UAV Design - Part II
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Lecture - 09
Longitudinal Static Stability - Wing Contribution

Hello friends, welcome back. So in our previous lecture we witnessed for a UAV to be to possess longitudinal static stability it has to satisfy two conditions, which is C m naught has to be greater than zero and C m alpha has to be less than zero right. So and we also discussed about stability right.

So it is a property of an equilibrium where the, if the system is able to return to its initial equilibrium once disturbed from it then we call the system is a stable system, right. And we also discussed about equilibrium and its properties, right. So now let us proceed ahead to figure out how various components of a UAV contribute towards stability of the system.

And we also studied that the stability of a system is bifurcated into two parts. The first one is the static stability and the second part is dynamic stability. Where the static stability is the initial tendency of the system to come back towards its equilibrium once disturbed from it, right. So in case of dynamic stability, it talks about time history of motion of the system once disturbed from its equilibrium.

So if the system is able to return to equilibrium over a period of time by damping out the disturbance that was created right, which has in fact disturbed from its equilibrium. So if it is able to return to its equilibrium then we can say the system is both statically as well as dynamically stable.

So and we also concluded that dynamic stability will guarantee static stability but static stability may not guarantee dynamic stability, right. And we are looking at longitudinal static stability of a UAV here, right.

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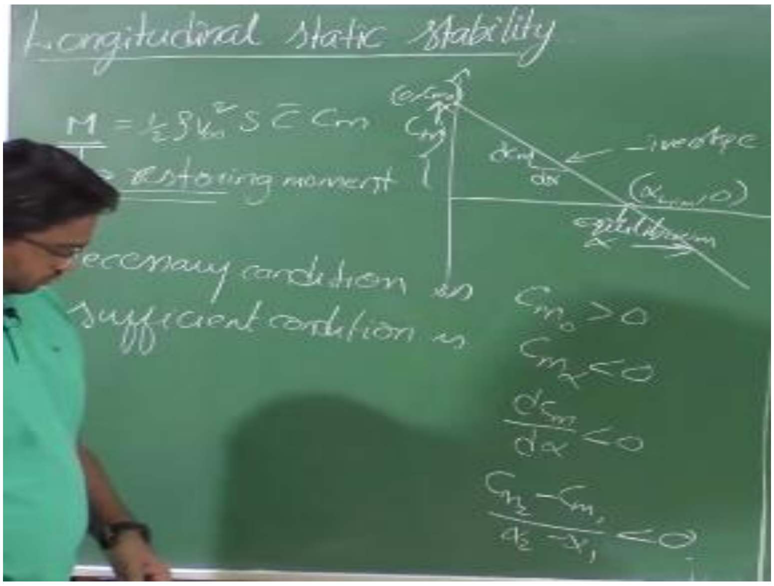

And we have considered an example. So we know for longitudinal case the only moment that contributes that can contribute towards stability is the pitching moment, right. So which is half rho V square S C bar times C m, right. So this is what is going to contribute towards restoring moment, right. So which is very important for a system to be stable.

So for a so we need a restoring moment in one form or the other. So in the first case we considered a pendulum pivoted about a point O, right. And this pendulum of mass m is deflected from its or disturbed from its equilibrium. And once we leave once it disturbed from its equilibrium, it tries to oscillate about the equilibrium about this pivot point.

So and finally, if there is damping, it will be able to reach that particular initial equilibrium right, is it not? So that happens only because of there is some restoring moment with respect to about point O by the system, right. So M g sin theta multiplied by l, the length of the pendulum is what helping your system to yeah restore to its equilibrium, right.

So that moment which helps the system to restore towards equilibrium is called restoring moment and restoring moment is essential for a system to be statically stable here, right. So for a longitudinal case of UAV we have pitching moment. Yeah, we have only one moment which is a pitching moment. And with this pitching moment, we should be able to generate that restoring moment.

So for this for the, for a UAV to be longitudinally stable, we have witnessed that C m variation with alpha has to be negative, right. This is dC m upon d alpha. So this is with negative slope, right. So if the system have this particular variation in the linear range of angle of attack, so if this system have this C m variation with angle of attack right, this particular trend that is shown here, so then we can say the system is statically stable.

And the other condition that is required is C m naught, that is C m at alpha is zero, right? This is C m naught. This particular point corresponds to 0, C m naught. And this particular point corresponds to alpha trim, 0. That is lift C m moment is 0. So which can be considered as trim state or equilibrium state here, right equilibrium, okay.

So this C m naught has to be greater than zero to trim the aircraft at a positive angle of attack and C m alpha has to be less than zero to have the tendency to produce restoring moment, right. So the necessary condition is, condition is C m naught has to be greater than zero. And the sufficient condition is C m alpha has to be less than zero, which is dC m upon the d alpha have to be less than zero.

Or in other words C m 2 - C m 1 upon alpha 2 - alpha 1. For each positive change in alpha there should be negative change in C m. That is there should be a pitch down moment when there is an increase in angle of attack, right. Okay. So now let us consider this wing. Let us consider this particular wing. It is a rectangular wing, right. So it is just a wing you know.

There is nothing attached to this. It is just a wing, simple wing straight wing. So if we consider this wing itself as an aircraft right? So the total lift of this aircraft is nothing but lift of the width, is it not? And whatever the moment produced by the wing is nothing but the moment produced by the aircraft itself, right. So by the way, where this moment will act? About the C.G here, right, is it not?

It is about the C.G. That is how we defined. We considered a body axis system with its origin at the C.G. And about each of those axes, which is passing through the origin, we have three different moments starting with rolling moment about the x axis, pitching moment about y axis and yawing moment about z axis, right. So if I simply throw this know, if I simply throw this what do you expect?

What do you expect this wing to do? It is just flipping is it not? So if I do this again it just flips, flips multiple times, right. So the position from which we started right is not able to maintain the same altitude or same state we can say. So we have released it from a level state in a equilibrium condition. Because it is not moving it is not rotating and it is static, right.

So I am throwing this at a particular equilibrium state, but it is not able to maintain. It is flipping, right. So such a motion is, so we can closely imagine this as a unstable state of this particular C.G location, right. So if I have to make this fly then I have to satisfy these conditions, right. C m naught has to be greater than zero, C m alpha has to be less than zero. Now what is this C m? It is pitching moment.

And this moment is acting about C.G, right? So now first let us look at where is this C.G. How can I find? So if I can balance this at a given location, I am taking a chord wise location, right. So let us consider I am taking this particular chord and I am trying to balance this at this chord wise location. So this is almost at this particular location, right.

The C.G is at this particular location. See, now it is balanced, right about this particular location C.G is balanced, I should say. Yeah. So this is the location. So almost mid chord. Almost, not mid chord I should say. Yeah. So close to mid chord. So this is the location at which the current C.G location is. So the moment about the C.G location is making it unstable, right.

We will see why it is making it unstable in the first place, right. So if I somehow can alter the C.G. I should know I do not know now, I do not have the information, whether should I take the C.G behind or should I take the C.G head to make to satisfy this particular condition C m alpha. So in order to derive those conditions, let us first look into the contribution of a wing towards stability for a given UAV, right.

So let us start with that. So let us assume from now we will try to represent the wing while dealing with the stability by the side view, just this particular aerofoil will draw a particular side view of this wing and then the characteristics of this like the lift drag and aerodynamics, moment about aerodynamic center will talk about the rest of this like characteristics of this wing, right. And they are represented at the aerodynamic center which we have discussed earlier.

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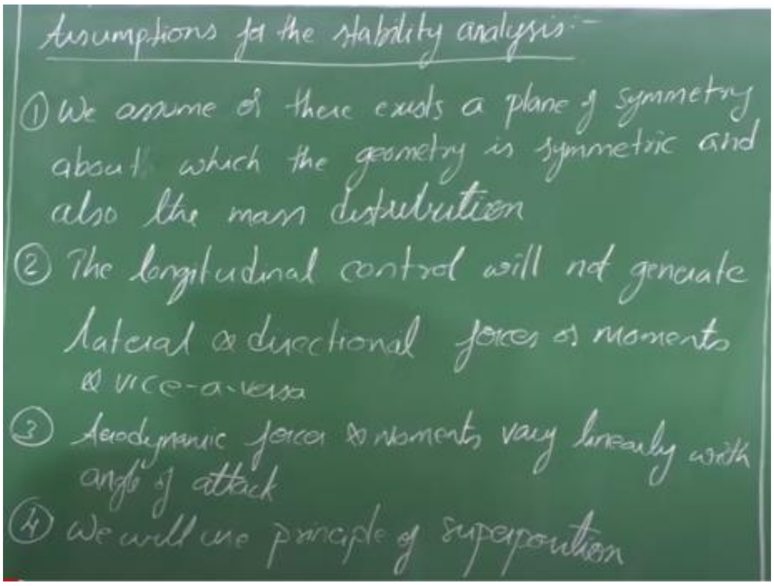

So before going to this, I would like to postulate this assumptions that we are considering for the rest of this analysis or analysis or say you can say the criteria for this entire stability analysis these are the following assumptions where the first one is we assume or there exist a plane of symmetry. So about which the geometry is symmetric and also the mass distribution.

Yeah and also the mass distribution, okay. So we have a plane of symmetry. For example, if I take this wing say at the center right at the mid portion of this span I can say this particular left side wing is symmetric right, is symmetric is it not? Or the left side wing is a mirror image of the left side wing is a mirror image of right side wing if I consider a plane here.

If I cut this wing at the mid portion of the span with the vertical plane right. So about that plane, the left side of the wing is mirror image of right side of the wing. Not just in geometry, but also in mass distribution. That is what it is here, right. The second thing is so longitudinal controls will not generate lateral directional and directional forces or moments. So what do you mean by that?

Let us say so the longitudinal control we know is an elevator. Let us say this is my wing and say there is a tail in the back, right. So if I deflect the elevator, that is only going to produce the change in the longitudinal forces, which is the lift, overall lift of the aircraft is going to change and then the moment as well. But it is not going to produce any rolling moment or yawing moment or any side force right, the force along y axis, and vice versa, right.

And yeah, so or the lateral, for example, if there is an aileron deflection, if you deploy ailerons that are those are not going to produce any yeah longitudinal moments like it is not going to produce any pitching moment or it is not going to alter the overall lift of the aircraft, just because of the deflection, right. That is the same case as a same case with the rudder as well, right.

And then the third assumption is so aerodynamic forces and moments vary linearly with angle of attack, right. So that is what we assume a linear variation of aerodynamic forces and moments throughout this analysis, right. And we will consider this description or these derivations, what we are going to come up with will be valued only for linear range of angle of attack, okay.

And the final assumption is, so we will use principle of superposition, right. Yeah, so we will use principle of superposition where the forces the total forces acting on a UAV or an aircraft will be sum of the forces produced by the independent components and the same as same for moments. Let us say, if this is my this is my wing, and this is my tail, let us assume that right.

So this these two together when attached forms a aircraft and we assume it as a rigid body, right. And then the total forces acting on this aircraft is due or the moments acting on aircraft is due to the forces and moments created by this wing, and also forces and moments created by this tail. So we will have a vectorial sum of this, right forces and moments from the individual components.

So these are the four assumptions that we are going to consider, right for our derivations here, right. Now let us look at the wing alone contribution. So I will try I will just erase this part. So I hope you have noted down these assumptions.

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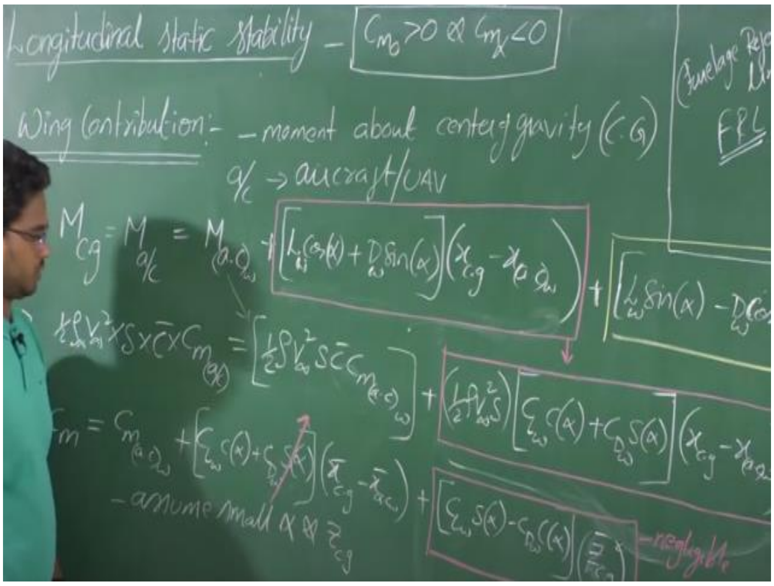

So from now I would like to deal this it in two halves, right. So why because we have bit lengthy equations during this derivations, right. So for a longitudinal static stability, we know C m naught has to be greater than zero and C m alpha has to be less than zero. So we know these two conditions are necessary and sufficient conditions here.

And also we have one important thing is aerodynamic forces and moments they vary linearly with angle of attack and we have a plane of symmetry and also we use the principle of superposition. At the same time, we assume that the longitudinal controls are not going to affect lateral directional dynamics, okay. So let us look at wing contribution. So what do you mean by this wing contribution.

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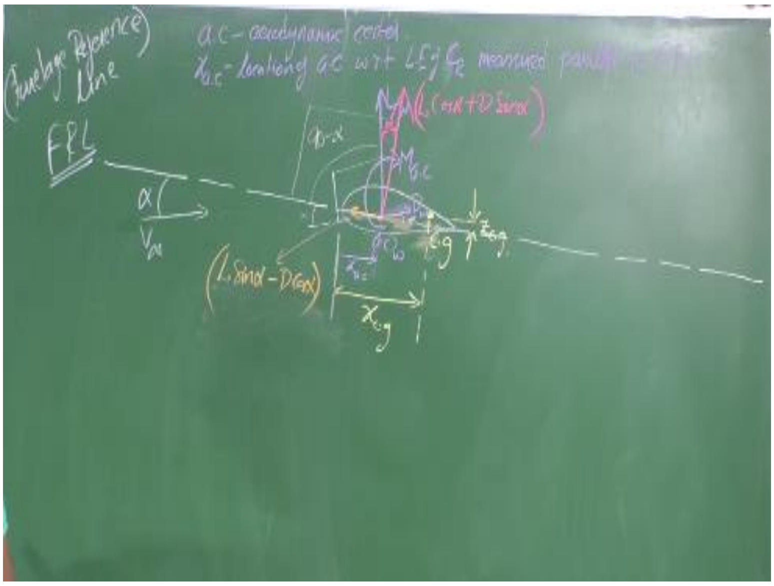

So let us consider the aircraft itself is a wing, right and we have mounted this wing with respect to fuselage reference line. For the time being, we assume that this wing chord coincides with mean sorry fuselage reference line. Let us say this is my wing and this is my chord, corresponding cord which is coinciding with the fuselage reference line. So the FRL here stands for fuselage reference line, right.

Let us say this is this aircraft is moving, which is with wing alone is moving at a velocity V infinity, which is making an, and this with respect to this fuselage reference line it is making an angle of attack alpha, right. So what else we have? So we have this object, right? We have this body which is wing, wing alone here, right. So we represented this wing. So what else do we represent, do we need to represent?

So in order to characterize this wing we need to define lift drag and pitching moment for the longitudinal case about a given reference point, right. If you define that completely, then we will be able to completely characterize this aerofoil or say wing at the same time, right. So let us assume with respect to leading edge, right this with respect to leading edge of this root chord, let us assume the aerodynamic center is located at a distance.

Let us say this is my aerodynamic center a.c, a.c here stands for aerodynamic center. You can take it down, a.c stands for aerodynamic center. So a.c subscript w stands for aerodynamic center of the wing here right. So let us say this aerodynamic center is located at a distance x a.c x a dot c subscript, x a.c is the location of aerodynamic center with respect to the leading edge of the root chord measured parallel to the fuselage reference line.

So what is this? x a.c is the location of aerodynamic center, right with respect to leading edge of C R of root chord right, leading edge of root chord, okay. This is what we emphasize. That is the reason why we solved three examples yesterday. So C R leading edge of the root chord measured parallel to FRL. So is it visible? Okay, yeah. Okay, now we know what is aerodynamic center, which is x a.c measured from the root chord here.

So at aerodynamic center, yeah so say if this is your free stream velocity then lift will be acting perpendicular to this, right which is L w is a lift of the wing is it not? At the same time you have drag of the wing acting along this free stream. So say this angle so this acting along the free stream direction and say we have a moment about aerodynamic center M a.c.

So pitching moment assuming that the y axis is into the board. So the positive pitching moment will be pitch up motion right. So this is the moment about aerodynamic center is represented by M a.c about this aerodynamic center here. So is that all or do we have something else as well? Now this is an object right with some mass. So it has center of gravity, right.

So we are talking about the a dynamic system is it not? When you throw it, so the resultant forces and moments will be acting about this center of gravity, am I correct or not? So we have to talk about C.G as well. Let us say the C.G of this object is somewhere here or say somewhere here. Say this is my center of gravity, okay. So we are now neglecting the z component, z offset.

So we are now not complicating the system, we assume that the C.G is along the fuselage reference line here. Now that is a pretty good assumption is it not? So the because the lateral distribution of mass is not much here, right. So we can and the contribution from the or say okay, you can assume so there is an offset in the C.G right. Let us do that part as well.

So let us say this is my wing and this is the aerodynamic center on this chord line and the C.G is at an offset with respect to this FRL, okay. So this is my z c.g, right. So this is my offset. And this is my offset here. That is my z c.g. Leading edge of the root chord? Let us assume this as x c.g, right. Again measured parallel to yeah so this is x c.g. So this is the location of center of gravity.

So this particular point is the location of center of gravity. And z c.g is the offset there and x c.g is the. So this is z c.g is a vertical offset with respect to fuselage reference line. And x c.g is the longitudinal offset here or horizontal offset along parallel to the fuselage reference line with respect to the leading edge of this root chord, right. Okay, got it? Now yes, these are the required variables to go ahead with this wing contribution analysis, right.

So for a wing, you have lift and drag produced with a wing about the aerodynamic center. And so yeah, the same can be represented at the aerodynamic center associated with the moment about aerodynamic center, right. Now, so the moments happen about C.G, because it is a free it is a rigid body right? If you throw a rigid body so the moment happen about the center of gravity, right.

So that is, so with respect to the center of gravity, we will now see when it flies, what happen to the moment, right. So moment about C.G. So first thing is moment about C.G, center of gravity C.G of this wing alone configuration. Let us say that moment about C.G as M c.g, which is equal to moment of the entire aircraft, a by c here talks about entire aircraft here.

So is equals to so a by c, a by c stands for aircraft or UAV, entire aircraft or entire UAV here, right. Now if I want to find the moment about C.G, right. So now I have to so in the body frame here, so now I have to resolve this lift and drag along fuselage reference line and perpendicular to fuselage reference line. Why because I have the distance x c.g measured parallel to fuselage reference line as well as z c.g measured perpendicular to the fuselage reference line.

Am I correct or not? So this is my perpendicular to fuselage reference line, right. These are the resultant forces which are acting perpendicular to fuselage reference line, okay. So there will be a contribution from lift as well as drag, right. So and then this is the direction in which this is the direction for forces right, which are acting parallel to the along the fuselage reference line. Am I correct or not? Now, so perpendicular to fuselage reference line what do you have? So this is separated by an angle alpha, right. So the offset is angular offset is angle alpha here. Why because L is perpendicular to V infinity, right.

And this direction of this normal force is perpendicular to the fuselage reference line. So these two are making an angle alpha and these two will also make angle alpha. You can do it by simple mathematics, right. It is a simple transformation. So because see, this is this is 90 degrees here, right. So let us say this is my V infinity. So this is 90 degrees, and we know this angle is alpha.

So what will be this angle? 90 minus alpha right? Am I correct? So and we know this fuselage reference line and this pink line, which is perpendicular to fuselage reference line makes an angle 90 degrees right, they are perpendicular. So what will be this angle? So we know this angle and we want to find out this angle, right. So this is 90 and this is 90 minus alpha that is 90 minus 90 minus alpha, which is alpha right?

So that is what alpha is here. Similarly, here alternate angles are equal. So here this is alpha, right. So alpha is a angle of separation between drag and the fuselage reference line here, okay. Now what we have is a drag component and the lift component. So the total forces acting perpendicular to fuselage reference line is L cos alpha plus D sin alpha, right.

So these are the two forces acting perpendicular to fuselage reference line. And the so the forces acting along this let us say this is the positive direction, right. So let us say this is z c.g. And positive direction of x is towards the nose of the aircraft. Here towards the leading edge of the root chord. So let us say this is my positive direction and positive direction z forces will be in the downward direction.

So I have a component L sin alpha acting in this direction. So along this particular direction what I have is L sin alpha and minus, is a negative direction in which it is acting right, minus D cos alpha, okay. If I consider this as the forward direction and I consider this as positive, so then L contributes lift of the wing contribution is the positive contribution here L sin alpha minus D cos alpha.

D is acting in the along the free stream direction, which is in the opposite direction. A component of it will be D cos alpha, which is in the negative direction according to my convention, right. So this is L sin alpha minus D cos alpha, right. So now so I have one force perpendicular force and an offset between C.G and the A.C right. So this is the momentum for this perpendicular force, right.

So that will create a pitch up moment here, am I correct or not? So before that the moment about of this entire aircraft what I have is moment about aerodynamic center of wing, right. Am I correct? Plus the perpendicular force, which is L cos alpha plus D sin alpha. It is contributing towards pitch up, right. So that is why it is positive contribution right, times the momentum between C.G.

And see these forces are perpendicular forces acting at A.C here right? The pink which is represented by this pink vector right, which is L cos alpha plus D sin alpha multiplied by this momentum will contribute towards a positive pitching moment here. And the momentum is x c.g. I know x c.g. I know x a.c. So if I subtract x a.c from x c.g I will have the corresponding momentum x a.c of wing, right x a.c of wing, is it not?

At the same time I have a horizontal component here, right. So the component which is parallel to fuselage reference line is L sin alpha minus D cos alpha. So multiplied by this vertical offset will give me the corresponding moment here. So again this force is acting towards the nose. So about C.G this is going to contribute towards pitch up motion, right.

Because y axis is into the board and the pitching moment is about y axis, correct. So the curl of my fingers will give you the positive pitching moment. So the positive pitching moment is nose up. And this force right is trying to give me a nose up motion here. So this is like plus the force L sin alpha b cos alpha right, multiplied by what is the corresponding momentum is z c.g, right?

It is clear, is it not? This is because of the horizontal force and the vertical offset of C.G with respect to fuselage reference line. There is a moment contribution because of the vertical offset by the horizontal force, right. And it is a positive moment and we have the vertical like vertical force and horizontal offset with respect to C.G. So this entire component is going to contribute towards a pitch up motion again, right.

And we have moment about aerodynamic center. This is clear I guess. So now let us further non-dimensionalize this. What I have is half rho infinity V infinity square is the reference velocity of the wing is considered as a reference velocity of the aircraft multiplied by reference wing area is considered as reference wing area of reference area of this aircraft right.

So times C bar is the mean aerodynamic chord of the wing is considered as reference chord length here. Reference length for the longitudinal direction times C m of the entire aircraft. C m a.c here. C m a by c is nothing but C m of the entire aircraft here.

So half rho V square is the dynamic pressure times the area on which it is acting will get us the dimension of force multiplied by the momentum, which is the reference length here will get us the moment dimensions, right moment or torque dimensions here. And multiplied by the non-dimensional moment coefficient, aerodynamic moment coefficient, which is equals to so this is for wing, right.

So this is this moment is due to wing which is half rho V infinity square C bar times C m a.c of a.c of wing. C m about aerodynamic center of wing. This is this first term. So plus so we have sorry, I am sorry. Please make a correction here. This is L w and D w. Similarly, this is from the wing right? So L w D w and similarly, what we have is the lift contribution towards this normal force to fuselage reference line, okay.

Plus half rho V square S. This lift and drag are generated from the wing. So we know V infinity is at the wing and S is the reference area of the wing times half rho V square S times. So C L of wing times cos alpha. I am writing cos alpha as C alpha here, right. And similarly, C D of wing times sin alpha. I am writing sin alpha as S alpha times x c.g minus x a.c of wing, right.

This is this second, the complete second term is given by this okay. Plus similarly, what I have is half rho V square S times C L of wing times S alpha minus C D of wing times C alpha multiplied by z c.g, okay. So this is this corresponds to this particular second term. So if I non-dimensionalize this, if I divide this equation entire equation by half rho V square S C bar what I have is C m of aircraft.

I simply write it as C m now. C m itself is the C m of entire aircraft, which is equals to C m a.c of wing plus C L of wing times cos **cos** alpha plus C D of wing times sin alpha multiplied by x bar c.g minus x bar a.c of wing right, okay. And then plus what I have is C L of wing times **cos** sine alpha minus C D of wing times cos alpha multiplied by z bar c.g, okay.

So let us, see this contribution is very less. Further the z c.g value will be very less and assuming a small angle of attack, right if you assume small angle of attack sin alpha will be alpha multiplied by z c.g small alpha and alpha is already small z c.g will be further small. So we can neglect this particular contribution and then including this if you can notice C D w right.

So drag is far less than the lift coefficient here right, at least one tenth of one tenth order. So it will be 0.0 kind of. So multiplied by this small number will further make it small. So we can neglect this vertical contribution. So we are now neglecting this contribution. So and also if you assume small angle of attack, so the C D alpha so cos alpha is 1 and sine alpha is alpha.

So C D times alpha will be further a small quantity. So we can also neglect this particular quantity. So how we are neglecting? We assume small alpha and z c.g, right. So the offset will is very small comparatively.

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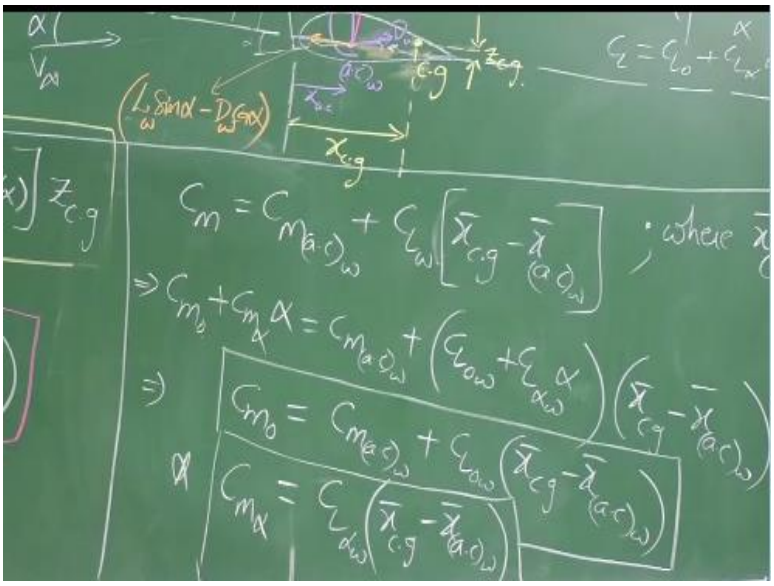

This by neglecting these components what we have is what we end up with is C m is equals to C m a.c of the wing, aerodynamic center of the wing plus C L of wing times x bar c.g minus x bar a.c of wing where are all x bar subscript anything is equals to S subscript upon C bar, right. Let it be x c.g, x c.g bar is equals to x c.g upon C bar. x a.c is x a.c bar is x a.c upon C bar, right.

And z c.g bar is also like z c.g upon C bar, fine? So we now got what is the pitching moment equation and we know the pitching moment variation if we have a linear variation, then we can express the aerodynamic coefficient, moment coefficient here as a linear function of angle of attack by C m naught plus C m alpha into alpha. So this is a straight line equation right, y is equals to m x plus c.

Where m is the slope here x is the alpha in our case, x is alpha in our case and C m naught is the offset or we can say y intercept there, right. Do you remember this plot and also remember our assumption, one of our assumptions where the aerodynamic forces and moment have a linear variation with angle of attack. So what do you mean by linear variation and angle of attack?

We can write this as a straight line equation y is equal to m x plus c to where so say this is my C m and this is my alpha. So the slope is that C m alpha and this y intercept is C m naught, right. So y is equals to m x plus c is represented as C m naught plus C m alpha into alpha here. Is equals to. So C m a.c of wing plus what is the C L of wing? How can we express in the linear regime?

It is a total lift coefficient of the wing variation of lift coefficient with angle of attack is it not? So in general so this is your C L alpha okay. So in the linear regime you have C L naught here, right. And you know C L alpha D C L upon D alpha. So similarly, C L is equals to C L naught plus C L alpha into alpha. y is equals to m x plus c.

So C L of wing will be C L naught of wing plus C L alpha of wing times alpha of wing multiplied by the corresponding momentum which is x c.g bar non dimensional momentum which is x c.g bar minus x a.c bar of wing. So you can see this is non-dimensional. X bar is non-dimensional. X is dimensional. Yesterday we derived in terms of x right? We solved those three examples in terms of x, which is a dimensional quantity.

But here x bar is a non-dimensional quantity. We divided this by a characteristic length. For longitudinal case it is aerodynamic, mean aerodynamic chord and for lateral directional case it will be span of the aircraft, okay span of the wings here span of aircraft is span of wing. So here we have arrived at an expression where which is at least having the terms that we are interested in, is it not?

What are we interested in? To figure out what will be C m naught and C m alpha of this UAV is it not? So yeah, here the UAV is having only wing, right. So we want to know whether this wing is giving C m naught positive and C M alpha negative. If that is the case, then we can say wing is contributing towards static stability, longitudinal static stability, right. So now let us look at so by comparing the coefficients of alpha, so alpha of wing is nothing but alpha.

We assume the wing is not in does not face any interference here and further there is no incidence angle of the wing with respect to fuselage reference line. So in that case the angle of attack at the wing is equals to angle of attack of the aircraft itself, right. So where alpha is the aircraft angle of attack, which is also equivalent to wing angle of attack here in this particular case, right.

So by comparing the constants and coefficients, what I arrive is C m naught is equals to C m a.c of wing plus C L naught of wing times x bar c.g minus x bar a.c of wing, right. Take this to here. Otherwise I end by comparing the constants and coefficients of alpha, right. C m alpha is equal to C L alpha of wing times x bar non dimensional momentum, which is x bar c.g minus x bar a.c of wing, okay.

So we have arrived at the required two conditions. C m naught condition and C m alpha condition. These are the two conditions.

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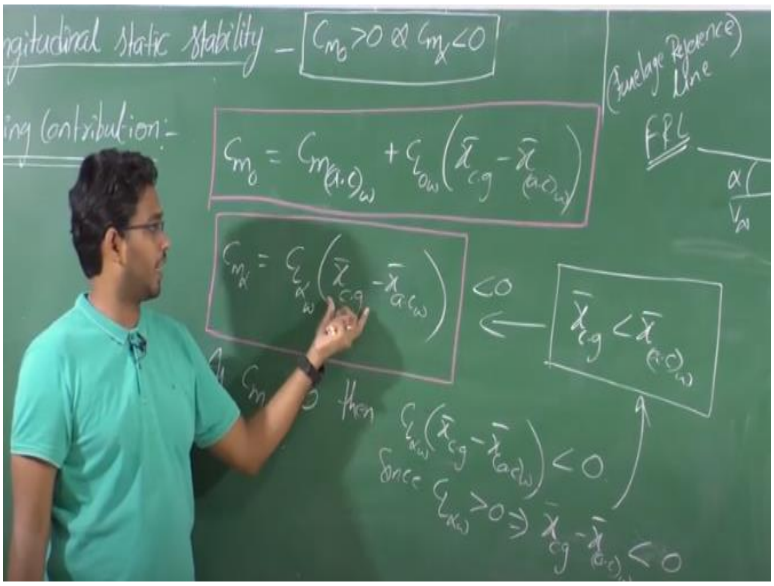

So what we have arrived it as C m naught is C m a.c of wing, right. So it depends upon moment about aerodynamic center. So it depends upon the type of aerofoil you are using, whether it is a symmetric aerofoil, whether it is a cambered airfoil or a reflex aerofoil, right plus C L naught of wing, right times x bar c.g minus x bar a.c of wing, am I correct?

So this is what C m naught is depend upon. And C m alpha is equals to C L alpha times of wing times C m alpha of the entire aircraft is because of C L alpha of the wing times the corresponding x bar c.g minus x bar a.c of wing, right. So okay, now we want C m alpha has to be negative, right, is it not? How can we make C m alpha negative? So these are the conditions for longitudinal static stability, which we have derived.

C m alpha has to be negative. So we have C m alpha as C L alpha times of wing times the momentum. When this can happen? If let us look at this first condition. If C m alpha has to be less than zero, then C L alpha wing times x bar c.g minus x bar a.c of wing should be less than zero. Am I correct?

Since C L alpha is since C L alpha wing is always positive right, as we increase the angle of attack, we have the lift will increase irrespective of whether it is a symmetric aerofoil, cambered aerofoil, or a reflex aerofoil. Am I correct or not? So C L alpha is always positive, which implies x bar c.g minus x bar a. c of wing must be greater than must be less than zero.

So this particular quantity has to be negative here, which means the x c.g should be less than x a.c of the wing here. So that is what the conclusion is. From here what I can say is x bar c.g has to be less than x bar a.c of wing, right. So this is the conclusion to make this C m alpha less than zero. Okay, is it clear?

What I mean here, the first the sufficient condition we try to arrive only when this particular quantity here has to be less than zero, which means the C.G distance has to be less than aerodynamic center, right? Less than the, so it should lie before the aerodynamics center. What do you mean by x c.g less than x a.c. So the C.G is here, it should be less than x a.c, which means it has to lie ahead of this aerodynamic center.

Am I correct? So if I place this aerodynamic center, sorry center of gravity ahead of this aerodynamic center, I do not have much control with aerodynamic center. Once I have, I have a wing, I have an aerodynamic center fixed to it, right? It is I cannot variate. If I have to variate I have to change the entire planform geometry. So the variable that I have in my hand is the C.G right.

So here if I have to achieve this particular condition, or if I have to achieve the stability, just like normal pendulum, right so it oscillates about the equilibrium point. If I have to achieve such a strategy, then I have to make the C.G, force the C.G to lie ahead of this aerodynamic center, which is the variable in my hand, right? So let us see how that variable how can I claim that that variable is in my hand?

Before just proceeding to that, let us also look at the other condition where C m naught has to be positive, right? If C m naught has to be positive so let us first talk about this. If moment about aerodynamic center right of wing, this is zero for a symmetric.

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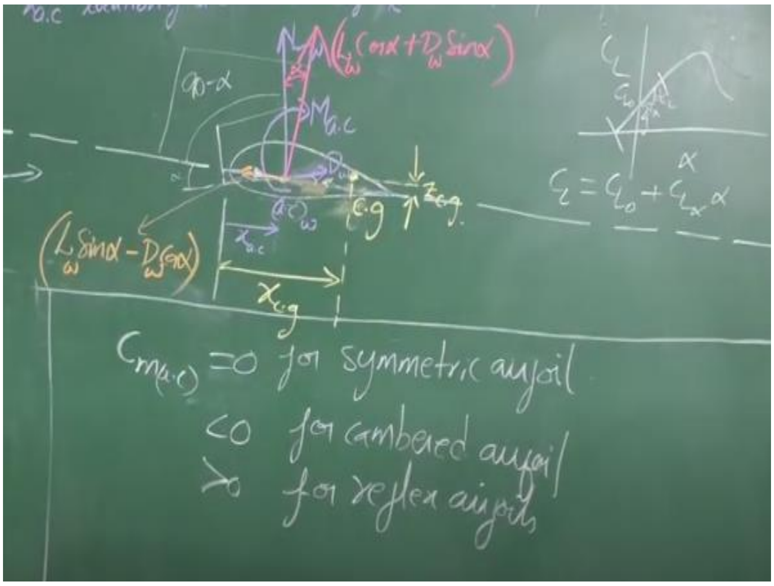

So C m a.c of wing is equals to zero for symmetric aerofoil. So it is less than zero for cambered aerofoil, positively cambered aerofoil. So I am taking that liberty. So cambered aerofoil is positively cambered aerofoil right in my opinion. So and is greater than zero for reflux aerofoils, which we have discussed earlier right, okay.

When you talk about symmetric aerofoil, right irrespective of this first thing is that the C.G has to be less than aerodynamic center, right. So this condition we have arrived irrespective of whether the wing is made out of cambered aerofoil or a reflex aerofoil or a symmetric aerofoil, right. So irrespective of that geometry of the wing, wing cross section, we have to make sure that C.G is head of the aerodynamic center for a wing alone to be which means the difference is negative, right.

So C L naught if you consider a symmetric aerofoil this is zero and this is zero, C m naught is zero, right. Am I correct? So and if you consider a cambered aerofoil this particular quantity is negative. C L naught is positive for a cambered aerofoil positively. But x c.g minus x a.c is negative. If I want to fly this in a stable configuration, then this C.G has to lie ahead of the aerodynamic center, right.

This particular quantity is negative. So two negative quantities contribute towards C m naught negative. That means, the aircraft will immediately dip down to a negative trim angle of attack. Yeah, if I consider a reflex aerofoil this will be positive and this contribution will be negative again. Even for reflex aerofoil C L naught is positive.

But you have to make sure you have to then operate at a very less value of this x c.g minus x a.c so that the resultant C m naught remains positive for you for the UAV, right. So is it clear? So for me to make this C m naught positive, I need to first look into whether which kind of cross section I am using. Whether it is cambered, symmetric or reflex. If it is symmetric, there is no point C m naught will automatically become zero, right.

Then I have to continuously create an additional moment from somewhere else. So let us consider this right. So this is made out of symmetric aerofoil. If you can see this particular wing and boom, right, so I have a wing and boom setup here. So this is made of symmetric aerofoil here, right?

So then C m naught will be zero altogether, because the C m about aerodynamic center, which is one fourth of this chord, which is at this particular location that we have marked here, right. So this is the aerodynamic center location. And it is a symmetric aerofoil. You can see the distribution is even about this particular chord line, this is the chord line you can see, right?

And yeah, this becomes zero. Now I may not be able to generate a moment from this particular surface. So what I need to do is I have to place some other surface either ahead or behind to make sure that this particular wing alone configuration along with that particular additional surface produces C m naught positive, right? That is what right and if I locate a particular surface, it is at an angle here.

If I place that surface, lifting surface at certain angle, and place it keep it like that, right? Then it will produce an additional moment about C.G that may help to achieve this C m naught positive, okay. Got it? Alright. So that is where we need a tail right? For a cambered aerofoil we need a tail. Even for a cambered aerofoil let us say, if I take a cambered aerofoil here C m a.c is negative and this particular quantity is negative.

So I need C m naught to be positive. But how can I achieve by adding one more surface which can make this force this system to produce that C m naught positive, right. That we will see how to achieve that particular C m naught. Now coming back to this example. So I have this. Yeah, wing. It is a cambered airfoil. You can see it is a cambered aerofoil, right. I just happened to pick it up from my lab, right.

So I do not even know the details but it looks as if it is more a flat bottom and then maybe a six series aerofoil I guess here, right. So yeah. Or a 5 series aerofoil. So initially, do you remember? So when I throw, when I was trying to throw this object right, so it is trying to flip, right? Is it not? It is trying to flip. Why because here, we witness that the C.G is almost close to the midpoint of the chord, mid chord of this wing right.

Maybe at least behind the quarter chord. So this is the location where I am able to balance this object, right. So that means the that is this so the C.G is close to this particular location here. If you notice, the C.G is close to this particular location. So that means the C.G is behind the aerodynamic center. So we know aerodynamic center yesterday we calculated for different configurations, right?

So aerodynamic center, so for a rectangular wing, we do not have any taper here. So it is located at close to quarter chord of this particular cross section, is it not? So quarter chord, maybe close to this location, right? So but the C.G is behind this quarter chord, because of which your wing alone is behaving unstable, right, is it not? So let us just try to shift this C.G before this aerodynamic center, right okay?

And try to fly this again, and see whether our derivation makes sense or not? No, whatever so it is highly non-intuitive, is it not? Why because see, we were able to say this condition that C m alpha has to be less than zero only after deriving this particular equation. This is where we were able to make a decision that, okay, the C.G has to be ahead of aerodynamic center.

We cannot directly looking at this aerofoil and I know where is the aerodynamic center I cannot comment about what should be the C.G location directly, is it not? That is what we discussed just before starting this derivation. So now let us try to shift the C.G ahead of this aerodynamic center and fly this again, and see whether this derivation makes sense or not.

Yeah, so now we have added some weights here in the front of this wing, just to make sure that the C.G is shifted ahead of this aerodynamic center. Now let us see how to locate this aerodynamic center. Let us quickly I need I need some support. Yeah, Prabijit can you please help me? Yeah. So I need some support here. I am taking I am using this scale to measure what is a chord length here in the first place.

So the chord length is about 22.5. So here it is 22.5 centimeters is the overall chord. I am measuring it from root to tip here, right? So it is 22.5. So 22.5 upon 4 is approximately 5.6. Yeah, right. So 5.6 is your location of the aerodynamic center. So if I take 5.6 from the leading edge, right, so towards the trailing edge. So I will be able to find out the aerodynamic center.

So the 5.6 here is close to this marking. So we have a marking here, this is the aerodynamic center. Now let me just see whether the C.G with this addition of weights have shifted, has shifted ahead of this aerodynamic center or not. At least I expect it to be close to this aerodynamic. See, so it is almost close to this aerodynamic center. With the addition of this weight, it has shifted close to the aerodynamic center here.

So this is the aerodynamic center. And the C.G is almost close to the aerodynamic center, right? So I will tell you the reason why I have done that, almost close to reason. So now so can I throw this? But this time, I expect this to fly, because our derivation should hold true. For the derivation to hold true this should fly, is it not? So Prabijit is going to help us.

So we will have a similar demonstration what we had in our previous lecture, yeah lecture series. So I will try to check this out from my hands, right. So Prabijit let us hope this will fly, right? Is it not? I will try to repeat this, right. So right? It is able to fly, is it not? It is not by chance. And let us try this one more time. So I am again trying to throw this with the same altitude.

Yeah, right. It is not at least behaving in a weird way. It is not flipping back, right? It is not behaving unstable. So one more time. This is funny. So perfect. Do you accept? Do you accept that this equation holds? Yeah, fine. Good. It is come here. So without this weights so the C.G yeah the C.G is almost close to this point. So which is aft the aerodynamic center here.

See we have located with this blue pen, we have located the aerodynamic center with this blue pen. So my finger points the current C.G location without any additional load. This is just a wing alone without any additional load. So it is aft the aerodynamic center here. So when I try to throw the body, right in this particular configuration, so this will try to flip, right? It is behaving unsteady.

So let me do it again. I will try to throw it with a higher velocity. So I cannot help, right. Yeah, this equation holds true then.