UAV Design - Part II
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Lecture - 02
Thrust Generation and Power Required

Hello all. Welcome back to this course, Design of Fixed Wing UAV.

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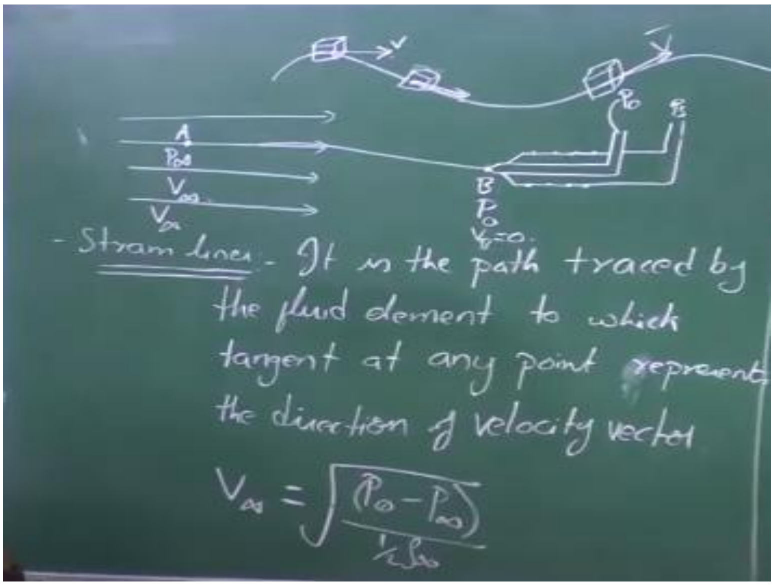

So we consider air flow which is nothing but ordered motion of this air and we have represented the flow by means, this ordered motion by means of the straight lines called streamlines, right. So what are these streamlines? By definition we can say it is the path traced by the fluid element to which tangent at any point represents the direction of velocity vector.

So what does it mean? Let us say if I draw the path traced by the fluid element in this particular fashion. Then say at this particular point consider a fluid element, a small fluid element here. Let its position be here. And then if you draw a tangent to this, so at this particular point, the velocity, the direction of the velocity of this fluid element is horizontal almost, horizontal along this particular line.

So if you look at, at this particular point it is the tangent is more or less in this particular direction to this curve. So this gives you the direction of velocity of this fluid element. That is it is going forward and downward, right. So at this particular position it is going up. Say this is the tangent here. So this is a velocity direction of this velocity vector.

So if we consider a fluid element here, so this gives the tangent here gives the corresponding direction of this particular fluid element here. The streamline itself is the tangent here. So the direction of velocity vector is along this particular lines. So let it be V infinity. What we considered is we so in our previous class to find out velocity is we considered a steady streamline flow.

And say let A be the point in the free stream which is far ahead to this particular sensor, right. So this particular sensor or this particular measuring unit, we call it as pitot tube and there is a static tube around this pitot tube from which we measure the static pressure. Let P s be the static pressure. So this static pressure is measured by means of holes on the periphery here.

We made this tube to interact with the free stream by means of this holes. And this total pressure is measured by means of this pitot tube. Let P naught be the total pressure measured here. So the incoming fluid element which is at point A, this free stream conditions with the pressure P infinity velocity V infinity is brought to rest isentropically. Say this particular fluid element is brought to rest isentropically at point B called the stagnation point.

And the corresponding pressure is P naught and the velocity at this particular point is zero. So from here we already we are able to deduce using Bernoulli equation what we deduced is P in P naught minus P infinity upon half rho infinity where rho infinity is the density. So if you know the differential pressure and the corresponding density we will be able to figure out what is velocity here, right.

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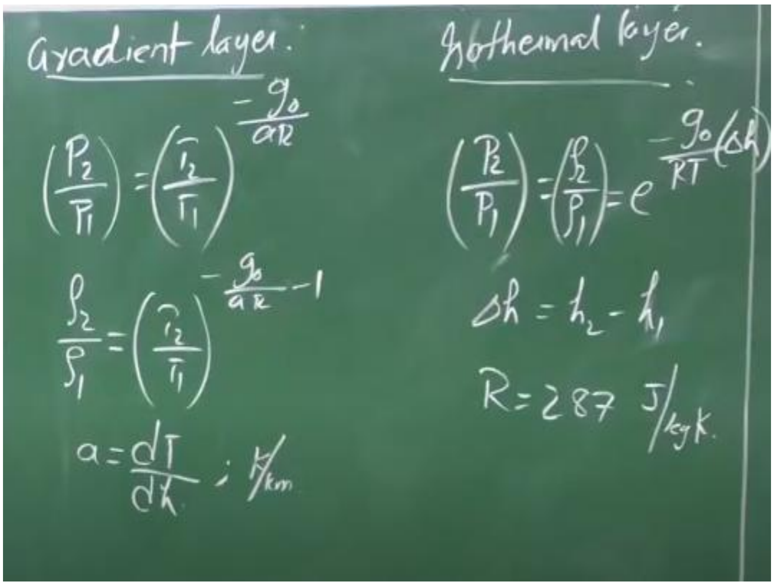

Pressure and density and temperature as a function of altitude from which we have derived this gradient layer equations and isothermal layer equations, where P 2 upon P 1 in a gradient layer is equals to T 2 upon T 1 raised to the power of minus g naught by aR, right. And rho 2 upon rho 1 is equals to T 2 upon T 1 raised to the power of minus g naught upon aR minus 1 where a is called lapse rate dT upon dh which is Kelvin upon kilometer.

So and also we have come up with isothermal layer equations where there are constant temperature zones in which we assume the temperature is almost remained constant in that particular layer. So for those, so for any two points in that particular isothermal layer we can relate using this two equations. e raised to the power of –g naught by RT times delta h with delta h is equals to h 2 minus h 1.

And R is equals to 287 joule per kg Kelvin for air, right. So we have used these two equation and solved one example problem, where we want to find the velocity of the aircraft at 12 kilometers altitude, where we are able to measure the total pressure. Is that what it is given? Right? So and we have solved using the geopotential altitude. So let us now look at what happens when we create a pressure difference.

So we have ambient conditions here like atmospheric conditions here. Let us assume we create a sudden pressure difference. Can you give one such example? Let us consider a fan. When you switch on the fan, what happens immediately you start feeling the wind right, some stream of air. That is because the fan is creating some pressure difference here, right.

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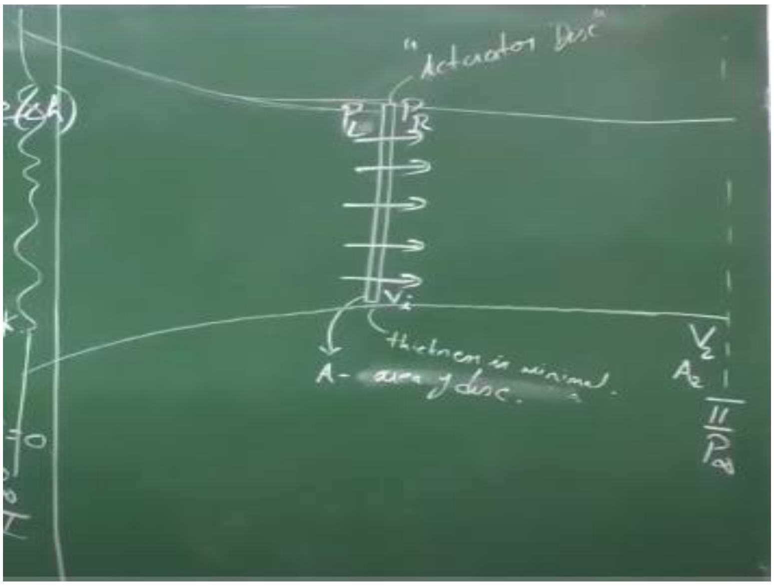

Let us assume this is one such disc called an actuator disc. Let us say this is called actuator disc. So this is called actuator disc, which is capable of creating pressure difference. Let us say this is the upper surface or let us say to the right, pressure to the left, left and this is pressure on the right side. If I take this is my left and this is my right. So I will label them as pressure on left and right.

So this actuator disc let us assume the thickness is minimal or very small. And let A be the cross-sectional area of this disc. Let capital A be the cross-section area or area of this disc, simply area of disc right. So what happens when there is a pressure difference here? What exactly a fan does? It will try to draw air from backside and it used to throw air at us, is it not?

So but let us assume this wind it is trying to this particular disc is trying to draw the air from ahead of it and try to throw it behind it, right. So in fan it is reverse case, right? It will draw air from behind and throw it ahead. So it is a reverse case here. And so let us now first look at, at far ahead. Let us say if I start if I place a fan here, so somebody who is sitting at hundred meters from here, is he going to feel any difference because of this, right.

So definitely no, most likely no with the ordinary fan that we generally use. So let us say at far ahead, so let us say this is this particular at far ahead we this disc, which we call it as an upstream of this disc, what we have is the atmospheric conditions which is P infinity is one atmosphere here. Let us say this is station 1 that I am considering.

At far downstream of this say again, if I stand behind hundred meters of this fan, do I feel any different because of it. So let us say this is my station 2, which is again at atmospheric pressure conditions P infinity is equals to P atmospheric, right, which is one atmosphere here. What we can read there is static pressure, right. So but here at some distance this has come close to P infinity, which is atmospheric pressure here.

And then yeah, so far ahead I do not. So the velocity is 0, right. And these two particular lines represents the boundary conditions of this or nothing but the streamlines, which are on the boundary or the circumference of the stream tube. What is stream tube? If we have a bunch of the streamlines together, then we can consider it as stream tube, right. So at station 1, I have P naught P infinity and V is 0.

And at station 2 this disc is inducing certain velocity to this flow. So which is nothing but V i here. Let V i be the velocity induced to the flow because of this disc, right. And let V 2 be the velocity at the station 2 where pressure is equals to P infinity and then let A 2 be the corresponding area of this particular stream tube, cross-sectional area of this stream tube here right.

So for this A to be 0 this has to be infinite, close to 0 let us say. This has to be the area here of the stream tube to very high right.

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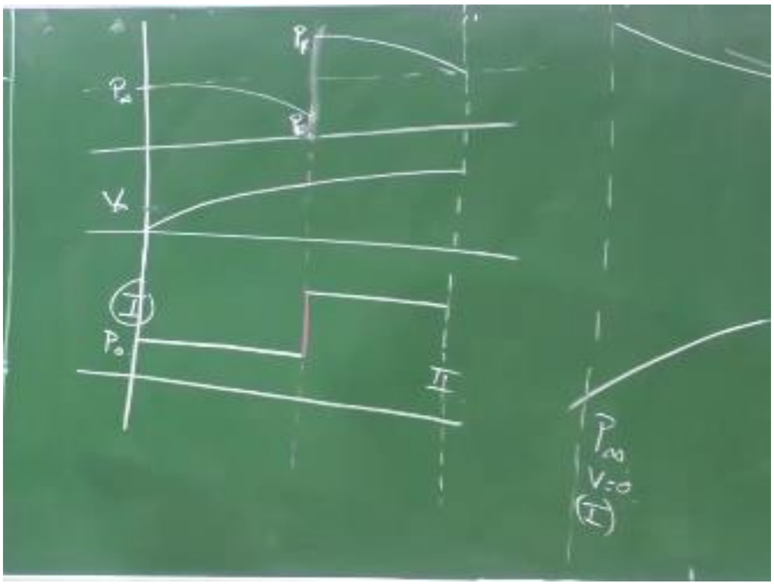

So now okay now let us look at so what we know is about the pressure here right. So let us see how the pressure is varying here. So if this is the location of the disc, right. So what is happening initially say this is my static pressure or P infinity, which is atmospheric pressure. So it drops as we go close to this disc, is it not? Let us say this is the location of this disc, right.

So at this particular location, so it drops the pressure drops nothing P u the upper surface sorry the left pressure to the left of this disc is dropped right. And then there is a sudden increase. Why because, here the there is an induced velocity here, is it not? What is happening the see in this particular portion there is no addition of energy is it not? So according to Bernoulli what happens?

If the pressure from static condition it is now converted to the pressure some static pressure plus dynamic pressure here, is it not because the flow is set into motion here and it is moving. So almost close to this disc it is ahead of disc let us consider it is moving at a velocity V i. And we consider it is thin right. So this velocity remains constant across this particular section here, right.

Why because the area of the disc is very minimal here. So according to continuity equation this velocity remains constant across this particular disc. So what is happening? We are importing some additional energy, right. This is this energy increases the pressure here, right and then it so the nature will take care of the rest once you have increased the pressure here.

So now the air or the fluid, which is at high pressure tries to accelerate so that the pressure become the raised pressure becomes equal to the ambient conditions, right. So here, so it is trying to get back to this. So let this be P on right side pressure, which there is a discontinuity here, right. Now let us see what happens to the velocity. So we know that so let us say this is the initial velocity or say it is at rest.

Now so this is V infinity. So this is at rest at station 1. Say this is my station 1. This is the disc and this is my station 2. So what happens here? The velocity increased to V i and we do not know what is happening after that, is it not? That we will see. So the pressure there is a sudden increase in pressure because of this disc and this pressure needs to be equal to the ambient condition after a certain distance like downstream of this disc, right.

In the slipstream if you, then the slipstream has to, in the slipstream, this particular air has to the pressure has to drop, right then it has to accelerate. Am I correct or not? Within this particular portion, again there is no addition of energy. Only at this disc there is an addition of energy here. So within this particular portion, we can still apply this Bernoulli's principle. So P plus half rho V square is constant.

So if P has to drop down dynamic pressure has to increase, right. So let us see will it follow the or not? So the velocity increases. So what do you think is the variation of total pressure? So till disc it is P naught is constant. And at the disc there is a addition in P naught, right. There is an addition in pressure because of this particular disc. This is the total pressure here.

So let us now look at the mathematics. So again we are trying to look at what the pressure difference does to a to this particular disc right. So what is happening here the air is pushed back by this propeller disc and accounting to Newton's third law. So the air will also push right? How it is pushing? This disc will be in the opposite direction, right. So what that push must be equal to?

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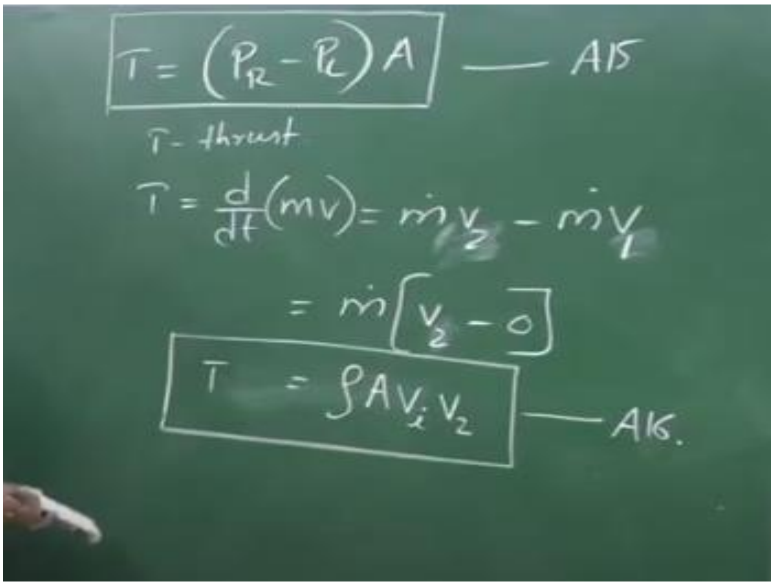

So there is a force because of this pressure difference across this particular disc right. So what is that pressure difference? P on right minus P on left times the cross-sectional area of this disc or the area of the disc is it not? This is the force with which the air is pushing this disc forward. So let us say this particular force is known as T which is thrust, right where T is the thrust acting on the disc, right.

For example, if I rigidly hold this disc, rotating disc, I will also experience the same force T, right? If it is more than my frictional force, right, is it not? So then I will try to move in that particular direction. So T is thrust here, right. So let us say this is our equation number A617? A15. This is our equation A15. And so what is happening here? So there is a force that is generated here, right is it not because of the flowing mass of air.

Am I correct or not? So this mass of air is entering this stream tube at a velocity V naught right and leaving the stream tube at a velocity V 2. That means there is a change in momentum. This change in momentum is what is going to create this force. Am I correct or not? This can also be related by this. So m dot at station 1 times velocity at 1 minus m dot station 2, right.

So this mass flow rate is due to this disc, rotating disc is it not? So this disc is what creating that mass flow rate. So what is that value? Mass flow rate is equals to rho AV, right? M dot times sorry V 2 – V 1 let us say. So what is V 2? So V at velocity 2 minus 0, right. Let V 2 be the velocity at this particular location. So okay second station this is V 2. This is at V 1. Am I correct? Okay.

Let 0 be the velocity with which this mass is entering this particular stream tube. What we have is rho A. So the mass flow rate is due to this disc. So at this disc we can have rho A, rho AV is the mass flow rate m dot where A is the area of the disc, V i is the velocity induced in the disc times this velocity V 2, right. This is the thrust generated because of this mass flow rate, change in moment.

So now equations A15 and A16 represents one and the same, the same thrust that is being generated. Am I correct?

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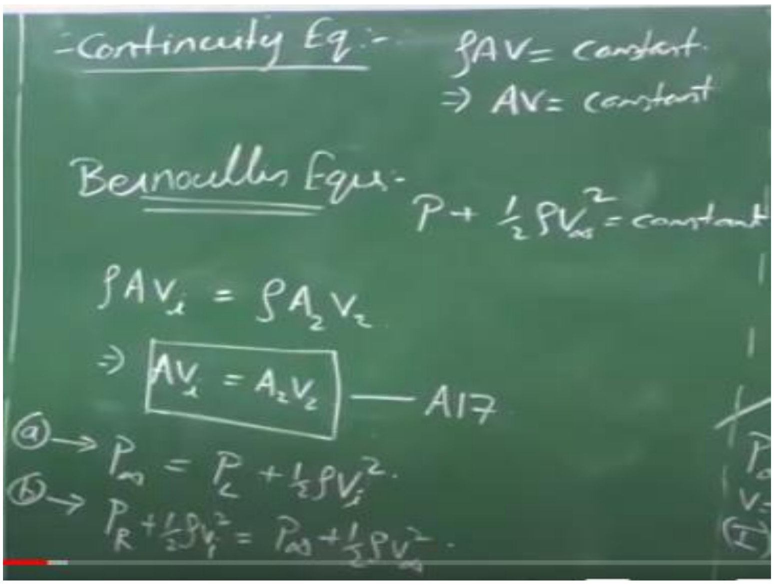

Now let us also look at what is so what so according to let me just note it down. So what we have is P plus half rho V square is equals to constant right along a streamline here. Okay, now let us look at this mass conservation here. So it should be rho AV i should be equals to rho A 2 V 2, right. Which is equals to AV i is equals to A 2 V 2 where V 2 is the velocity measured when P is equals to P infinity, right the static pressure equals to P infinity, okay.

Now let us apply Bernoulli's equation ahead of the disc and behind the disc because here there is an addition of energy, which we cannot use because of which we cannot use this Bernoulli equation throughout this right. We can use it in the section 1 or section 1 is given. So part a section a and section b right. So what we have here is P infinity, which is the static pressure when the velocity here is 0 right is equals to the velocity to the left side of this disc which is P L just immediate to the disc.

And then the corresponding dynamic pressure is due to the velocity induced velocity V i, right. So similarly, if you apply this Bernoulli's equation to the part b, from a from b what we have is P R plus half rho V i square is equals to P infinity plus half rho V infinity square sorry V 2 square okay. So just remember this A15 and A16. We will call back these equations.

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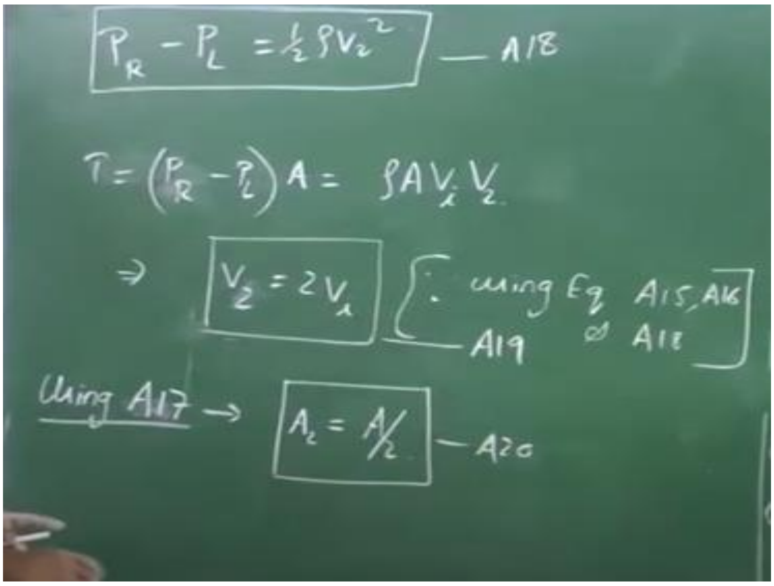

So using these two equations what we have is, subtracting these two this a equation from a from equation from equation b right. So what I have is P R – P L is equals to half rho V 2 square. Let us say this is my equation 18, A18. Now comparing this equation 18 and then previous 15 and 16 what we can write is the thrust generated here or so T is equals to P left minus P or say P right minus P left, right.

What was there P right minus P left times area, is it not? Which is equals to which one? So P R minus P L times A is equals to rho AV i times V 2, right using 16 and 17. So using 18 what I have is so V i is equals to or say V 2 is equals to 2 times of V i. So using equations, A15, A 16 and A18, right. Am I correct or not? Okay, so V 2 is twice the induced velocity here. That means the flow is accelerating here, right.

And what will be the cross-section area A 2 here? So using this particular A17 we can relate, using A17 what we have is, so A 2 is equals to A upon 2. So half the cross-section area of, half the area of the disc. So **so** area of this, cross-section area of this twin tube is half the area of this disc. That is why it is accelerating right. So yeah. So let this be A19 and A20, right. Clear?

We can also express this induced velocity in terms of this thrust loading. That is also a good equation to note it down.

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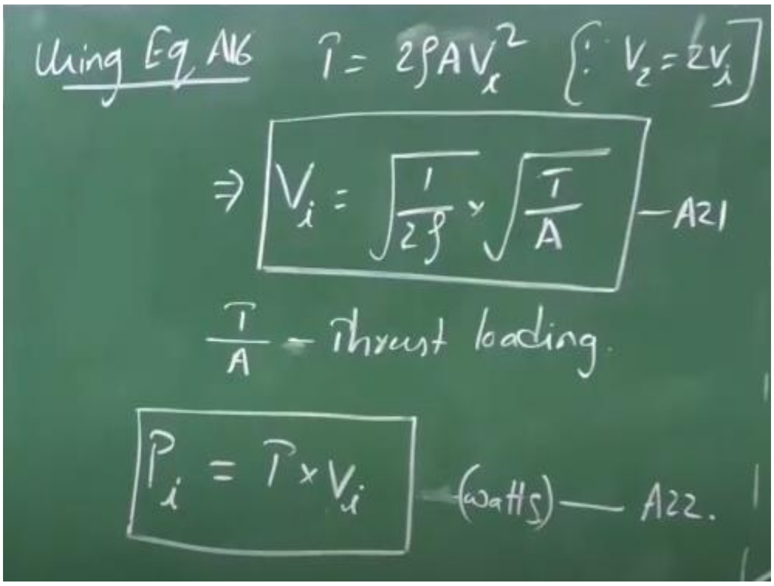

So from this 15 and 16 again using, so using equation 16, using equation A16 what I have is T is equals to 2 rho AV i square, right. Since V 2 is 2V i. So this implies, so the corresponding induced velocity depends upon root over T by A, right. So this particular factor T by A is known as thrust loading. So what is the amount of force generated across this particular area, area of this disc, right?

It is known as thrust loading which is Newton upon meter square. Yes, yeah too many equations yes I can understand, but still we have to note it down. So V i here is the induced velocity by this disc depends upon this particular thrust loading factor, right. And how much power we need to spend in the first case? How much power we need to spend in order to create so much thrust.

So is it clear? Is my question clear? So that means if this disc is generating thrust T that means this disc is also pushing the air with the same force but in the opposite direction right? And it is pushing that air at a velocity V i is it not? So the power induced or the power that we need to put in is equals to power induced is equals to thrust times V i, right. Correct or not?

The force times the velocity. So what are the units of this power which is watts, in watts. So this equation be A22 right. Fine? So that means in order to generate a thrust T I need to produce this much of pressure difference across a disc of cross-sectional area A. And if I produce this much of thrust or this much of pressure difference, it creates a velocity at a downstream where the pressure equals to P infinity, right.

So it creates it induces a velocity of twice that of induced velocity, right. It creates it accelerates the flow at this particular velocity right. So it is worth noting down non-dimensional analysis or non-dimensional parameters for this particular actuator disc theory, right. So with which we will be able to compare propellers of various configurations, okay. So what we are dealing with is Froude's momentum theory or actuator disc theory, right.

So that is what we are discussing here. So we know now, this thrust will propel your aircraft, so will take your aircraft forward, right even if it overcomes the resisting forces, is it not? So this is the main idea So thrust is this is how we can generate thrust. This is one of the means in which we can generate T to accelerate our aircraft.

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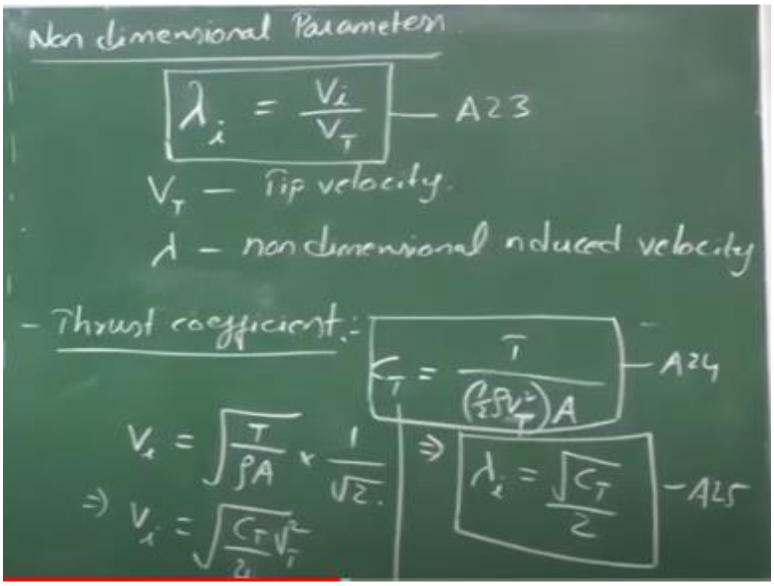

Now let us look at some non-dimensional parameters. So if I had to compare propellers of different dimensions and different geometric characteristics, right. If I have to compare them with to bring them onto a single platform for comparison, I need certain non-dimensional parameters, right. So let us first look at so one important factor that we look at is the induced velocity here, right.

Let us define a non-dimensional induced velocity, which is lambda i is equals to V i upon V T where V T is a tip velocity. Velocity at the tip of this disc right. So V keeps varying. Am I correct or not? Induced velocity keep varying along the disc. So V T remains constant for a given propeller, okay at a particular rpm again. And then let us define C T is so non-dimensional induced velocity.

So lambda i keep varying along the radius of the disc, right. As we progress towards the tip this lambda keep varying, okay. Whereas V T remains constant for this particular rpm. So similarly let us now define thrust coefficient which is C T is equals to T upon dynamic pressure right times the area of the disc. So dynamic pressure times the area will help you to like this is force, force upon force helps you to non-dimensionalize this thrust, right.

So this V is nothing but again here is the tip velocity here fine where C T is called non-dimensional thrust which is the thrust coefficient here, right. And now can we rewrite this equation? So can we look at this equation? So what is vi So V i is equals to 1 upon 2 rho, right. So let me do this, let me rewrite this equation. So root over T upon rho A times 1 upon rho 2.

So what I can do this with this is this is like C T by 2 V T square is it not? Root over C T by 2, which is 4 times V T square, right. So this equals to lambda i root of root over C T upon 1 or C T is 4 lambda square, right. A23. Where is other equation? So let this be 24 and this be 25, right. So can we also convert this? So similarly you can do this.

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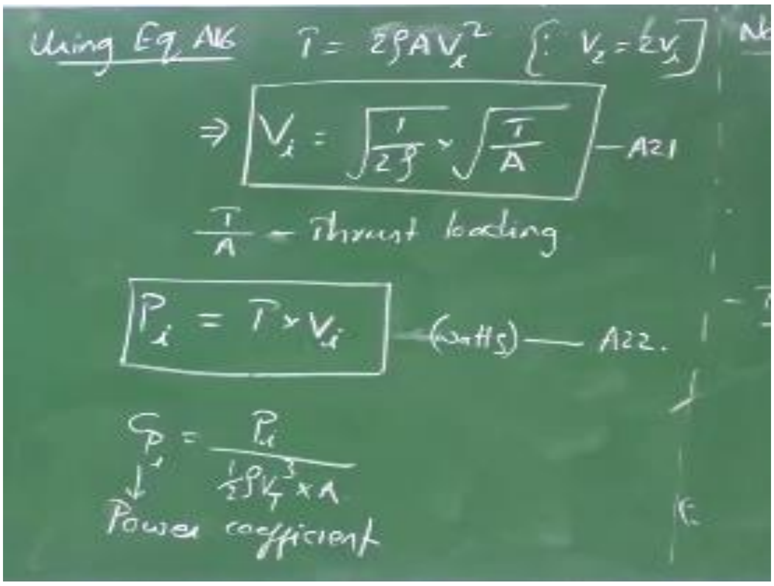

So let us say C P i is defined as P i upon half rho V T cube times area, right? Similarly you can relate them. So we can substitute this in this particular equation and then for T again you can convert in terms of coefficient C T, you will be able to arrive at this particular non-dimensional pressure coefficient C P, okay. C P is pressure coefficient which is a non-dimensional number, which is pressure, induced pressure, pressure coefficient.

So this is the amount of power that you need to sorry, it is not pressure coefficient, power coefficient, I am sorry. C P is the induced power coefficient, which is P i upon half rho V T cube times corresponding area of the disc. So C P i is a power coefficient, which is the induced power coefficient. So you need to spend so much of power in order to throw the air at this particular velocity with the thrust T backward here, right.

So what we can observe here, pressure difference across a disc created a force called thrust in the forward direction. Say if that forward direction say is nothing but the direction of our motion then if you hold or say, if you fix that particular setup, rigidly to our aircraft then the aircraft will also move with a thrust T, right will also experience thrust T in that particular direction, right.

So later on we will again come back to that point, right? So we will solve a example about this particular momentum theory. And then we will get back to that how thrust is going to help us right. So let us solve a small example.

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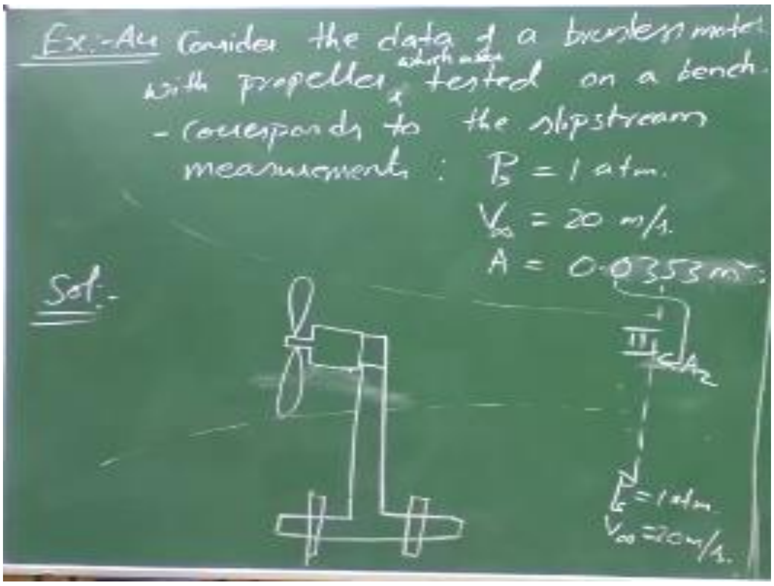

So this is our next example A4, example number 4 in this particular in this introductory topic. So consider the data of a brushless motor with propeller tested on a bench, right which was tested on a bench let us say, tested on a bench right. So the data corresponds to the slipstream measurements. What are these measurements? So P in the slipstream is equals to 1 atmosphere.

And V in the slipstream is identified as 20 meters per second, right and this measurements are effective within an area of say 0.0707 meter square okay. So these are the measurements here. So what do you mean from this slipstream measurement? So we have say a BLDC motor, brushless motor right, which is rigidly mounted on a bench and we have a propeller, right.

We have a propeller here. So this is mounted on a bench by means of an adapter, okay. This is the bench and we have a propeller here mounted on this motor and so when the propeller is operated, when this BLDC motor is operated the propeller rotates due to which the air, like there is change in downstream conditions here right. And it is identified at a far downstream.

So say let us say this is my particular location at which P is equals to 1 atmosphere that is measured here. And then V is equals to so static pressure here right, okay. And V infinity is equals to 20 meters per second, right, 20 meters per second. Then the corresponding area of effectiveness is 0.0707 meter square.

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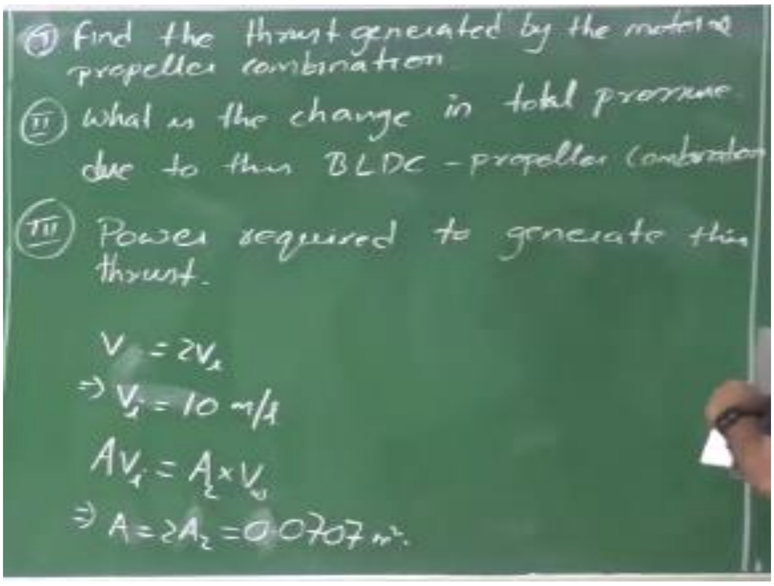

Okay, now what do you mean to find? So find the thrust degenerated by the motor and find the thrust generated by the motor and propeller combination. So when we have these measurements what is the corresponding thrust generated by this motor and propeller combination. That is what we had to find. And the second one is what is the change in total pressure due to this BLDC and propeller combination?

And also power required to generate this thrust, fine. So these are the three questions that we need to answer. Straightforward, right? So let us assume the interference is negligible with the flow, so with this setup and all. So with this attachment and the rest of the setup, let us assume that they are not going to interfere with the flow here. Find the thrust generated by the motor and the proper combination.

So with this information, so V at this particular station is about 20 meters per second. So what I can do is, like we know the relationship between this V i and V is it not? So what is V i here? V i is, V at this particular station is two times that of V i because the pressure is 1 atmosphere right. So **so** V i is equals to 10 meter per second right. So V is 20 meters per second so what V i is 10 meters per second.

Similarly, once we have this I know what is A i times V i at this particular disc is equals to A times V i, A times V at this particular location. So area of disc and area of area at this section, let us say this section 2 in our case right. So A 2 let us say this is

A 2 here. So A 2 times this particular V 2 here or V infinity whatever. So what is a A here?

So V i upon, you know A by 2 right A 2 by 2. So two times A 2 right, is it not? So V 2 is 2V i. So it becomes 2A 2, which is what is the area here? So there is a small correction here. Let us consider this as 0.0353 meter square. So what we have here is area is equals to 0.0707 meter square. So this is the area of the disc. Am I correct or not. So if I have to, now what is the thrust? Please make this correction here.

So the area of this disc this stream tube at this particular location is equals to 0.0353 meter square. So which is approximately, so here the using this relationships, so we figured out that the area of disc is approximately 0.0707 meter square.

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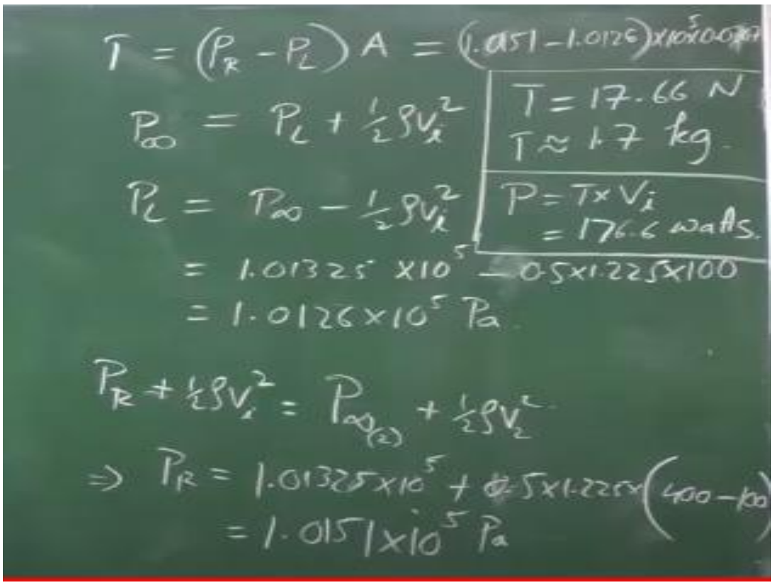

So the thrust generated is equals to P R - P L times the corresponding area right. So how can I get it? So you can do this way or you can also since you know what is V i, you can right, you can use this use the thrust formula, but I am not doing that. You better do this. So far ahead of this is equals to atmospheric pressure like right P infinity which is far ahead of this disc is equals to P L plus half rho V i square, is it not?

So P L is equals to P infinity minus half rho V i square. So what is the value here? So this is 1 atmosphere, which is 1.01325. So because far ahead of this disc is again 1 atmosphere and the velocity there is 0, right. 10 to the power of 5 Pascal minus 1.225

times 100. So what I have here is one point. Similarly, I can find out what is the pressure on the right side of this.

This is the pressure on the left side of the disc. And to the immediate right of the disc what I have is T R plus half rho V i square is equals to P infinity at station 2 plus half rho V 2 square, which is let us say this is my station 2, right. So what I have here is, so what is P R is equals to P infinity at station 2 is 1 atmosphere. Then 1.01325 multiplied by 10 raised to the power of 5 plus 0.5 times 1.225 times V T square minus V i square.

So this is like 400 – 100. So the answer here is P R. So the pressure on the right side of the disc, immediate right side of the disc is Pascal, right. So substituting these numbers we can get the corresponding thrust which is equals to so 1.0151 - 1.00126 times 10 power 5 times 0.707 right. So the thrust T is equals to which is 17.66 Newtons, close to 1.7 kg. This is the thrust generated by this particular engine, okay.

So this is the thrust generated by this engine and the corresponding power generated is equals to power required is equals to thrust times the velocity which is 176.6 watts. So this is answer for the third question. This is the answer for the first question, right. Am I correct? So what is the change in the total pressure? What is total pressure ahead of the disc? Total pressure ahead of the disc is nothing but atmospheric pressure here, right.

So 1 atmosphere and total pressure behind the disc is P L plus half rho V square. Am I correct or not? You can see that P L so P R plus P R plus half rho V square. You can see so pressure on the left side has dropped below the atmospheric pressure, is it not? It is P infinity minus this particular quantity and beyond the right side, like it is like yeah, P infinity plus the total increase in the velocity is the P on the right side, right.

So this particular quantity is the total pressure on right side. And this particular quantity is the total pressure on so P, so P infinity is nothing but 1 atmosphere, which is the total position on the left side, right.

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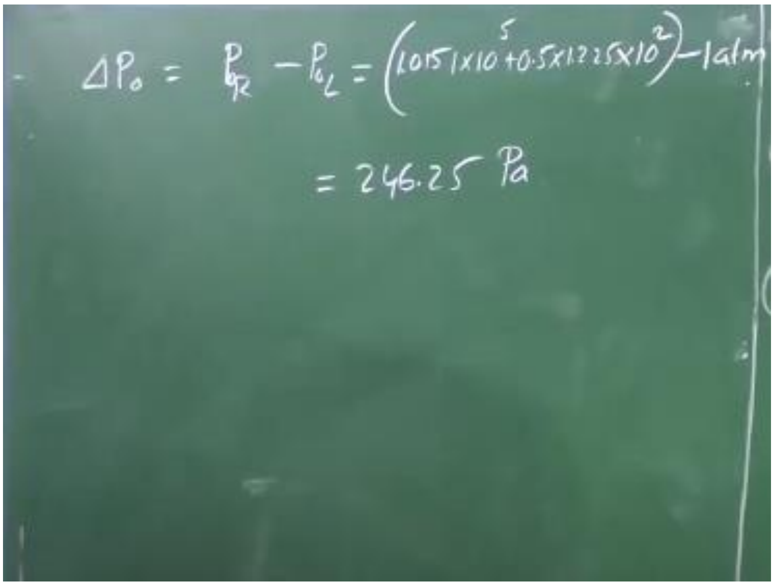

So the change in the total pressure delta P naught is equals to P R minus P on the right side P naught R - P naught L, which is 1.0151 times 10 power 5 plus 0.5 times 1.225 half rho V square, right. P R plus half rho V square. Half rho V i is 10 meters per second 10 meter square minus 1 atmosphere 1 atm, right. This is equals to 246.25 Pascal is the additional pressure that is added because of this particular disc.

So the power required to generate this thrust is close to 177 watts here, right. Okay, thank you.