UAV Design - Part II
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Lecture - 06
Aerodynamic Characteristics of Wings

Dear friends, welcome back. In our previous lecture we were discussing about wing platform geometry. So now we are going to talk about, is there a difference between lifting characteristic of a 3D object which is wing here and a 2D object which was aerofoil that we discussed earlier right, which is a cross section of wing, right.

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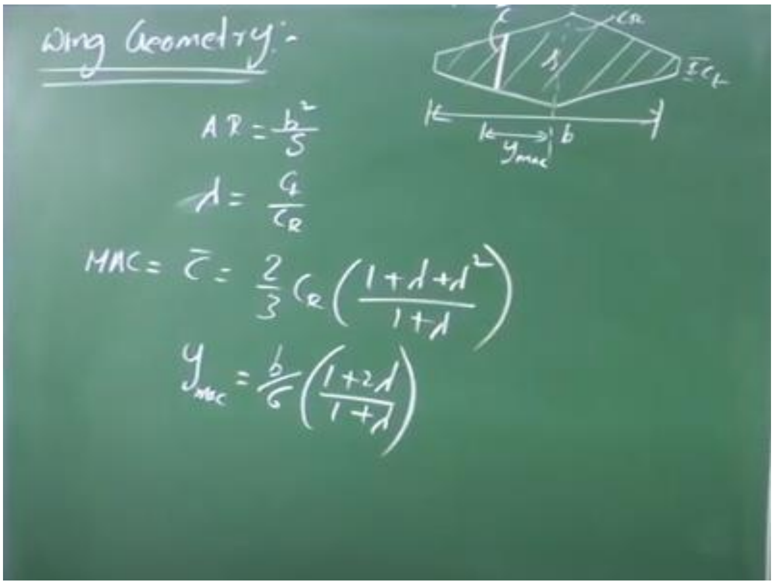

So what we discussed about is the wing geometry. So we talked about some of this non-dimensional parameters. So and so we talked about aspect ratio and taper ratio C t upon C R and then mean aerodynamic chord C bar which is MAC, mean aerodynamic chord is equals to two third upon C R times 1 plus lambda plus lambda square upon 1 plus lambda.

And then we also discussed about this location of this mean aerodynamic chord, location of MAC span wise location. Say this is the corresponding y MAC. So y MAC is b upon 6 times 1 plus 2 lambda, 1 plus lambda. Now how do we proceed? First let us look at the theoretical aspects here how to figure out the lifting characteristics of wing, right.

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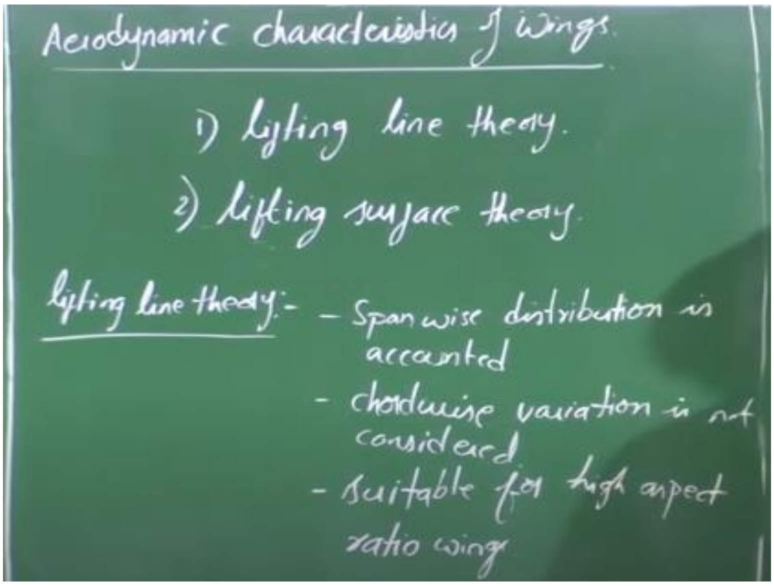

So they are we are talking about aerodynamic characteristics of right. So there are two theories that talks about this two main theories that talks about this lifting characteristics of wings. The first one is lifting line theory. The second one is lifting surface theory. So what lifting line theory talks about is high aspect ratio wings, right. So it considers the span wise variation of lift.

Whereas, it neglects the chord wise distribution of lift, right. So the span wise distribution is considered in this lifting line theory whereas the chord wise variation is neglected, right? In lifting line theory so span wise distribution is accounted, right. So chord wise variation is not considered and hence suitable for, so this theory is suitable for high aspect ratio wings okay.

Whereas lifting surface theory it accounts for chord wise variation as well but it is very complex in nature and right now we are not going to talk about that, right. So we are going to talk about this lifting line theory. It requires higher computational capabilities as well for this lifting surface theory. So let us consider this wing made out of Styrofoam. You can see there is, a cross section is aerofoil here, right.

You have an aerofoil cross section. And there is so it is about 0.75 meters approximately, right. And moreover it is a rectangular wing here, right. Now what happens what will be the major difference here? So how the lifting characteristics varies with from an aerofoil to that of a wing, right. So what is the difference here?

Now consider, so as we know there is a pressure distribution, if you consider a cross section, right.

At a given location what you have is an aerofoil. Similar to the pressure distribution on aerofoil you will have pressure distribution on wing as well. So there is low pressure and high pressure, low pressure on the top surface, high pressure on the bottom surface right. So this abrupt ending, so wing is a finite object here right, 3D object and of finite length. That means there is certain end here is it not?

So at this abrupt end, so the airfoil still have the same characteristics, right. There is a higher pressure distribution on the bottom surface and lower pressure distribution on the top surface. So because of this pressure difference across this tip, the flow tries to curl around from high pressure area to low pressure area. So that curling will form this tip vertices, right.

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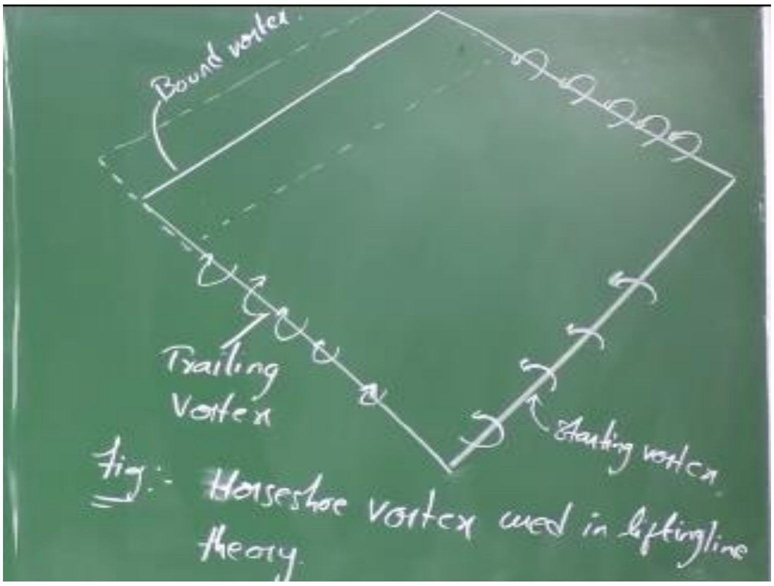

So now, according to this lifting line theory, this wing is replaced say this is the wing that we are talking about, right. So this according to this lifting line theory, this wing is replaced by or the lifting characteristics of this wing is replaced by a vortex sheet, right the horseshoe vortex, right. Now, the part of this vortex sheet which is on the surface of the wing is known as bound vortex.

This is called bound vortex, right. So this and it is followed in the downstream by a trailing vortex, two trailing vertices, right. This is how the trailing vortex will be. The flow tries to rotate from about the tips from, so from the region right from the regions of high pressure to low pressure right. So you have trailing vortex here and then this vortex sheet is closed by means of a starting vortex, right.

So we will see what is this starting vortex is. So what these two are the trailing vertices, trailing vortex and then what we have is a starting vortex. This is how a horseshoe vortex is found. So this figure is about okay? So do you remember our discussion about the two stagnation points right that forms on the airfoil? Do you remember the discussion? We have an aerofoil. So we discussed about this earlier.

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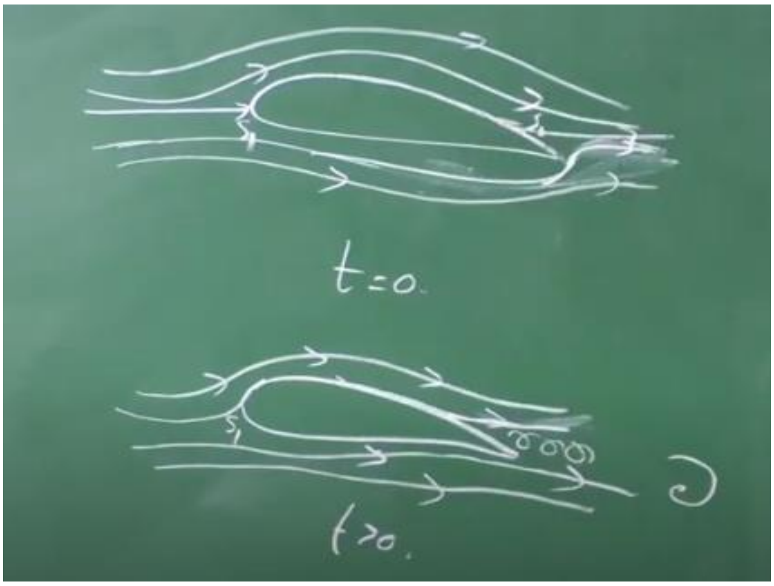

So you have two stagnation points s 1, s 2 on this aerofoil. So the streamline itself so the body itself is one of the stagnation, forms a part of the stagnation streamline and then yeah, let us say at time t is equals to 0. So this is what is going to happen at time t 0 what happens? The airfoil started moving ahead impulsively, right. That means, initially the fluid particles will try to flow around this, is it not?

It does not have any obstruction. Am I correct or not? So there is no flow earlier so there is no pressure distribution here is it not? So when as the time progress that will develop, right. At t is equals to 0 this will be able to smoothly pass over and then form these two stagnation points, which we discussed right.

So this will curl, according to the potential flow solution it will curl around this tip and then close this particular flow, right. Is it not? Now, at time t greater than 0, we discussed about boundary layer and all, right and adverse pressure gradient because of which the flow tries to separate here and the initial thing that has formed right the smooth flow that has formed, so this curl will be swept away in the stream.

For example, if you look at time lapse picture of this. So at t greater than 0 what happens is at t greater than 0 right. Because the flow develops after the initial fluid particles move across the airfoil, right. So and then the pressure gradients develop. So this initial vortex which was formed here so the starting vortex which is formed here was swept away in the flow right to the in the downstream of the flow.

So this particular vortex is the starting vortex that was considered here okay. So what is this doing, this bound vortex?

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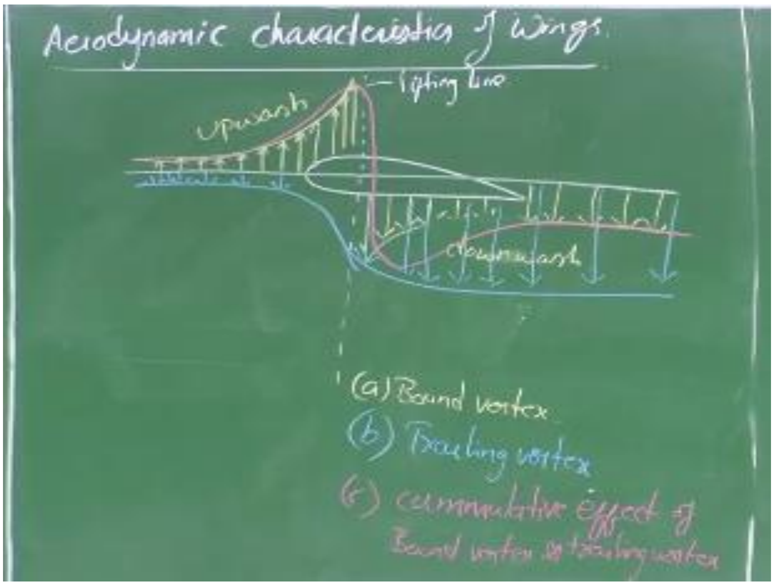

So when we have an airfoil right. So say this is my location of this lifting line. So what I have is upwash and also a downwash here. So this bound vortex will try to push the flow up which is ahead of this aerofoil right, ahead of this vortex and then also push, it will also push down the flow which is behind this airfoil is it not? So you have an upwash as well as downwash. So this is what this is due to bound vortex.

And this trailing vortex will try to push down, incorporate downwash right. It will try to induce downwash ahead as well as behind the flow, right. It will also induce, it will induce everywhere including on the wing, but the influence of this downwash is negligible in presence of this bound vortex right. So it will try to induce, it will try to induce downwash. So do not consider these arrows as the magnitude here, right.

So this is just for understanding the concept about this downwash and upwash induced by this bound vortex and trailing vortex okay. So a cumulative effect of this two, bound vortex and the trailing vortex will have this upwash and downwash effect. So the second color code is for this trailing vortex in the cumulative effect, right. Okay. Now this is a cumulative effect by these two.

So what is the immediate consequence of this lifting line theory? So at a given span wise location, it will try to disturb the flow is it not? It will try to, so it is inducing an upwash, which means it will make the flow to move upward and there is a downwash behind in the downstream is it not? So that push the flow downward about this lifting line. Say this is my lifting line, right.

That means, the direction of the flow is altered at various span wise location. Am I correct or not here? So that is what we have to take away from this particular theory. So if the direction of this flow is altered, which means the angle of attack at that particular span wise location will also alter right.

So and we know the lift generated at this particular location will always act perpendicular to that free stream velocity or local velocity, not the actual free stream velocity right. So now, the lift at a given span wise location will be perpendicular to the local velocity, local velocity vector not the free stream velocity vector. So to elaborate it more we will consider this picture again.

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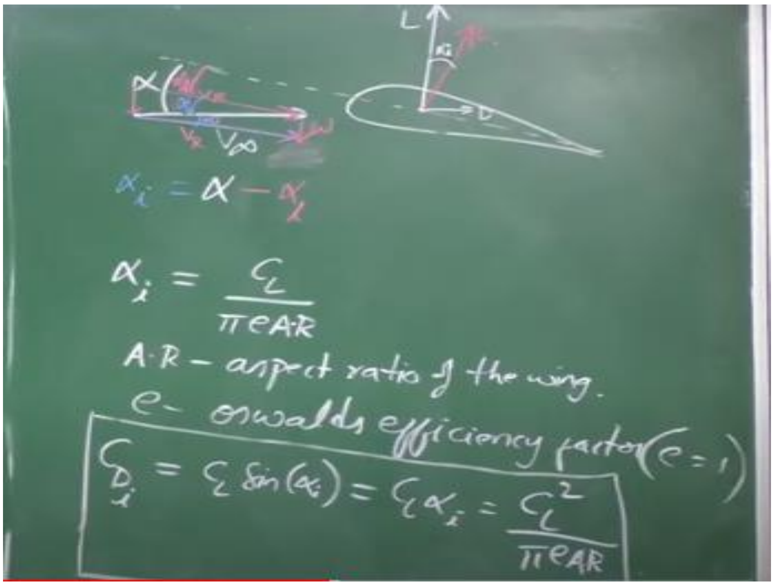

So say this is an aerofoil of our interest such as span wise location, right. So we have taken a span wise location across this wing. So let that be the corresponding span wise location say yeah. We have wing here. So if you take any of this span wise location here, right. So what you have is aerofoil everywhere is it not? So this particular aerofoil is represented here.

Now, let us say this is my free stream velocity V infinity, V infinity and the corresponding angle of attack is alpha right. So there is a downwash is it not? So there is some downwash. So may not be of this magnitude say so just to make it scale more realistic. So let this represent the downwash W here right.

See in general yeah W at the lifting line, right so because of which the resultant velocity or the local velocity will be tilted down right. So this particular change in angle of attack is a induced angle of attack by the downwash here right. So how I can represent? I can represent it by the same arrow W and then this will be my resultant velocity V R, right V R.

And this angle is alpha i and this particular angle is called alpha l, is a local angle of attack, right. Okay, let me write alpha i. So this is alpha induced and this particular angle is alpha i, fine. So alpha i is equals to alpha minus sorry this is local angle of attack, right. This is alpha l. So this is minus alpha l is a local angle of attack. So what is happening here?

Say this is the actual lift direction of this entire wing because, we define lift is perpendicular to V infinity, right? So force component of the force resultant force which is acting perpendicular to free stream velocity is lift and along the free stream direction is drag right. But now, so because of this local angle of attack right. So the lift has to act perpendicular to this resultant velocity, right or the local velocity here, which is V R here right.

So this is the direction of this local lift here, is it not? So it has a component along the direction of actual lift, overall lift as well as along the direction of overall drag. So this additional drag due to this lift is known as induced drag, induced by the lift right. So you so this particular angle is what, alpha i is it not? So this is alpha i and this is alpha i.

These two are perpendicular and these two are perpendicular. So the corresponding included angle here is alpha i, right. So alpha i is equals to alpha minus alpha l. So according to this lifting line theory this induced angle of attack is equals to C L upon pi e A.R. A.R is the aspect ratio of the wing and e is the Oswald's efficiency factor. So for an elliptic wing e is equals to 1.

That means, it has minimum induced drag right and for rest of the wings it will be less than 1. So how the elliptic, how the lift distribution is deviating from the elliptic lift distribution. This factor talks about that particular parameter right. So what is the drag that is induced because of the lift is equals to so you have lift here. You have a component in the direction of drag is it not sin alpha, right.

So C L times sin alpha i. For small angles of attack what we have is C L times alpha i right. So if I substitute alpha i here what you get is C L square upon by pi e A.R. So this is an important result, which we will be using later on when we discuss about drag, right. So this is the induced drag because of this local variation in the, because of this upwash and downwash effect right is it not?

That which varies the local velocity vector and hence the lift at that particular location, right. So it has a component along the overall lift, direction of the overall lift as well as overall drag right. So the corresponding component of this lift in the drag direction is the lift induced drag, right. So which is equals to C L square upon pi e A.R.

So this is what we intended to use this theory right for this, we intended to use this theory for this particular derivation, right. So apart from this we can also look at how yeah how the lift coefficient is varying with respect to angle of attack for finite wings. So what do you mean by finite wings? How it is characterized by definite length is it not? So when there is definite length there is aspect ratio, different aspect ratio here.

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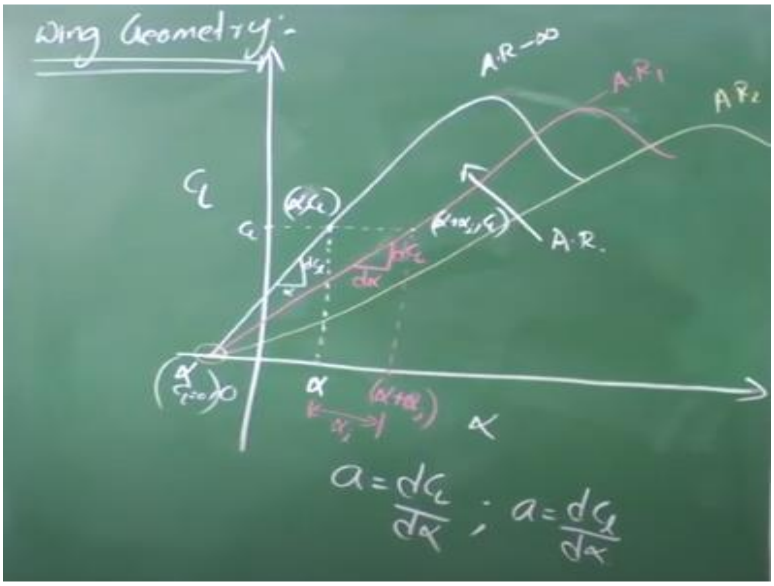

So let us say C L, it is 3D, 3D C L is equals to so sorry is varying with angle of attack here. So let us say this is for an airfoil okay. So this is for an airfoil. This is how C L is varying with angle of attack. So let us assume alpha at with C L is equal to 0, right. Again coming back to this concept when there is no lift that means the pressure distribution on the top and the pressure distribution on the bottom has to be same.

So when there is no pressure difference on the top and bottom, do you think the flow will be curling around? It may not be able to curl around right? There is no driving force to driving force for the flow to curl around the tips, right? That means, so at zero angle at zero lift condition, so the pressure distribution over the wing and a particular cross section should be same, is it not?

So that means, the angle of attack for wing at with C L is equal to zero and for the airfoil at which C L is equals to zero can be same, is it not? Because there is no induced angle of attack there. Am I correct or not? Is it a decent assumption? Is it not a decent assumption here, right? So for with the increasing in aspect ratio, right what happens is there is a drop in this C L alpha.

So the slope is decreasing, right. So more or less this is more or less a decent assumption here according to me, right. So right so this is four aspect ratio is infinite right. So this is for aspect ratio 1 say this is aspect ratio 2 and this is the increasing direction of aspect ratio, fine. Now let us relate the lift produced by aerofoil and the lift produced by the wing made out of same aerofoil right.

So let us consider a particular lift coefficient value C L here, right. So this say this is my C L interest of my interest and then the same the lift by this aerofoil is produced at an angle of attack alpha here right, okay. But, whereas, because of induced angle of attack what happens is let us assume it is produced by the wing at an angle of attack, which is alpha plus delta alpha, otherwise alpha i, right.

So this particular difference in airfoil 2D section and the 3D section is because of this induced angle of attack alpha here. Am I correct or not? So let us assume this three dimensional slope here D capital C L upon D alpha right. So b a is equals to let a is equals to dC L upon d alpha and a naught is equals to dC L two dimensional upon d alpha right.

So where so the same lift again is produced by different angles of attack for aerofoil and wing respectively, right.

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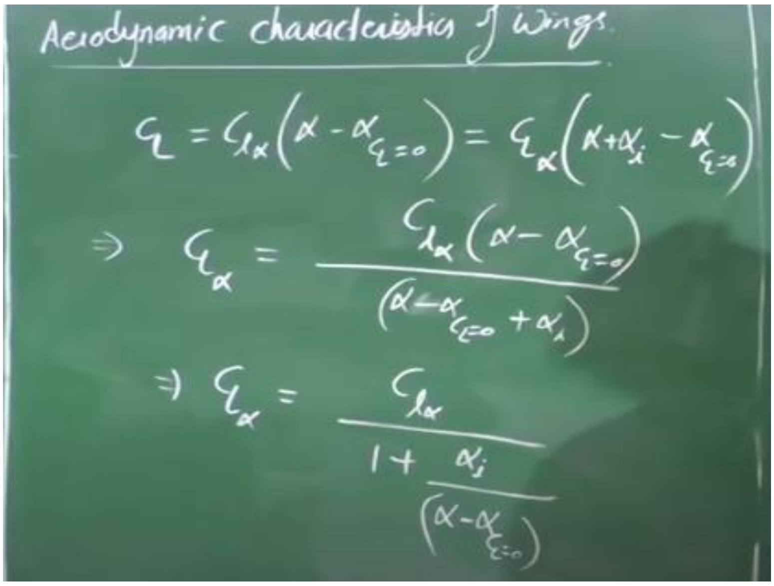

Which means the C L here is equals to so I know one point here which is alpha, C L and I know the other point which is alpha + alpha i, C L right and I know the other point here, which is alpha at C L 0, 0. So this particular C L is equals to C L alpha times right what can I write? Alpha minus alpha at which C L is equals to zero, is it not? Am I correct or not? So what is the slope, y 2 - y 1 upon x 2 - x 1.

So this small C l alpha is equals to what is y 2? C L – 0 upon small yeah alpha minus alpha at which C L is equals to 0. So this is C l alpha 2D, small C l alpha here. Now I am differentiating with wing and aerofoil right. Earlier I have not. So this implies C L is equals to C l alpha times alpha minus alpha at which C L is equals to 0. Am I correct? Right.

Similarly, this must be equals to what? C l alpha which is 3D for a finite aspect ratio wing let C l alpha be the lift curve slope here in the linear regime C l times alpha plus alpha i minus alpha at which C l becomes 0. So can we relate these two expressions? So C L capital alpha is equals to C l 2D, C l alpha 2D multiplied by alpha minus yeah, this is a total angle of attack because alpha C l 0 is negative.

So this is the total angle of attack to achieve that C l with a C l alpha as slope of this curve and then alpha minus plus alpha i. So this implies C l alpha is equals to 3D is equals to C l alpha 2D upon 1 plus alpha i upon alpha minus alpha at which C L is equals to 0.

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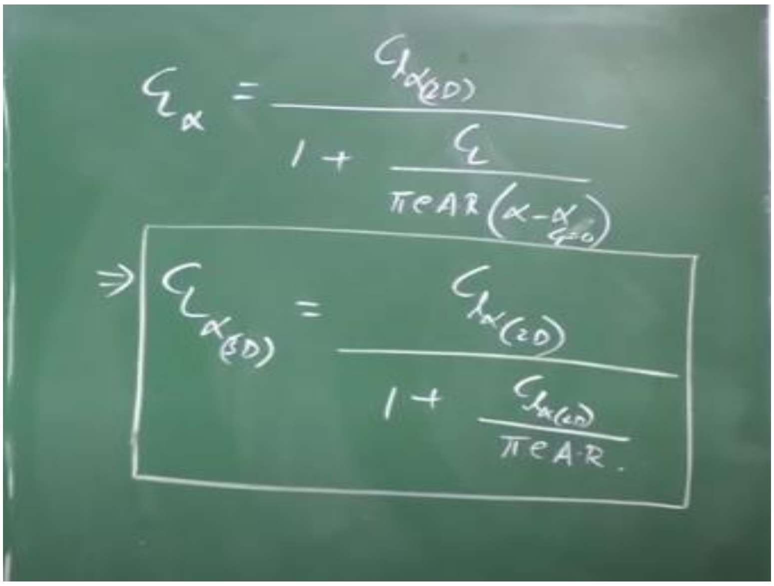

So according to lifting line theory what is alpha i? Yeah, this is C L right, I can write as this C L upon pi e A.R times alpha minus alpha i right. So alpha minus alpha at which C L is equals to 0. Am I correct or not? So what is this C L again? This C L with aerofoil, I will be able to generate by using this expression C l alpha times alpha minus alpha at which C L is equals to 0.

So substituting this in this equation, what I have is C L alpha 3D is equals to C l alpha 2D upon 1 plus C l alpha 2D upon pi e A.R. So this is the relation between lift curve slopes of finite wing and infinite wing, right. Thank you.