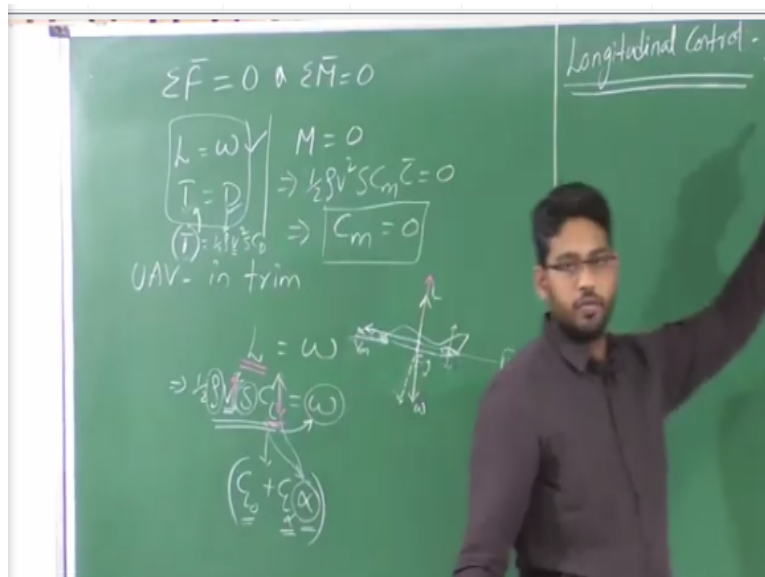


UAV Design-Part II
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Lecture-18
Trim Requirements of UAV

Hello friends, welcome back. So, we have now completed longitudinal static stability part for both wing alone and wing and tail combination. We have solved few example problems, how to figure out the neutral point for a given configuration. And then we also studied about static margin which is positive for in order to achieve statically stable flight. And then we have also demonstrated the wing and tail combination, right in stable and unstable mode, so flight of wing and tail combination. Now let us proceed till now we are talking about an equilibrium condition, right. So, what is an equilibrium condition?

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So, we have the flight in which of the state about which the resultant forces and moments are 0. So, for longitudinal case what will be an equilibrium condition for longitudinal flight? So, when lift is equals to weight and thrust is equals to drive and the pitching moment is equals to 0 which means half rho V square is C m times C bar has to be 0, this implies the pitching moment coefficient has to be 0 for equilibrium condition.

So, we call when the aircraft satisfies these 3 equations then what we call is the aircraft or the UAV is set to be in trim condition, right. That means what exactly trim means we were maintaining a constant angle of attack, is not it. So, the aircraft is flying at a constant angle of attack where there is no resultant pitching moment acting and then the lift is balanced by weight of the like lift generated by the UAV is balancing the weight of the exactly balancing the weight of the configuration.

And then thrust generated by the engine is satisfying the drag that is required by the system, ok. So, ok as per as the particular trim is concern that is fine, that means we are flying at a given angle of attack. So, what if I want to change that angle of attack, ok, so what I need to do when I have to fly at different angle of attack? So let us closely look at this equation lift is equals to weight which is equals to half rho V square S times $CL = w$, right.

So, when you talking about a level flight this particular parameter is constant at that particular altitude. And the for a given UAV see these parameters are constant more or less constant, ok, weight we consider there is no change in the weight of the UAV due to fuel consumption. So, when I have to fly at a particular velocity I need to develop a particular CL from the wings, right, how I develop that CL ?

CL as a linear function we assumed a linear function of angle of attack, right, CL variation with angle of attack. So, in order to change this CL I need to change the angle of attack because CL_0 is constant, you cannot vary, right. And CL_α you cannot vary it depends upon the profile of the aerofoil, once you fixed it then this is fixed. So, the only variable that I have in my hand is alpha, right, so if I have to change the velocity I need to change this angle of attack of flight, right.

For example say this is my reference axis or fuselage reference line, ok FRL coinciding with my body axis, right, say when there is forward velocity V_∞ and say it is a level flight. That means this V_∞ is horizontal, right. So this particular angle with respect to this V_∞ is angle of attack alpha and I have lift perpendicular to V_∞ and weight acting. So, this axis is

perpendicular to fuselage reference line or yeah and weight is acting perpendicular to the local origin, so w is the weight.

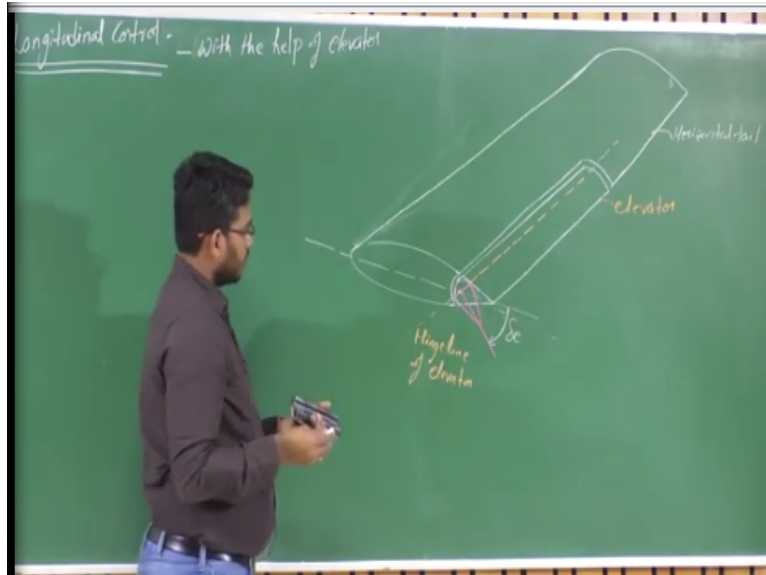
So, now when lift is balanced with the weight you are flying at a particular velocity, right. So, if I have to increase the velocity let us say, if I want to fly faster then I need to decrease the CL value to generate this weight or to generate a force that can balance this weight. That means what I need to do? I need to change this angle of attack, I need to decrease this angle of attack, how can I decrease it?

Or say if I have to decrease the velocity I want to fly slower, then what I need to do is? I need to increase this CL value, so that this equation is balanced, right, still producing. So, the force is still equal produced from this lift, right, or the wings is still equivalent to the weight of the aircraft of the UAV. Now when I have to increase this I need to change this angle of attack again, say I have to increase the angle of attack in this case.

That means I need to hold the aircraft at a particular orientation, see when I say it is flying at a particular angle of attack, which means that the aircraft is oriented with respect to the floor at particular orientation, right. So, now when I have to increase the angle of attack, that means I need to change the orientation of the aircraft and I have to hold that orientation, right. So, how can I do that? by means of a longitudinal control surface, right.

So, we have a control surface which is located on the horizontal tail, alright. Let us say there is a horizontal tail here by deflecting a small surface which is attached to this horizontal tail, I will be able to either produce force upwards or downwards, right. So, with the help of this force that is produced aft of the Cg, right aft the Cg. So, say this is my Cg, so this force multiplied this by this momenta will produce a corresponding moment that helps me to change this angle of attack, right. So, that means for each and every angle of attack I need to produce different force at the tail, right.

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So, let us see what is that what we are going to talk about is longitudinal control, ok. So, how we achieve longitudinal control with the help of elevator? So, let us get back to these equations again, so when I have to increase the velocity what does it mean? I need to produce the thrust which should be greater than the drag at that particular velocity, is not it or not. So, say the engine is producing thrust D which is equals to drag that is nothing but $\frac{1}{2} \rho V^2 S C D$.

So, this is equals to $\frac{1}{2} \rho V^2 S C D$, right. So, initially say I am flying at 30 meters per second, now I want to change it to 40 meters per second. So, the initial engine setting or the throttle setting of my engine is producing only the force that is required to fly this at 30 meters per second, right. Now, suddenly the force from the engine has increased, so that the engine output is higher compared to what it was initially, ok.

So, now with a higher throttle setting will be definitely more than the initial thrust. So, I have an excess force that will accelerate my aircraft wing new velocity. So, if I do not change my CL, if I still fly at a higher velocity, right. So, then I am producing additional force even in the perpendicular direction of flight, right, here in this direction. So, that will take the aircraft away from this level flight condition, the aircraft will not remain in the equilibrium and then we cannot claim that the aircraft is in trim, right.

So, if we have to maintain the trim condition or say level flight condition, then what we need to do is? we need to decrease this CL, ok. So, immediately what is happening with the increase in velocity there is a change in CL, there should be a change in CL. So, that we need to incorporate, so how we are changing this CL? by changing the alpha, how this alpha is change with the help of elevator? right.

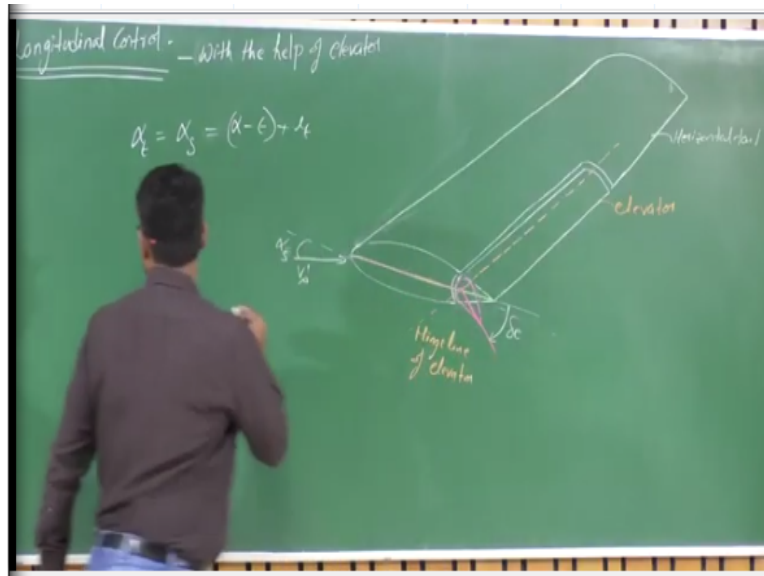
Let us see what exactly elevator is, so elevator is a small control surface which is attached to the horizontal tail, right, for longitudinal control, so which we have discussed earlier. So, let us say as we discuss the horizontal tail is fabricated or made out of symmetric aerofoil, right, ok. So, let us say this is my fuselage reference line, let us say this is my chord line of this horizontal tail, ok.

Now there is a small control surface and it is a part of the symmetric tail, right. So, this particular portion, which is a part of this tail can be deflected about a hinge point, right. So, this particular portion can be deflected about this point up and down, right. So, this portion is known as elevator, say, ok, so this is my horizontal tail and this particular portion which is a part of this horizontal tail is called a elevator, right.

So, this particular and there is an axis about which this elevator can rotate, right. So, let us say if I hinge my elevator about this particular axis, so this axis is coming out of this board, so this elevator can rotate up and down about this particular axis. So, this is known as hinge line of control surface in this case it is hinge line of elevator control surface. So, this particular portion is elevator, ok.

So, this is horizontal tail and as we discussed earlier thumb along the positive axis of this hinge line in the corresponding curl of your fingers will give you the positive rotation of the elevator, ok. So, now positive rotation of elevator is when the trailing edge of the elevator deflects downward. So, this particular angle is δe elevator deflection, ok. So, this is positive when you deflect the elevator up that is negative deflection. So, a positive deflection, that means what is happening with a elevator deflection?

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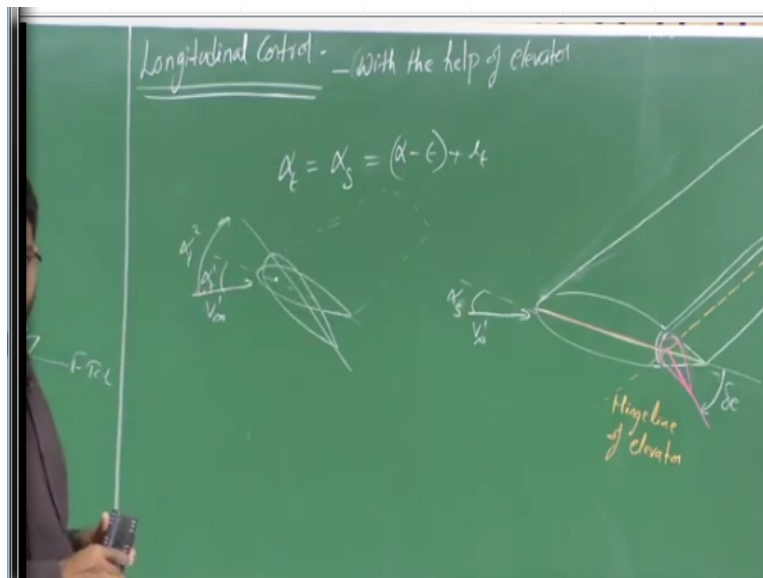
So, as we mentioned this is my chord line, right, so let me draw it a bit clearer. So, say this is my chord line which in the earlier case is the line joining leading edge and the trailing edge, right. And which is nothing but the mean as well, is not it, for a symmetric aerofoil this is nothing but the mean camber line. Now when there is a deflection, what is happening? this chord line is up to this portion is fine is same but there is a change in the mean camber line at here.

So, the mean camber is changing here, so when there is a change in mean camber line that means there is change in or the CL alpha of this aerofoil gets affected, am I correct or not? So, because of the change in the mean camber line there is a upward force the CL alpha is either increasing or decreasing. Let us say initially the aircraft is trim or flying at a particular V_{∞} prime, this is horizontal tail, right.

So, the tail is seeing certain alpha t initially, fine. So, which includes both i of t as well, right. So, let us assume that as stabilizer angle of attack S_t , ok. So, horizontal stabilizer and the corresponding angle seen by this horizontal stabilizer. So, when there is no deflection, when there is zero deflection, the total angle of attack seen by this tail is equals to stabilizer angle of attack just stabilizer angle attack which is equivalent to alpha - epsilon + i of t, this we discussed earlier, is not it.

But when there is a deflection with the elevator what happens is? So, **so** in order to understand this properly, let us assume that the entire tail horizontal tail itself is a elevator, ok. So, that means, if I deflect this about a hinge line, right. So, a elevated deflection will change the corresponding angle of attack, am I correct or not.

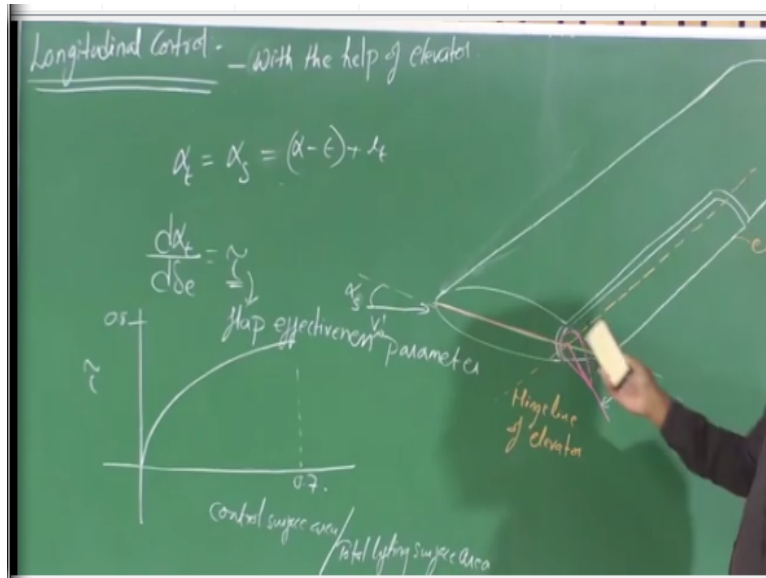
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Let us assume a case where, so say this is my horizontal tail and the entire horizontal tail is my elevator, ok. So, this is my horizontal tail and the entire horizontal tail is elevator I am rotating about certain hinge line, right. So, now when I rotate this what happens? So the aerofoil still remain same. So, this is like and giving a delta here which is actually changing the, so let us say this is my V_∞' . So, this is my initial alpha at tail 1, right, so this becomes alpha at tail 2, right.

So, the elevated deflection when the elevator is a entire horizontal tail it is actually affecting this angle of attack that means you are actually changing the angle of attack with the elevated deflection. So, there is an effect on the angle of attack due to elevated deflection, right.

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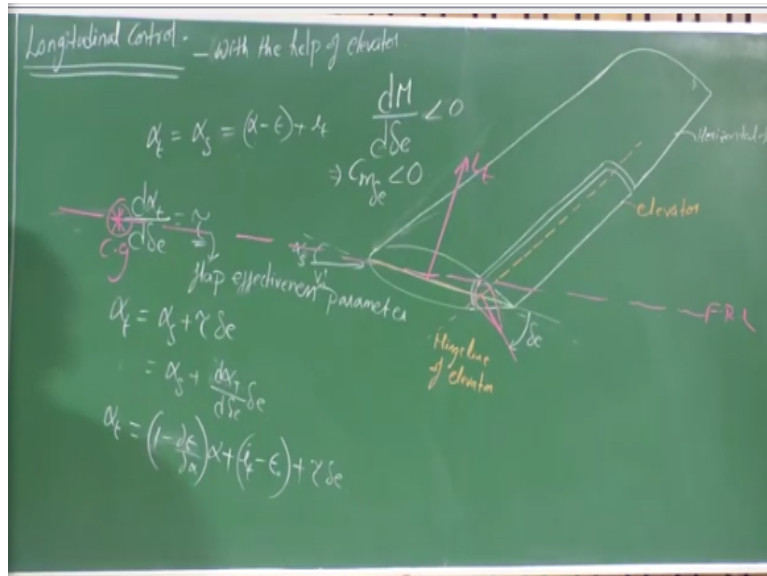
So, that factor is generally given as, so the change in the angle of attack of the tail due to elevated deflection is given as tau, elevator effectiveness parameter, ok. So, even here in this case that may not be explicitly visible, but still certain portion of the wing is being deflected that is changing the local angle of attack, right. So, that the change in the elevator angle of attack at the tail due to elevated deflection is given as tau which is flap effectiveness parameter or control surface effectiveness parameter.

So, this from the historical database you will be able to figure out this parameter tau given the control surface area upon total lifting surface area, ok. So, if the elevator control surface is equals to the total tail area let us say, that means it has to become 1, tau has to become 1. Because change in elevator let us say positive deflection is rotating it downwards, right, a degree of rotating it downwards is nothing but degree increase in the angle of attack just now we have witnessed, right in the previous figure.

So, that is when tau becomes 1, so it is not exactly linear variation, right, so up to 0.8 when this is at 0.7, ok. So, we will give you that data, how elevator control surface area if it changes you know with respect to the total if this ratio changes how this tau changes. So, it is fixed for a given aircraft, is not it, because you have a fixed control surface area and the fixed tail lifting area, right.

So, the total lifting area is nothing but the elevator area here or say I am sorry, the total lifting area is the horizontal tail area here.

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So, now when there is an elevated deflection, what happens is? The lifting the tail angle of attack will now be a function of this parameter tau times delta e, ok. So, this tau is nothing but, so alpha S + dou alpha of tail d delta e times delta e, fine. Where alpha of S is still 1 - dou epsilon upon dou alpha times alpha + i of t - epsilon 0 + tau times delta e, ok. This is the angle of attack at the tail when there is an elevator deflection, when delta e is zero this becomes alpha S, this particular equation will be equal to alpha S.

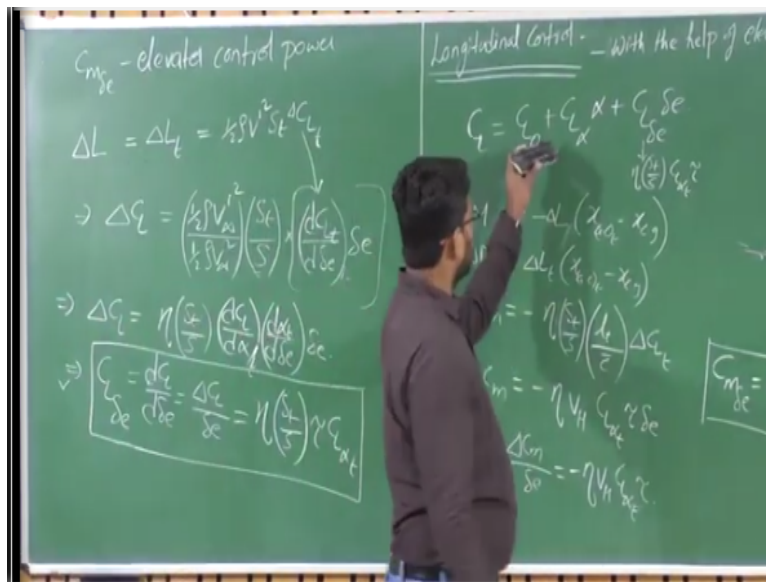
Now, again coming back to this, this figure when there is delta e deflection there is change in camber, that means the change in lift, am I correct or not? So, in a layman language what we can see is, it is trying to obstruct the flow here. So, the flow will exert a force in the opposite direction. So, now a positive elevated deflection when the elevator is deflected downwards there is an upward force at the tail, right.

So, this upward force is at the tail lifted the tail. So, say if I extend this if I consider this as my fuselage reference line FRL, right. And say the Cg of the aircraft is somewhere here, this is my Cg. Now the lift produced by this tail, right, so what effectively happening is a change in the

pressure distribution because of the change in the control surface deflection. So, the change in the pressure distribution is creating a change in the force at the tail, right.

So, that creates a moment about Cg. So, when there is a positive deflection, there is an upward force that creates a negative moment. So, the change in the (C_m) (20:56) or say the change in the pitching moment due to change in elevator control surface is less than 0, right, negative or in other words the $C_m \Delta e$ is less than 0 here, ok. So, we will see what is the $C_m \Delta e$ here, we will discuss about that in detail here.

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So, this $C_m \Delta e$ is known as elevator control power, so elevator control power, how good you can control your aircraft using the elevator that is there depends upon this parameter $C_m \Delta e$. So, higher the value greater is your control, so higher the negative greater is your control, $C_m \Delta e$ is negative for a yeah is negative here, right, $C_m \Delta e$ is less than 0. Now when there is an elevator deflection, there is a change in total lift of the aircraft, is not it?

So, ΔL represents the total change in the lift of the aircraft which is due to change in the lift at the tail, right, am I correct or not? So, this equals to half rho V prime square S of t tail lift is due to wing tail lifting area, is not it? S of t times ΔC_L change in lift coefficient at the tail, ΔC_L at tail. So, this equals to ΔC_L of the total aircraft is half rho V prime square V infinity prime square S t divided by half rho V infinity square upon S is the wing reference area.

Because aircraft reference area is wing reference area and then what you have is change in lift coefficient at the tail. So, this change in lift coefficient at the tail is due to elevated deflection, right, am I correct or not? So, that is nothing but dC_L upon $d\delta_e$ at tail, right. So, ΔC_L of the total aircraft is equals to η times S_t upon S η of horizontal tail, ok, efficiency of horizontal tail times C_L change in the C_L at tail due to δ_e deflection, right.

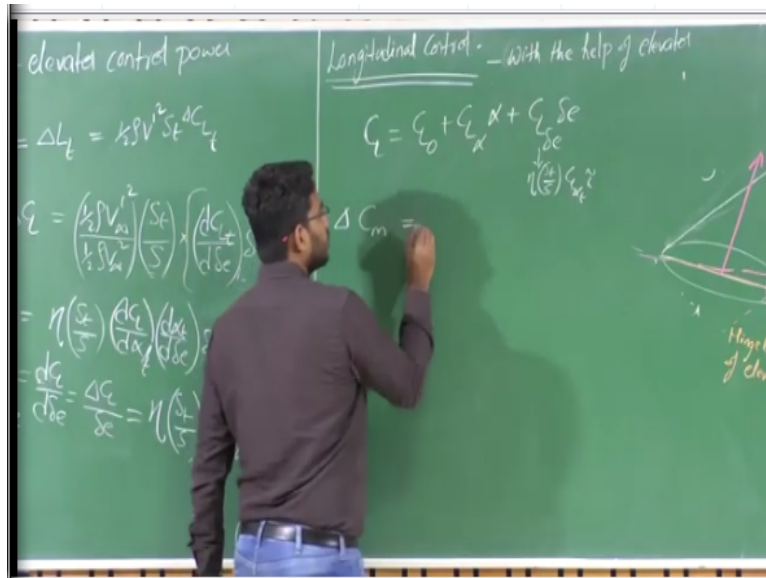
So, what we observed when there is change in δ_e , there is change in camber which in turn affects the C_L α , am I correct or not, when there is a change in camber there is a change in lift curve slope, am I correct or not. So, this can be modeled as dC_L upon $d\alpha$ of tail times the $d\alpha$ times $d\delta_e$, right. So, this is times δ_e here I am sorry please make a correction there should be a δ_e here.

So, this talks about change in lift coefficient for δ_e deflection, ok, fine. So, this is nothing but C_L α of tail here. So, now ΔC_L δ_e of the total aircraft is equals to dC_L upon $d\delta_e$ which is ΔC_L upon $\Delta\alpha$ is equals to or $\Delta\alpha$ is nothing but δ_e here. So, is equals to ηS_t upon S times we know $d\alpha$ upon $d\delta_e$ $d\alpha$ of tail I am sorry $d\alpha$ of tail upon $d\delta_e$ is nothing but tail effectiveness parameter τ which we have discussed earlier.

So, τ times C_L α of tail, ok. So, if you have the details about C_L α of tail and τ which means which can be deduced if you have the geometric details about control surface area of the control surface and the area of the horizontal tail taking the ratio of that, you will be able to figure out what is τ here. And then yeah of course, once you have the details you know what is S_t upon S .

And knowing η you will be able to figure out what is ΔC_L δ_e of this aircraft, right, ok, so this is the ΔC_L δ_e of the entire aircraft.

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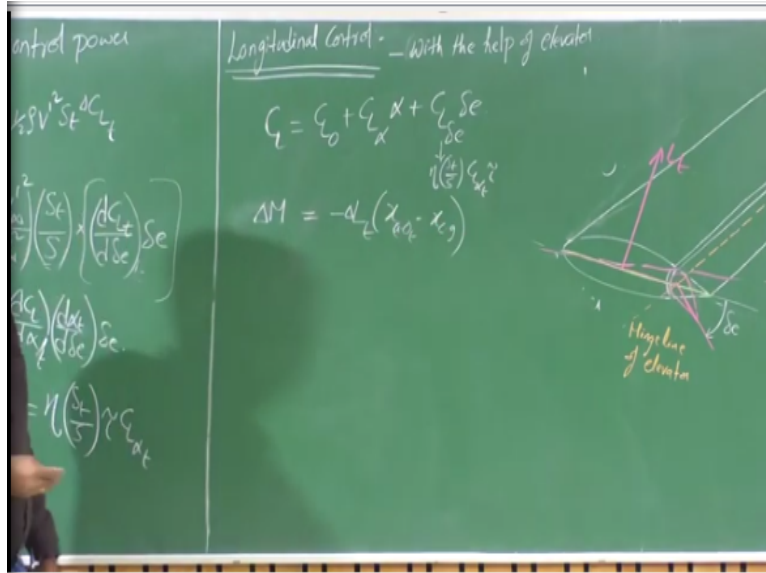


So, ΔC_L , now that total C_L of the aircraft, right, lift coefficient of the aircraft is due to C_{L0} lift independent like lift coefficient at 0 angle of attack and its variation with angle of attack $C_L \alpha$ into α and then ΔC_L into Δe . So, this particular ΔC_L is what we have derived here which is $\eta \frac{S_t}{S} \tau C_{L\alpha}$ of tail times tail effectiveness parameter τ , ok, fine.

So, initially it was just $C_{L0} + C_L \alpha$ when there is no Δe but there is change this C_L here ΔC_L is due to ΔC_L times Δe here where ΔC_L is nothing but $\eta \frac{S_t}{S} \tau C_{L\alpha}$ of tail, ok. Similarly there is a pitching moment, when there is change in the lift at the tail there is a change in pitching moment, the change in pitching moment apart from C_m of wing and tail combination, so when it is in trim we know C_m is 0, right.

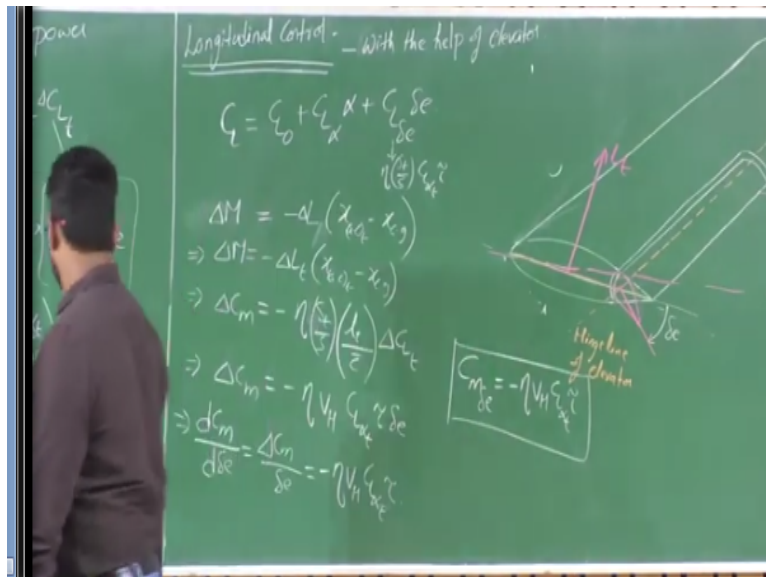
So, apart from that, so during the case the contribution is only from C_{m0} and $C_{m\alpha}$, when there is a Δe deflection, there is a change in C_m total pitching moment which is due to the lift produced at the tail, right.

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So, the change in the pitching moment ΔM is due to the lift produced at the tail, right, which contributes towards negative moment let us assume, if there is a positive lift there is a negative moment. So, let us assume L of t is positive by convention, so this produces a negative moment and the negative moment is because of lifted tail or change in the lift at the tail, right, times the distance between C_g and the aerodynamic center of the tail which is X_{Cg} or X_{ac} of tail - x_{Cg} , right, that is a momentum, we know it.

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So, or change in the lift of the aircraft which is acting at the tail part, the changes near the tail, right. So, our aerodynamic center of the tail multiplied by X_{ac} of tail - X_{Cg} , right. So, this equals to minus lift at the tail times x_{ac} of tail - X_{Cg} ΔM . So, this implies ΔC_m , a

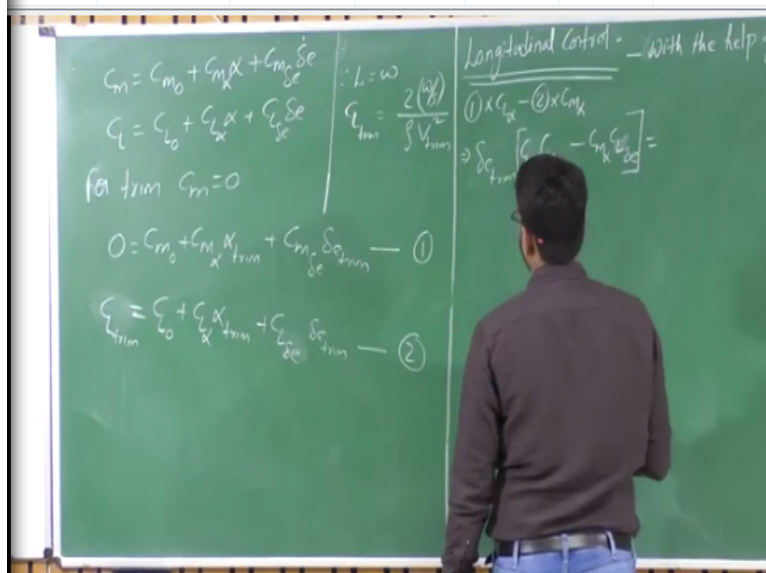
change in the pitching movement of the total aircraft because of the elevator deflection is due to the change in the lift at the tail because of the elevator deflection which is equal to ηS_t upon S times.

So, this particular parameter is called l_t length of tail upon c bar times change in the lift coefficient at the tail, ok. So, we know that change in the lift coefficient at the tail, this change in the pitching moment coefficient is equal to $-\eta V_H$, correct, V_H is the tail volume ratio, this particular product is the tail volume ratio. Area of the tail times length of tail upon area of the wing times mean aerodynamic chord of wing.

So, this particular product is a tail volume ratio V_H and ΔC_L at the tail is due to change in lift coefficient at the tail, right. This is ΔC_L at the tail, this particular parameter, so you can simply substitute it there, right. So, this is nothing but $C_L \alpha$ of tail times η , ok, so times $\Delta \epsilon$ here η times $\Delta \epsilon$. So, the total pitching moment or change in pitching moment coefficient of the aircraft due to $\Delta \epsilon$ deflection equals to $-\eta V_H C_L \alpha$ of tail times tail effectiveness parameter, ok.

So, $C_m \Delta \epsilon$ is equal to $-\eta V_H C_L \alpha$ of tail times tail effectiveness parameters. So, this is one equation and this is the other equation that talks about $C_L \Delta \epsilon$ here $C_m \Delta \epsilon$ here ok. Similar to this the total pitching moment of the aircraft is now can be expressed as $C_{m0} + C_m \alpha$ into α and $C_m \Delta \epsilon$ times $\Delta \epsilon$.

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So, when you just have wing and tail combination, right, what you have is $C_{m_0} + C_{m_\alpha}$ into alpha and there is no elevator deflection. But when there is an elevator deflection, we have additional terms $C_{m_{\delta_e}}$ that elevator control power comes into the picture times delta e. And what is $C_{m_{\delta_e}}$? $C_{m_{\delta_e}}$ is equal to, so earlier we discussed $C_{m_{\delta_e}}$ has to be negative is not it, whether it is true or not, eta is positive we know tail volume ratio is positive C_L of tail is positive and tau we witness it is positive.

So, this entire positive term multiplied by minus what we have is $C_{m_{\delta_e}}$ as negative, ok. Now you have C_{m_0} C_{m_α} $C_{m_{\delta_e}}$ and similarly for trim conditions before talking about trim we have $C_{L_0} + C_{L_\alpha}$ into alpha and $C_{L_{\delta_e}}$ into delta e in the linear regime, right, this is what we have modeled, right. So, we have come up with this aerodynamic model for the linear regime which is a function of alpha as well as delta e for both the equations.

Now we are talking about how to change angle of attack from one trim condition to the other trim condition. Or let us say if I want to trim aircraft at certain angle of attack, what should be the corresponding elevator deflection that I require? Or trim the elevator deflection to trim the aircraft at that particular alpha trim, ok. That means we witnessed for trim C_m is 0, right.

So, if you substitute that there what I have is $C_{m_0} + C_{m_\alpha}$ times alpha trim + $C_{m_{\delta_e}}$ times delta e trim, ok. Similarly, C_L is equal to this C_L becomes $C_{L_{trim}}$ is equal to, how can

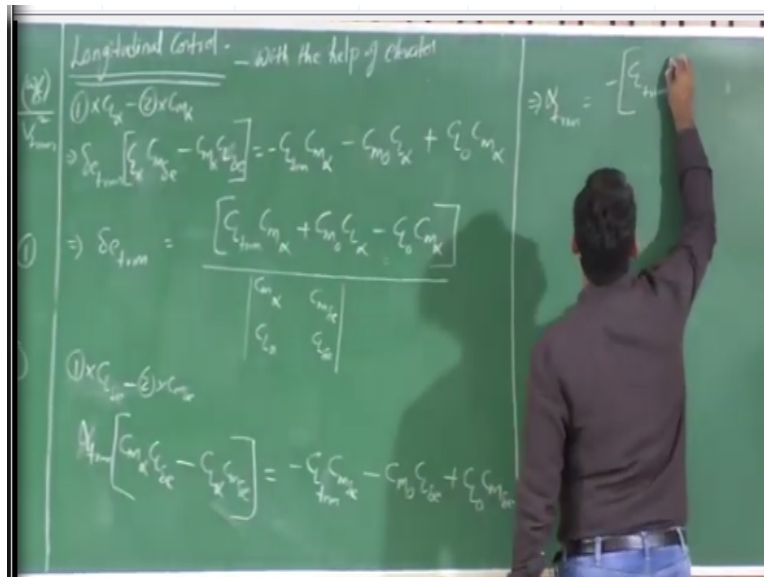
we estimate CL trim? From the level flight equation $L = W$ from $L = W$ when you want to fly at a particular velocity then you will be able to find out what is it the corresponding CL trim.

If you know that CL trim you will be able to find out what will be the resulting alpha trim and delta e trim to fly at that particular velocity, right, that satisfies the level flight condition, ok. So, this is equals to $C_L 0 + C_L \alpha \text{ times } \alpha \text{ trim} + C_m \delta e \text{ times } \delta e \text{ trim}$, ok, say this is my equation 1 and this is my equation 2 ok. So, I have 2 equations, see C_m is 0 here and we know CL trim, so how can we find CL trim?

So, from lift is equals to weight, ok, so CL trim is equals to twice the wing loading upon rho times V infinity square or V trim square velocity for that particular trim condition V trim square. So, given the data about this and the flight velocity that you want to fly, you can find out what is the corresponding trim CL lift coefficient that you require. So, for this lift coefficient, what should be the alpha trim and delta e trim?

So, you have 2 equations that mean left side is known here, right, and you have a 2 unknowns here in these 2 equations, you can simply solve it, is not it, you can solve for what is alpha trim and delta e trim, can you do that.

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So, say 1 multiplied by, so if I want to find out delta e trim in the first place, 1 multiplied by CL alpha - 2 equation 2 multiplied by C m alpha, right. So, this implies c delta e trim, so what I am doing? I am multiplying this equation 1 with CL alpha, right and then equation 2 with C m alpha and subtracting 2 from 1, ok. So that this alpha trim terms gets cancelled out, I have this equation with one variable and I can rearrange the terms, so that I can get to know what is delta e trim.

So, this delta trim is equals to C m delta e times CL alpha or say CL alpha times CL delta e C m delta e - C m alpha times C m delta e, this is a mistake, this is CL delta e, ok. So, please correct this equation, so this is CL delta e that is the reason why I am. So, this is C m alpha times CL delta e this equals to. So, minus of CL trim time C m alpha CL 0 times C m alpha, ok.

So, delta e trim is equals to minus of CL trim C m alpha + C m 0 CL alpha - CL 0 C m alpha upon like we can see C m delta e, right. So, C m this is like C m alpha C m delta e CL alpha CL delta e, you can take the discriminant of it. So, in that case this minus will not be there, ok, fine. So, similarly what is this C m alpha CL e - CL alpha C m delta e. Similarly alpha trim in order to find out alpha trim equation 1 multiplied by CL delta e - equation 2 multiplied by C m delta e.

So, if I do that what I have is? So, alpha trim is C m alpha times CL delta e - CL alpha times C m delta e = - CL 0 times C m delta e - - C m 0 times CL delta e + CL 0 times C m delta e ok.

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Handwritten derivation on a green chalkboard:

Control - With the help of charnel

$$-C_{m\alpha} \epsilon_{trim} = -C_{trim} C_{m\alpha} - C_{m0} C_{\alpha} + C_{l0} C_{m\alpha}$$

$$\frac{[C_{trim} C_{m\alpha} + C_{m0} C_{\alpha} - C_{l0} C_{m\alpha}]}{\begin{vmatrix} C_{m\alpha} & C_{m\alpha} \\ C_{\alpha} & C_{\alpha} \end{vmatrix}}$$

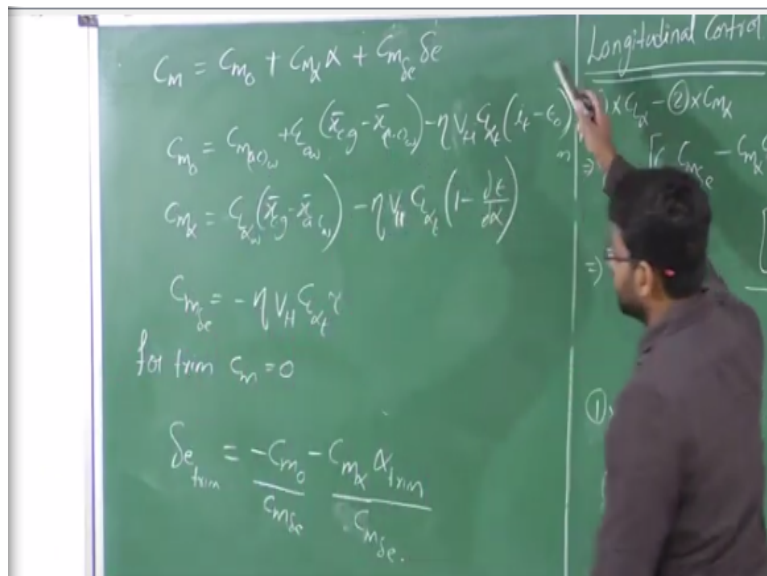
$$\Rightarrow \epsilon_{trim} = \frac{[C_{trim} C_{m\alpha} + C_{m0} C_{\alpha} - C_{l0} C_{m\alpha}]}{\begin{vmatrix} C_{m\alpha} & C_{m\alpha} \\ C_{\alpha} & C_{\alpha} \end{vmatrix}}$$

$$-C_{m\alpha} \epsilon_{trim} = -C_{trim} C_{m\alpha} - C_{m0} C_{\alpha} + C_{l0} C_{m\alpha}$$

This equals to alpha trim is equals to minus of CL trim this is CL trim not CL 0 this is CL trim, right, CL trim multiplied by C m delta e here. So, CL trim multiplied by C m delta e, so plus C m 0 CL delta e - CL 0 C m delta e upon you can find the determinant C m alpha C m delta e CL alpha CL delta e, ok. So, with these two equations you can find out what is delta e trim and alpha trim.

So, we now got delta e trim or say what should be the alpha trim and delta e trim if you want to fly at this particular CL trim condition, right which corresponds to the velocity V infinity for a given UAV, right. So, when you want to change from one velocity to the other velocity, you need to change the CL trim, the CL trim changes which can be achieved by elevator control, right, how the CL can be changed? by change in the alpha trim which is achieved from elevated trim, right. So, let us look at what we have done till now, so this particular equation.

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The C m equation which is C m 0 + C m alpha into alpha + C m delta e times delta e, right. So, what should be the control surface deflection to trim this particular aircraft delta e trim? So, in this the C m 0 again see we have wing and we have a tail combination and in the tail we have a elevator right now, ok. So, this C m 0 will still remains the same like C m 0 is equals to C m ac of wing + CL 0 of wing times X bar Cg - X bar ac of wing - eta S t upon S.

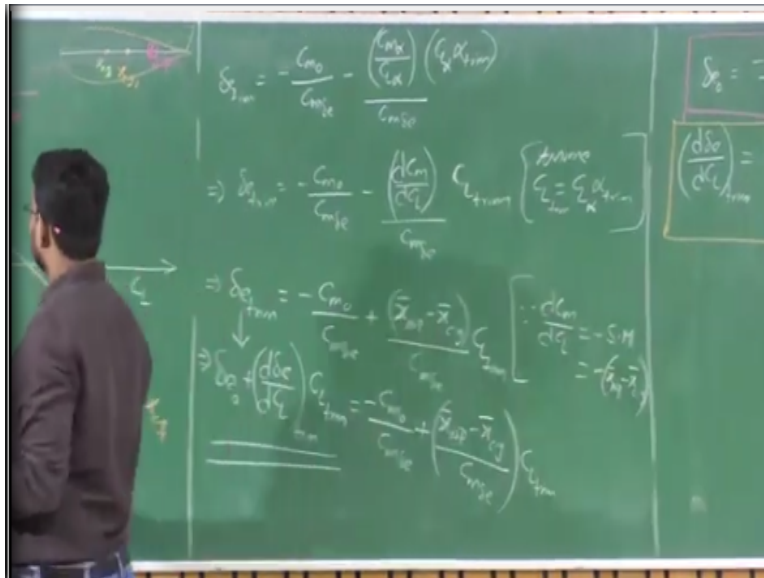
Or you can say V_H tail volume ratio V_H times C_L alpha of tail times i of $t - \epsilon_0$, ok. So, this still remains the same and the C_m alpha that we are using in this equation is C_L alpha of wing times $X_{Cg} - X_{ac}$ bar of wing - ηV_H tail volume ratio which is S_t upon S times X_{ac} bar of tail - X_{Cg} upon, right, yeah X_{Cg} that particular product is a tail volume ratio $S_t L$ T upon $S c$ bar ok.

So, tail volume ratio multiplied by C_L alpha of tail multiplied by $1 - \epsilon_0$ upon ϵ_0 change in the downwards you to angle of attack change in angle of attack, ok. So, these two C_m 0, C_m alpha still remain same and $C_m \delta \epsilon$ we derive, right, we just derived the for solving for this alpha trim and $\delta \epsilon$ trim which is - η tail volume ratio times C_L alpha of tail times ϵ_0 , ok.

So, $C_m \delta \epsilon$ is negative for a statically stable aircraft $C_m 0$ is positive and C_m alpha is negative, alright. So, for now for trim condition, so we will now see what will be the change in that trim angle of attack with the change in the trim lift coefficient. So, when I have to fly at different velocities I need to change my trim lift coefficient, right, that is what we discussed.

So, what will be the change in the trim? elevated deflection or what should be the change in elevated deflection when there is a change in trim lift coefficient, ok? Let us derive that. So, for trim there is a reason to derive it we will soon discuss about that, so for trim C_m has to be 0, so substituting 0 in this equation $\delta \epsilon_{trim} = - C_m 0 -$ or say - C_m alpha times alpha trim upon $C_m \delta \epsilon$ upon $C_m \delta \epsilon$, am I correct? So, just substituting C_m is equals to 0 in this equation and trying to find out what is $\delta \epsilon_{trim}$.

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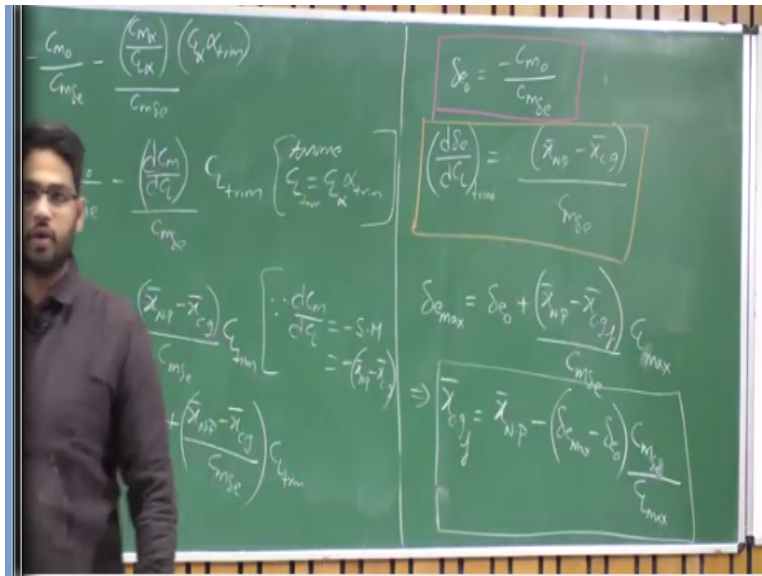


So, delta e trim is equals to minus of C m 0 upon C m delta e, ok, minus of this C m alpha upon CL alpha of the entire aircraft times CL alpha times alpha trim upon C m delta, ok. So, delta e trim = - C m 0 upon C m delta e times d C m upon d CL, right. So, C m alpha upon CL alpha is d C m upon d CL, right, so multiplied by let us assume the total trim angle of attack is dominated by CL alpha times alpha trim, ok.

So, let us have that assumption then what we have is? this is nothing but CL trim upon C m delta e, ok. So, what is d C m by upon d CL? This is nothing but negative of static margin, right, do you remember this? So, delta e trim = - C m 0 upon C m delta e + X bar NP - X bar Cg upon C m delta e times CL trim. Since here assume CL of total aircraft is dominated by CL alpha of the aircraft times alpha trim, ok, assume CL 0 is very small, ok.

Since d C m upon d CL is equals to minus of static margin, ok which is minus of X bar NP - X bar Cg of the aircraft, just refer our previous lectures, you will be able to figure out this. And now so can I express this delta e trim like this in this form? So, delta e 0 + change in delta e due to change in CL trim condition, ok, change in CL trim. When there is change in CL trim there is a change in delta e trim, ok, times CL trim, ok is equals to minus of C m 0 upon C m delta e + X bar NP - X bar Cg upon C m delta e times CL trim, ok. So, by comparing the constants and coefficients of CL trim, what I have is.

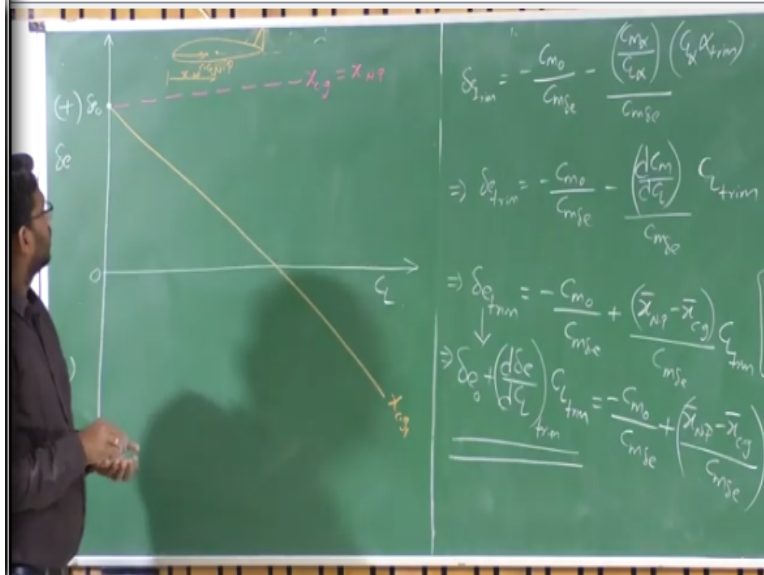
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$\delta e_0 = -C_{m0} / C_{m\delta e}$, ok and $d\delta e / dC_L$ for trim, right is equals to change in δe for change in C_L trim, ok, is equals to minus of $\bar{x}_{NP} - \bar{x}_{CG}$ I am sorry $\bar{x}_{NP} - \bar{x}_{CG}$ upon $C_{m\delta e}$, ok. So, there is lot to learn from this equations, so the elevated deflection δe_0 mainly governs when like is required when your C_g is at the neutral point, am I correct or not?

If your C_g is at the neutral point, so that means you do not have this trim coefficient anymore here, so you need to satisfy this δe_0 for this C_{m0} and $C_{m\delta e}$. So, we know C_{m0} is positive for a statically stable aircraft and $C_{m\delta e}$ is negative, so this particular δe_0 becomes positive, right. So, this is how the variation of C_m with δe .

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Variation of CL and delta e, right, so delta e is positive here and delta e is negative and this is 0, ok. So, what is delta e positive? We just discussed about it deflecting elevated downwards, right, is delta e positive which gives a negative moment, that decreases angle of attack. The decreases your CL it has to be, is not it? That is why if you see this particular equation dou delta e upon dou CL trim.

So, when there is for a positively or for a aircraft or a UAV with positive static stability or static margin, you have $X_{NP} - X_{CG}$ is positive, right. So, $C_m \delta e$ is negative, so this particular slope is negative. So, when there is a positive or downward deflection in elevated deflection, so the trim that results the CL trim that is going to result is negative or decreased or when there is a positive increase in delta e there is an increase in CL trim.

Or say when you have to decrease your CL trim you have to increase that delta e or deflect your elevator downwards, ok. So, that means the slope is negative for this plot, if I have to plot delta e versus CL trim this particular equation this is nothing but is equals to delta e, right, this is delta e trim, right. So, $\delta e_0 + \text{slope} \times \text{CL trim}$, is not it, slope is negative here and delta e is positive.

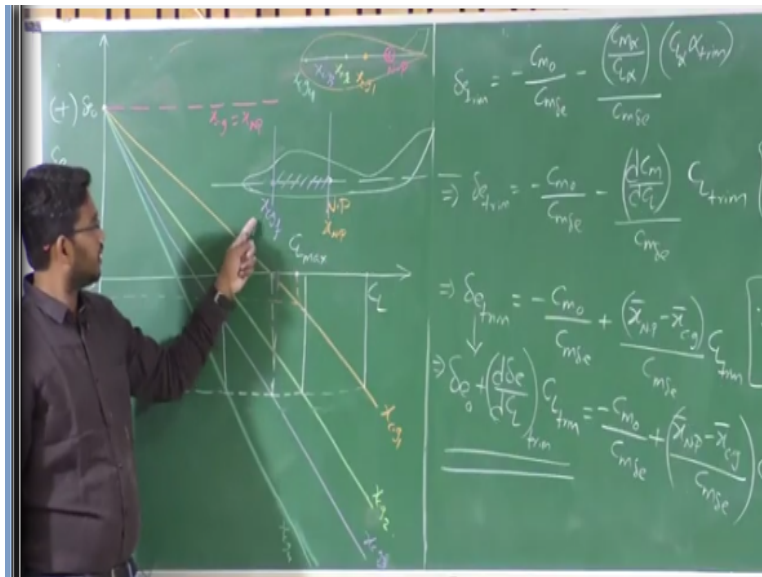
So, that means delta e 0 is positive let us say this is my delta e 0, right, so this is positive. And for a given Cg location for the same UAV for a given Cg location that means the you have

placed the weights and distribute at the weight all other components in such a way that you achieve at this particular the C_g will result in this X_{Cg} , right. For that particular C_g location, so this $X_{NP} - X_{Cg}$ upon $C_m \delta e$ is negative, right.

For that C_g location the slope is negative here for example. So, $\delta e = 0$ corresponds to what? If X_{Cg} is at X_{NP} $X_{Cg} = X_{NP}$ neutrally stable condition, ok. And now when there is when the C_g is ahead of the neutral point, right, what does it mean? If $X_{Cg} - X_{NP} - X_{Cg}$ is positive what does it mean? For example if this is my aircraft, ok, let us say this is my aircraft, ok, so wing alone aircraft say this is my reference line.

So, say this is my NP, right, neutral point, now if it has to be positive that mean this distance X_{NP} this is X_{NP} should be greater than this distance C_g . Say this is my C_g distance, so this should be greater than otherwise the C_g should lie ahead of the X_{NP} , right, close to the leading edge here, let us say. Now, so for this particular location of C_g I have this slope constant, is not it? The slope is constant, ok. Let us say this particular line represents X_{Cg1} position, ok, X_{Cg1} .

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Now consider a second location, so what can I do for that is this corresponds to $X_{Cg} = X_{NP}$, ok. So, say this is my reference line, so this is my NP neutral point, ok, now this is my X_{Cg1} location 1, ok. So, say I have an aircraft or a wing alone aircraft something like this, ok, this is

my X Cg 1. Now let us say I take my Cg a bit more or forward or towards the leading edge here, right, towards the nose of this aircraft.

So, let us say this is my X Cg 2, ok, so that means the distance between this neutral point and Cg location is increased which makes this slope more steeper, is not it? That is more negative here, am I correct or not? So this makes the slope more negative and δe_0 is independent of this location of center of gravity is not it? That is what we figured it out here, ok.

And then let us assume the third case where this is like X Cg 2, ok, now let us assume another case where the Cg is more further up further towards the nose which is X Cg 3 location. So, that means it is more steeper now compare to the previous case. So, what I have is this particular slope, let us say this corresponds to X Cg 3. So, how far I can move this? question is how far I can move this?

Let us consider there is another location X Cg 4, ok, so let that be X Cg 4, ok. So, what do you mean by this? I am changing CL trim here, is not it? The CL trim or what is this CL trim, I am changing the CL value, right, is not it. So, at each and every when there is a change in elevated deflection, that means for each and every CL there is a particular δe value which means there is a particular α trim, am I correct or not?

So, if for the same Cg if I take some other CL value, so there is a particular δe negative here in this particular case. If I have to achieve this particular CL value then I have to give δe positive here, ok. So, this is what this curve talks about it, is not it, am I correct or not? And if you might have noticed that at CL trim or say this is your desired CL let us say this is called design CL where you there is no need of δe trim, right, δe should be 0, you should be able to achieve this CL trim CL design, right.

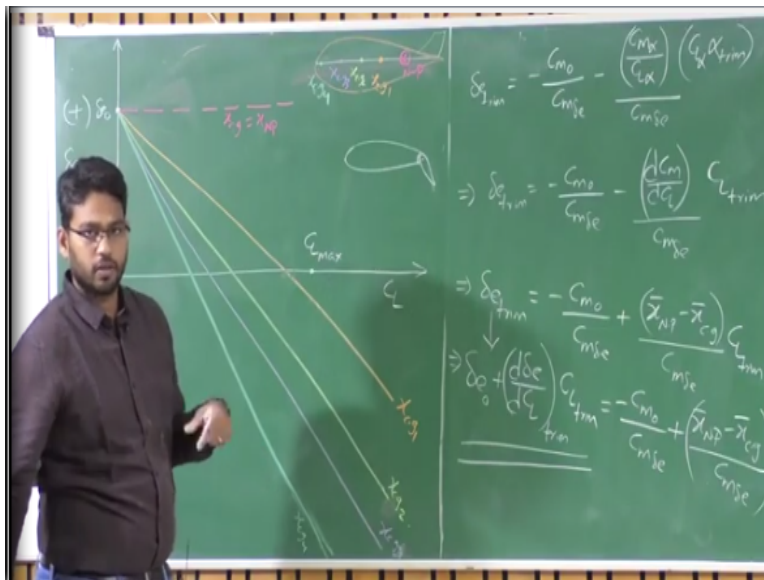
CL design without any control surface deflection, right, so that CL design yeah corresponds to a particular Cg location, so corresponds to a particular curve here, am I correct? So, yeah coming back to this, now let us say there is CL maximum ok, for any aircraft or we know we have CL

maximum for the wing, right. So, how can I achieve CL maximum? It is again at a particular angle of attack, am I correct or not? am I correct?

So, how can I achieve that angle of attack by deflecting the control surface, right, so that means in negative deflection will give me positive angle of attack, what is negative deflection? So, deflecting elevator upwards will produce a downward force that creates a positive moment which increase as the angle of attack, right. So, if I have CL max is at higher angle of attack or say angles of attack higher positive angles of attack.

So, that I, so it is clear that I need a negative deflection here, right. So, there is a maximum negative deflection for the elevator, is not it or not?

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So, let us say if this is my symmetric aerofoil I may not be able to deflect it 90 degrees down. So, apart from the mechanical constraints I am talking about aerodynamic constraints, the flow may separate all together, right. So, there is certain value of this control surface deflection where is the flow is still effective on the control surface, right. So, that particular value is known as delta e maximum, so let us say this is my delta e maximum, right, negative delta e max or maximum upward deflection of the elevator, right.

So, with this δe_{max} I will be able to achieve CL_{max} let us assume that, ok, so this δe deflection up and downward, right, otherwise upward maximum deflection will achieve α_{max} or α_{stall} ok. For with which I can achieve this CL_{max} , ok, now the Cg location as the Cg location varies the slope is changing here, right. So, if I look at this particular curve, right, Cg_4 I may not be able to achieve with maximum δe deflection.

I may not be able to achieve this CL_{max} because CL_{max} is somewhere here, am I correct or not? I am not able to trim my aircraft at CL_{max} I have to satisfy myself with this particular like with the limited regime of CL , I may not be able to use the entire CL versus α that my wing possesses, do you understand? So, if you take it more forward, right, so this is a constraint that you face, you cannot be able to use the CL_{total} available.

So, now say if I take this Cg a bit backward, right from here I shifted my Cg here, so that means I am able to increase yeah still I am not able to achieve this CL_{max} , right. I am able to right yeah increase the available CL regime but I am still not able to achieve this CL_{max} , right. Now say I have taken my Cg further backward, so with this I am able to do this, right, do you get this point or not?

So, I do not need this δe_{max} here, for example with this δe_{max} I can achieve something else here more than CL_{max} . I do not even require δe_{max} if my Cg is within this limit, right. So, for this particular curve orange curve our Cg_1 location, I just need some negative δe to achieve that CL_{max} , some upward deflection that is it, ok.

So, now the constraint on the forward Cg location, the most forward Cg location is from this δe_{max} condition. So, the elevator control surface with or the elevator control deflection with which you still be able to trim your aircraft at CL_{max} . So, the Cg location for which with the maximum elevated deflection upward elevated deflection, you can still be able to trim your aircraft at CL_{max} , right.

So, that is the most forward Cg location, so by substituting that in this particular equation, ok, so what is this $X_{NP} - X_{Cg}$ most forward this becomes CL_{trim} become CL_{max} , alright. So,

δe becomes $\delta e_{\text{maximum}}$, ok, by substituting those parameters in this equation we will be able to find out what is the most forward Cg location, ok, how can I do that? So, I have δe_{max} , δe_{max} here is maximum upward deflection, right, negative deflection is equals to δe_0 , right + $X_{\text{bar NP}} - X_{\text{bar Cg f}}$, Cg f talks about most forward Cg location, ok.

$X_{\text{bar Cg f}}$ upon $C_m \delta e$ times CL_{maximum} , now you have trimming aircraft at maximum CL, ok. So, by rearranging this equation, so what I have is $X_{\text{bar Cg}}$ most forward Cg location is equals to $X_{\text{bar NP}} - \delta e_{\text{max}} \delta e_0$ multiplied by $C_m \delta e$ upon CL_{maximum} . So, you cannot simply place your Cg for ahead you know, if you want to still want a control, good control of your aircraft, you have to limit your Cg forward location, right.

So, what we have is a permissible, now with these details what I can say is so if I have an aircraft something like this, ok. Say if this is my reference line, say this point as my neutral point, ok and this is my $X_{\text{bar Cg}}$ most forward which is governed by this $\delta e_{\text{maximum}}$, right. So, this particular distance is a allowable Cg traverse, you have to design your UAV to lie within this particular distance, right, the Cg of the UAV should be within this limits X.

So, the most aft location is $X_{\text{bar NP}}$, so we are not talking about stick free and stick free condition. Because in the UAV there is no point of stick free, it is always control by a servo motor, right or an actuator. So, this Cg, so this is nothing but the most aft location of a see this is $X_{\text{bar NP}}$ corresponding location, most aft location is the neutral point for static stability.

And most forward location is $X_{\text{bar Cg f}}$ which is governed by this δe_{max} , ok. So, we have now covered good enough concepts in order to talk about performance analysis, right. We will consider a UAV data of a UAV and then we will do performance analysis for various. So, for example if I have to trim that aircraft at different angles of attack, right, within the linear regime let us say.

For different angles of at different angles of attack what should be the corresponding δe_{trim} , right for a α_{trim} what should be δe_{trim} . And then say what should be the velocity of the flight resulting from that and then what should be the thrust required, power required and

also CL by $C d$ and CL power $3 d$ by $C d$ for that particular configuration, how they vary with velocity or say angle of attack?

That is what we will solve using an iterative approach, so I wish you should like one of our TA Kasi made it tutorial on MATLAB. So, you can start getting comfortable with the MATLAB environment. So, we will be solving that problem using MATLAB, right, that iterative problem using MATLAB, thank you.