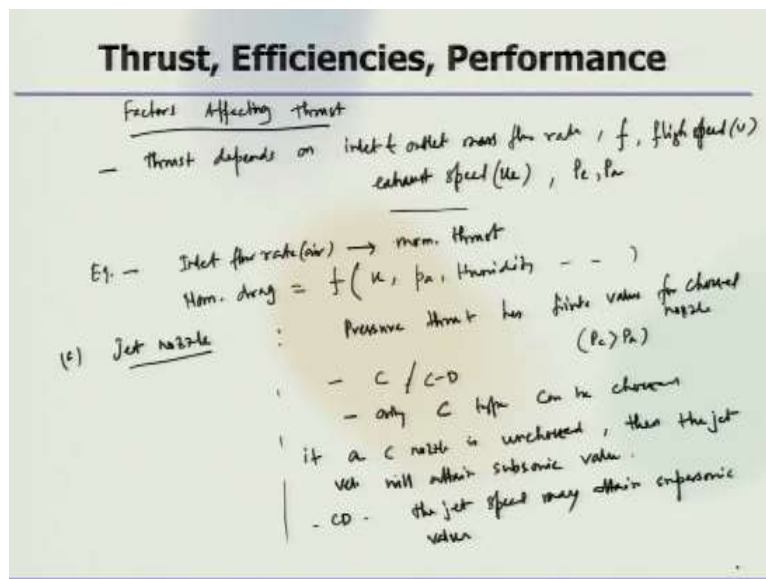


Introduction to Airbreathing Propulsion
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Lecture – 13
Introduction to Gas Turbine Engines (Contd.,)

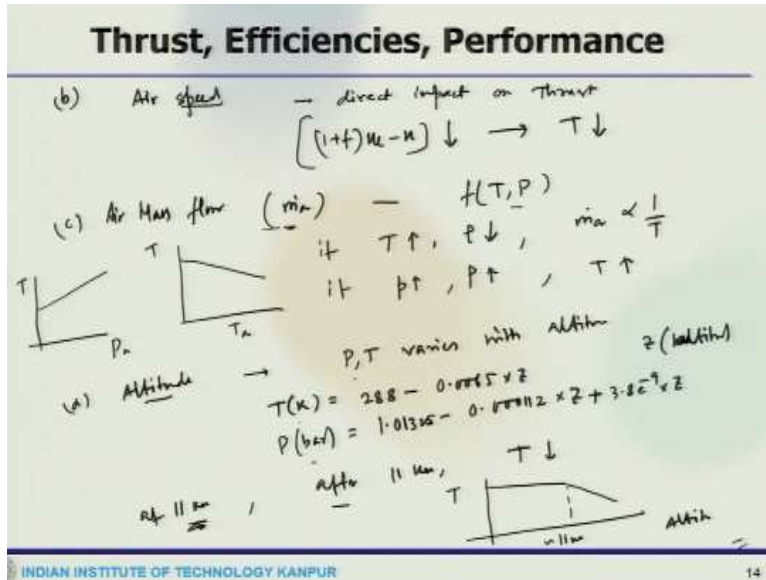
So welcome back to this discussion on the performance, thrust and efficiencies and all these. So, we are initially derived the equation for the thrust and that for turbojet and then we have also looked up on the equation for turbofan, but for example turbofan we have not gone through the detail derivation, but we will do while we will just go to the analysis of particularly turbofan. Now, one can look at the textbooks and he will find it and then we are looking at the different factors that impact the thrust and we just listed upon few of these factors here.

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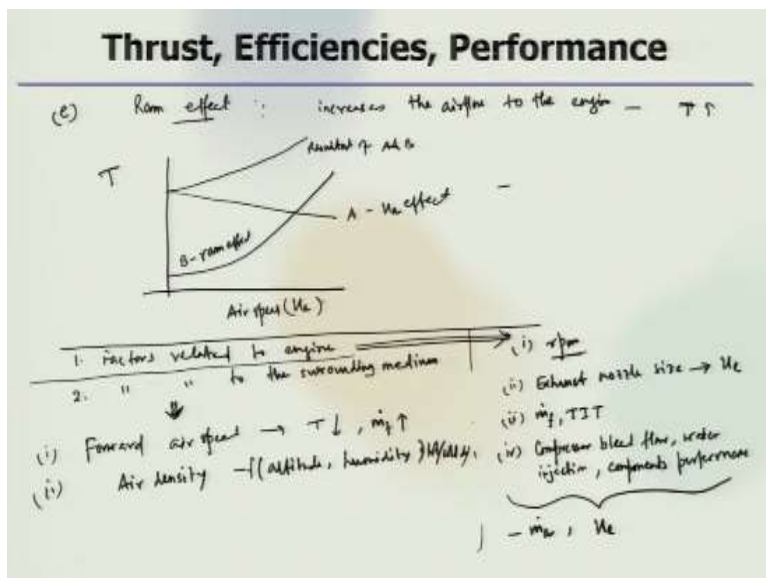
Like just to recall we started with jet nozzle that we discussed.

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And then we have talked about air speed and then air mass flow rate how that is connected with the pressure and temperature then the altitude obviously altitude variation that will have serious impact.

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Then we can have let us say another one in that continuation is the Ram effect. So, this is something that movement of the aircraft which is relative to the outside air which causes some air to be ramped into the engine inlet duct. So what it does the ram effect actually increases the airflow to the engine which in turn increases the gross thrust. However, it is not easy that ram effects combine two factors.

Namely the air speed increase and in the same time increase in the pressure of the air and the air flow into the engine because these are combined things. So, as you can see that increase of

the air speed reduces the thrust and so one can see some schematic of that so let us say this is u_a or air speed and this is what happens this is obviously thrust with this. So, one could go this could be A u_a effect this is B – ram effect and the resultant could be something like this.

So this is resultant of A and B. So, the increase of airflow will increase the thrust this is what sketched here in the B and this resultant curve which is the curve let us say C which is the effect of A and B. So the increase of thrust due to ram becomes significant as the air speed increases which will compensate the loss in thrust due to pressure at high altitude. So this is also ram effect is also important due to the reduced pressure at high altitude.

And also it is important in high speed fighter aircraft, but modern subsonic jet powered aircraft fly at subsonic speed and higher altitude to address this, make use of this ram effect. So, finally if we bring down all this effect or factors that affecting the thrust primarily into two group maybe something like factors group 1 could be factors, but related to engine and second one could be factors related to the surroundings or surrounding medium.

Now, in the first group so one can see what includes in the first group that includes number one the rotational speed or RPM which influence both the compressor both of the compressor pressure ratio and the turbine work. So the rotation speed is going to impact. Number two exhaust nozzle size because that is going to affect the exhaust velocity. So which in turn will have effect on thrust.

So then there could be fuel flow rate, turbine inlet temperature. So, these are effect the combustor heat generation because how much fuel is pumped into the system so accordingly how much reaction has taken place which will allow to have the temperature rise due to the energy release in due to combustion and something like that. So, now compressor bleed or compressor of bleed flow from water injection and also components on performance.

So components performance individual component performance of that which had also lead to the specific work. So these are the some of the important factors that actually affect the which are engine related factor and the second category which is the factors that related to the surrounding medium. So, in a nut shell if you see all this finally impact the \dot{m}_a air mass flow rate and jet velocity which is u_e .

Now when we look at the factors which are related to surrounding medium this could include like forward air speed which essentially leads to decrease in thrust and more fuel consumption increases. Second, air density and this guy is influenced so this is affected because of function of rather altitude, humidity, hot or cold day. So this air density has impact on finally the thrust.

So, it can increase the thrust or it can reduce the thrust depending on the situation of the condition. So, I mean if you sum up things together then one can see how this factors actually affect the thrust production in all this.

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Thrust, Efficiencies, Performance

Performance Parameters	Turbojet Performance
(a) Thermal efficiency (η_{th})	Rate of production of Propellant
(b) Propulsive efficiency	Kinetic energy = $m_e \frac{u_e^2}{2} - m_a \frac{u^2}{2}$
(c) Propeller "	$f = \frac{m_f}{m_a}$, $m_e = m_a + m_f$
(d) Overall efficiency (%)	$= (1+f)m_a$
(e) Take-off thrust	Total energy consumption rate: $m_f Q_R$
(f) SFC (specific fuel consumption)	$Q_R = \text{heat of reaction of the fuel}$
(g) Aircraft range, Endurance etc.	

$$\eta_{th} = \frac{m_e \frac{u_e^2}{2} - m_a \frac{u^2}{2}}{m_f Q_R} = \frac{[(1+f) \frac{u_e^2}{2} - \frac{u^2}{2}]}{f Q_R}$$

for a given f and forward speed (u), η_{th} increases continuously for decrease in $\frac{u}{u_e}$

K.E. depends on Q_R
 Higher $Q_R \rightarrow$ high $m_e \frac{u_e^2}{2}$ or higher u_e for turbojet/turbofan

Now we would sum up the engine performance parameter that we can categorize or look at some efficiency like thermal efficiency which is designated at eta h. Propulsive efficiency, propeller efficiency, overall efficiency then take-off thrust then SFC which is specific fuel consumption then obviously range and other factor. So we can put it aircraft range, endurance etcetera.

So these are the some of these performance parameter that one can look at and define those performance parameters. Now again let us start with the turbojet case. So, let us take with turbojet performance parameter so what we can first define the thermal efficiency. So, thermal efficiency essentially now they are in turbojet, the rate of production or propellant kinetic energy.

So this is

$$\eta_{th} = \frac{\frac{\dot{m}_e u_e^2}{2} - \frac{\dot{m}_a u_a^2}{2}}{\dot{m}_f Q_R}$$

So that is the rate of change of propellant kinetic energy where

$$f = \frac{\dot{m}_f}{\dot{m}_a}$$

$$\dot{m}_e = \dot{m}_f + \dot{m}_a$$

which is nothing but

$$\dot{m}_e = (1 + f)\dot{m}_a$$

So, total energy consumption rate which is $\dot{m}_f Q_R$. So Q_R is nothing, but is a heat of reaction of the fuel. So, kinetic energy of the exhaust – incoming air that is the propellant kinetic energy.

So, the thermal efficiency is defined

$$\eta_{th} = \frac{[(1 + f)\frac{u_e^2}{2} - \frac{u_a^2}{2}]}{f Q_R}$$

So that is how it can be defined. Now for a given f and let us say forward speed u. Thermal efficiency increases continuously for decreasing u / ue. So, one can see that it increases with the decrease u / ue ratio. Also kinetic energy depends on QR if you have higher QR which lead to the high value of $\frac{\dot{m}_e u_e^2}{2}$ or higher ue for turbopump and turbofan so that increases that thing.

Also for turbo pump the power output is the shaft power.

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Thrust, Efficiencies, Performance

Power output = shaft power (P_s) , $\eta_{th} = \frac{P_s}{f Q_R}$

Propulsion efficiency (η_p)

Thrust Power delivered to vehicle $\pm T u$

$$\eta_p = \frac{T u}{\frac{\dot{m}_e u_e^2}{2} - \frac{\dot{m}_a u_a^2}{2}} = \frac{T u}{\dot{m}_a [(1+f)\frac{u_e^2}{2} - \frac{u_a^2}{2}]} \quad \dots (1)$$

In General, $f \ll 1$ & $(P_2 - P_1) A_e \ll \dot{m}_e u_e - \dot{m}_a u_a$ (neg. term)

$$\eta_p \approx \frac{(u_e - u) u}{\frac{\dot{m}_e u_e^2}{2} - \frac{\dot{m}_a u_a^2}{2}} = \frac{2(u/u_e)}{1 + (u/u_e)} \quad \dots (2)$$

If $\frac{u}{u_e} = 1$, $\eta_p = 1$ (max.) , However, $T = \dot{m}_a (u_e - u)$
 & $\frac{u}{u_e} \rightarrow 1$, $T \rightarrow 0$
 To produce finite thrust, infinite large engine
 will be needed \rightarrow To maximize η_p .

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So the power output is nothing, but the shaft power which is let us say P_s then one can write thermal efficiency as $P_s / f Q_R$. So that means what is the power output and what is the energy

input, so essentially output by input. Now the second one can look at propulsion efficiency which is termed as η_p . What we can look at the thrust power which is delivered to the vehicle is Tu .

So, propulsion efficiency is Tu that is the thrust power delivered to the kinetic energy. So one can write

$$\frac{Tu}{\dot{m}_a \left[(1+f) \frac{u_a^2}{2} - \frac{u^2}{2} \right]}$$

Now in general f is small and $(P_e - P_a) A$ also that is small. This is the pressure term and this is the momentum term. So if we take this into account then we can write

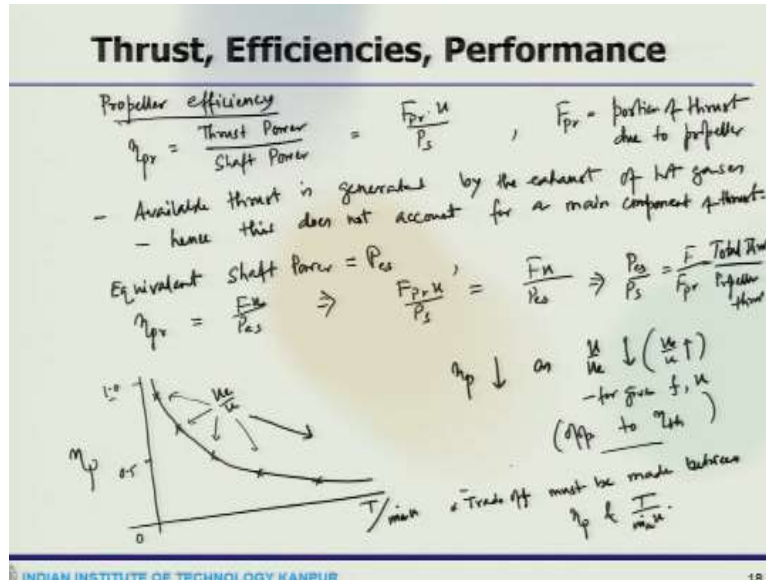
$$\eta_p = \frac{(u_e - u)u}{\frac{u_e^2}{2} - \frac{u^2}{2}}$$

$$\eta_p = \frac{2(u/u_e)}{1 + u/u_e}$$

So, if (u/u_e) is 1 η_p is 1 that is the maximum one can have.

However, thrust is $\dot{m}_a(u_e - u)$ if u / u_e tends to 1 that means thrust tends to 0. So that means it clearly says that to produce finite thrust infinite large engine will be needed. So which clearly says and message this is not a good idea as a designer to maximize η_p . So it is designed that may not be considered or may not be consider by setting an objective of maximizing the propulsion efficiency because that can lead to a really disaster situation.

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Now, propeller efficiency so the propeller efficiency shaft power is converted to thrust power of a moving aircraft by propeller. So η_{pr} would be the ratio of thrust power to shaft power which is

$$\eta_{pr} = \frac{F_{pr} u}{P_s}$$

So, F_{pr} is the portion of thrust due to propeller. So the available thrust is generated by the exhaust of hot gases. Hence, this does not account for a main component of thrust.

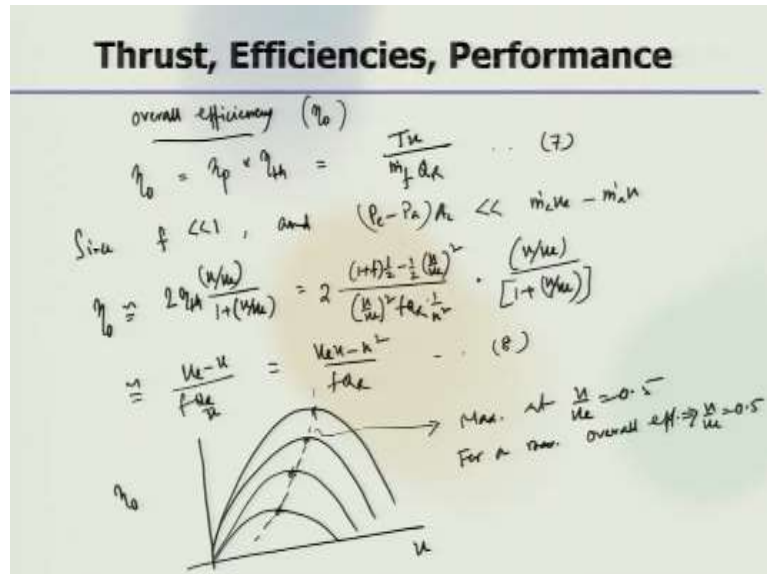
So, one can define an equivalent shaft power so equivalent shaft power can be defined which is P_{es} and then with that we can have $\eta_{pr} = \frac{F_{pr} u}{P_{es}}$ so which is total thrust by propeller thrust. So, you can just look at some situation this is $T / m \dot{a}$ and this is η_p and this is how it is going to vary. There could be different point on this curve and these are correspond to different values of u_e / u .

So these are different values. So obviously this is the direction that increases. So this is small then once it increases this is how propulsion efficiency actually changes and somewhere the curve here is 50% somewhere it is 100% I mean just to give you an idea how. Now propulsion efficiency versus this thrust per unit inlet air momentum with u_e / u as a parameter what we see η_p decreases as u / u_e decreases which means u_e / u increases.

So that means u_e / u when increases in this direction this is for a given f and u . This is clearly opposite characteristics to thermal efficiency. So this is opposite to thermal efficiency. So as a

designer it is a good idea to make a trade off which must be made between η_p and $T / m \dot{a}$. So, I mean you can see how things are quite complicated that it is not that if you change one parameter or I mean they are so non-linear lead to some extent couple.

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Now we look at the overall efficiency which is η_o . Now

$$\eta_o = \eta_{th} * \eta_{pr}$$

so that would be

$$\eta_o = \frac{Tu}{\dot{m}_f Q_R}$$

Now, since f is very small and $P_e - P_a$ that pressure component is small compared to momentum. So, overall efficiency could be

$$\eta_o = 2 * \eta_{th} \frac{\left(\frac{u}{u_e}\right)}{1 + \left(\frac{u}{u_e}\right)}$$

Now if you plot this like this side is u and this is overall efficiency then the curves looks like this, like that and these are the point which are going to so this is maximum at u / u_e is always 0.5. So this is how the overall efficiency also changes. Now just a quick note for a maximum overall efficiency that is u / u_e is 0.5 and that correspond to this different values of this things.

So this is how the different performance parameter I mean f is the efficiency parameter like thermal efficiency then propulsion efficiency and propeller efficiency and finally the overall efficiency and they are obviously linked with each other and it is not quite easy that once you

change one parameter the others are not going to be affected. Now, similarly we will look at the performance parameter for turbofan in the next session.