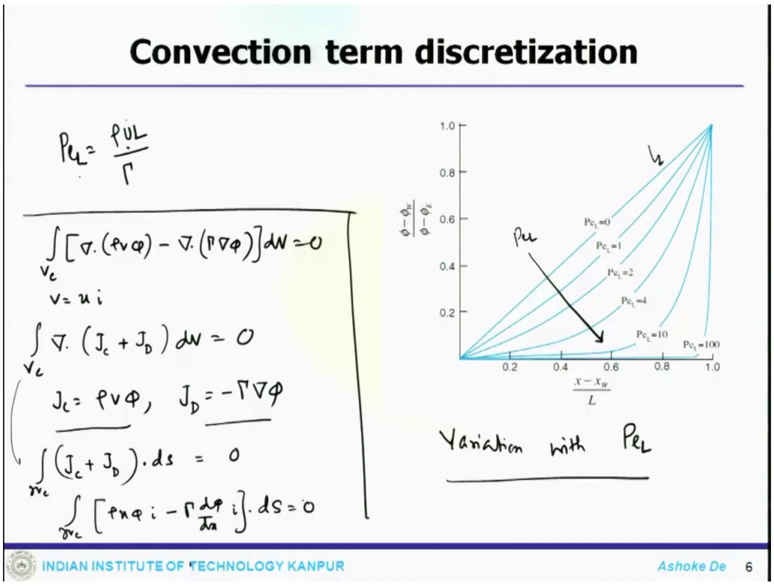

So, this is how the variation with the. So, this is variation with P e L. Now, if you see how the variation is done so, this is the P e L 0. So, you get completely linear variation then as you increase in the Peclet number then you see the normalised length how the profile is varying more and more Peclet number the profile is getting a this is the increasing Peclet number and what is happening the Peclet number is nothing, but the ratio of rho UL by gamma.

So, once this is 0 that means there is no contribution from the primarily from the convection. So, it was primarily diffusion. Then once you increase in the Peclet number your convection becoming important or the effect of convection term that U on the solution field is getting predominant and that is why with the increasing Peclet number the profile of the scalar is getting varied ok.

So, now we will start with the other portion is the solution of our convection diffusion system. So, once you do our integration so, delta dot rho v phi minus delta dot gamma del phi d V equals to 0 that is the equation and one-dimensional case V is ui that is the velocity vector. So, the conservation equation one can write from here that v c delta dot J

C plus J D dV equals to 0, where J C stands rho V phi and J D stands for minus gamma del phi. So, this is convection flux this is diffusion flux.

So, once you transform this one using the divergence theorem. So, this guy will become now essentially surface integral J C plus J D dot ds which is 0. Now, one can write that in rho u phi i minus gamma d phi by dx i dot ds equals to 0.

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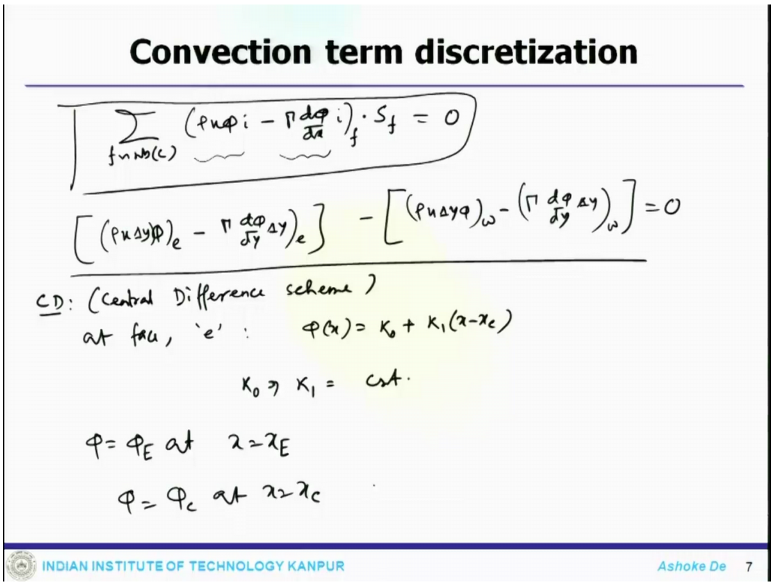

Now, the surface integral if you put over the fluxes over the faces this will become f nb c rho u phi i minus gamma d phi by dx i over face dot S f equals to 0. So, that is a very standard discritize previously when we did the diffusion term only we had this term. Now, you got the convection term in addition to the diffusion term.

So, if you look at the surface vectors and then put it back for this 1D element that we have shown this will become rho u delta y and so, this will become the rho u delta y phi e minus gamma d phi by dy delta y e this is and then rho u delta y phi w minus gamma d phi by dy delta y w which is 0. So, it is in the 1D case we get this kind of system or the now, we will see how we get the discretized equation.

So, we will start with them simplest one is the central difference scheme or CD which is called Central Difference scheme. So, we will start with that. So, obvious answer would be to assume some sort of a linear interpolation like what we did in the diffusion case.

So, in that case at face let us say e the phi x would be some K 0 plus K 1 x minus x e that is what one can assume ok.

Now, what are K 0 and K c. K 0, K 1 these are constants and can be obtained at face e. So, now, what one can use actually this face you can see this elements and using this face what you can do that phi equals to phi E at x equals to x E and phi equals to phi C at x equals to x C.

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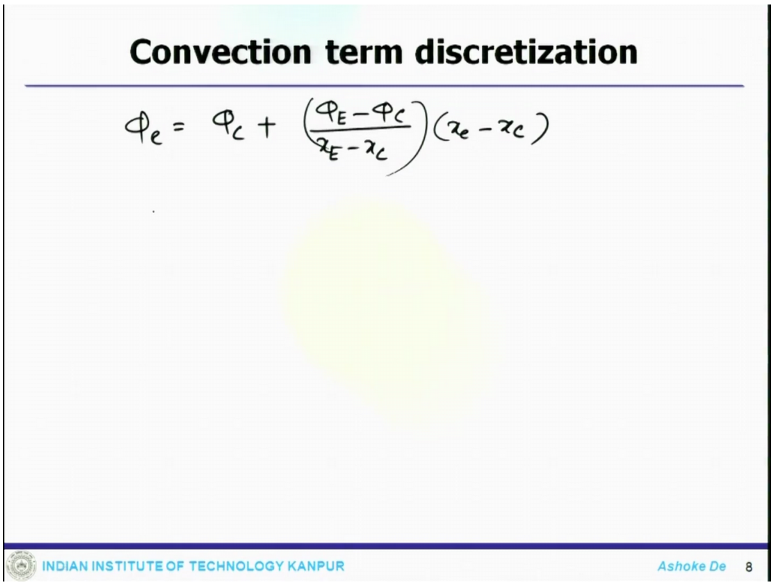

So, one can evaluate the phi e equals phi C plus phi E minus phi C. So, that is a linear interpolation x small e minus x capital C. So, this essentially one can use that Taylor series and find out that.

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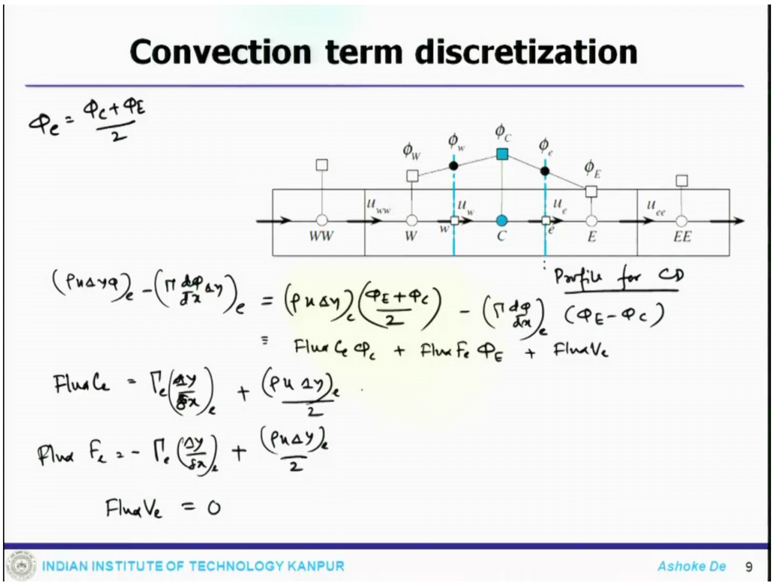

So, if you look at the stencil here which is shown so, is typically the profile for CD scheme, then my phi e at the face here. So, at this face it would be the arithmetic mean of phi C plus phi E by 2.

So, this is going to be now once you use this and use in the discretized equation or discretized equation will become like rho u delta y phi at e minus gamma d phi by dx delta y at e equals to your rho u delta y e plus phi E plus phi C by 2 then minus gamma d phi by dx at east multiplied with phi E minus phi C. So, which is essentially equivalent to flux C e phi C plus flux F e phi E plus flux V e and what are those components flux C e is nothing, but gamma e delta y by delta del e plus rho u delta y e by 2 flux F e minus gamma e del y by del x e plus rho u delta y e by 2 and flux V e is 0. So, the different coefficients you can obtain.

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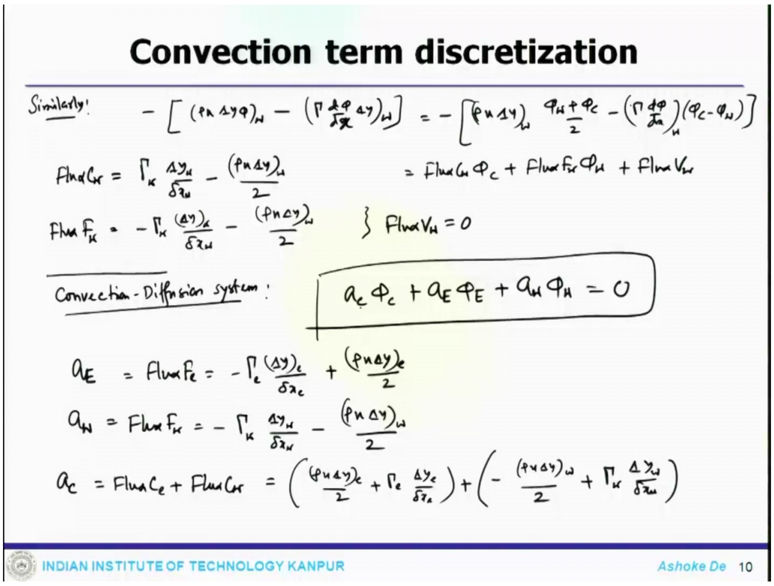

Now, similarly one can find out for the west face and the west face it was rho u delta y phi W minus gamma d phi by dx delta y W which one can write rho u delta y W phi W plus phi C by 2 minus gamma d phi by dx W phi C minus phi W which will be again flux C W phi C flux F w phi W flux V w and the term which will be there flux C W is gamma W delta y W by del x W minus rho u delta y by 2. Similarly, flux F W is minus gamma W delta y W del x W rho u delta y W by 2 and flux V W is 0 that is what it does.

So, what would be the solution? Now, once you put this things the equation of the discretized equation for the convection diffusion system. So, the equation of convection diffusion system would lead to a C phi C plus a E phi E plus a W phi W equals to 0. So, this is my discretized equation where you get a E equals to flux F e which is minus gamma e delta y e by del x e plus rho u delta y e divided by 2 a W equals to flux F W gamma W delta y W by del x W minus rho u delta y W by 2 and a C equals to flux C e plus flux C W which would be rho u delta y e by 2 plus gamma e delta ye del x e plus minus rho u delta y W by 2 plus gamma W del y W by del x W.

So, these are the coefficients for the discretized system.

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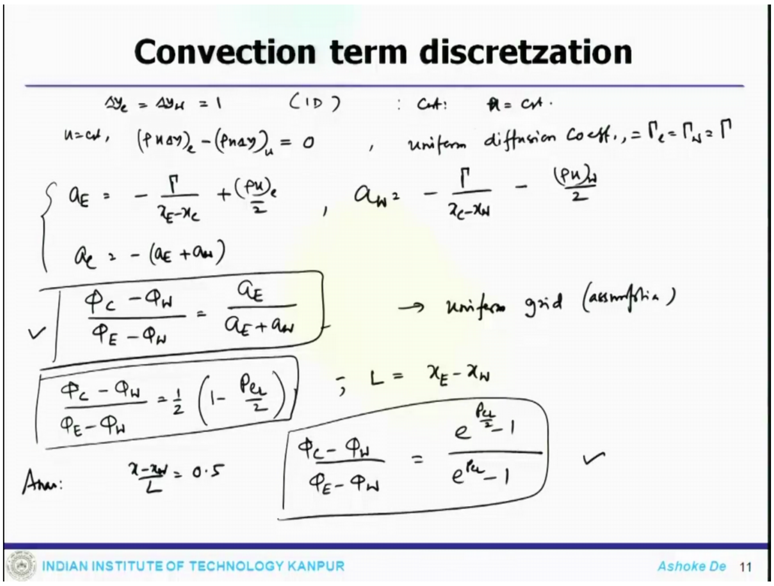

Now, once you get that another important condition because it is an uniform 1D stencil. So, you had delta y e equals to delta y W. So, now, without loss of generality you can say this is 1, because it is a one-dimensional system and also the continuity system provides, now the continuity provides u is also constant. Thus what happens that seems u constant rho u delta y at east face minus delta y at west face, this would be 0 and now, also what we can assume that uniform diffusion coefficient which will lead to gamma e equals to gamma W one can say gamma.

Then one can write that coefficients a E would be minus gamma x E minus x C plus rho u e by 2 a W equals to minus gamma x e minus x W minus rho u W by 2 a C equals to minus a E plus a W. Now, you put this coefficients back in your discretized system then you get the phi C minus phi W by phi E minus phi W equals to a E by a E plus a W. So, this is what we get.

Now, since we have assume if you assume the uniform grid. So, that is another assumption. So, the solution can be written in terms of Peclet number and like phi C minus phi W divided by phi E minus phi W equals to half of 1 by P e L by 2 and where L stands for x E minus x W. So, it is essentially the distances.

Now, the analytical solution of this particular problem was also obtained. The analytical solution one can obtained by x minus x W by L by setting 0.5 if they belong to the middle of the system, then the analytical solution was phi C minus phi W by phi E minus phi W is equal to e to the power P e L by 2 minus 1 e to the power P e L minus 1.

So, we will stop here today and we will take from here in the follow-up lectures.

Thank you.