Introduction to Airbreathing Propulsion
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Lecture – 16
Introduction to Gas Turbine Engines (Contd.)

So looking at this different performance parameters and we have reached to a stage where we are looking at range and how the range is kind of affected due to the overall efficiency and exit velocity. Now the other parameters which could be important to look at is the endurance and how the aircraft actually get affected due to other forces while climbing or descending because when you have looked at the range that time one of the biggest assumption that we have made that there is no climb up or there is no descent from the level flight. So, assuming the level flight we have looked at that, but let us continue with the discussion from there and then.

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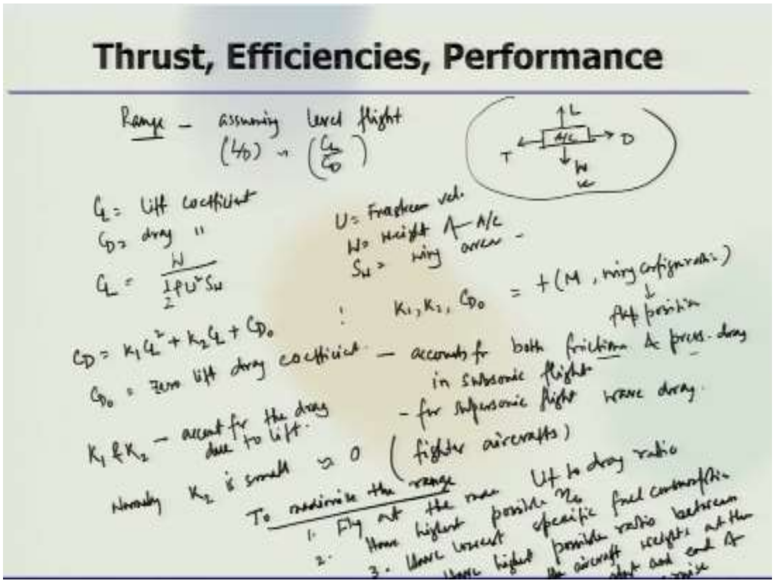

So, again we say that what we have looked at is the range assuming level flight. So, that is which means the level flight talks about that if this is your aircraft then thrust is balanced, this is weight and this is lift. So this is what you call by the level flight where all this thrust, lift, drag there would be. Now we have already looked at L/D ratio and all this. Now this could be also can be represented in the range formula where we had used as L/D

We can also replace that C_L/C_D and something like that. Now C_L is essentially the lift coefficient and C_D is the drag coefficient. Now, this we can represent like we can write C_L is let us say

$$C_L = \frac{W}{\frac{1}{2}\rho U^2 S_w}$$

where U is the upstream velocity or freestream velocity whatever you call it. The upcoming freestream velocity, W is the weight what we mentioned here weight of aircraft.

S_w is the wing area and the drag coefficient can be calculated like

$$C_D = K_1 C_L^2 + K_2 C_L + C_{D_0}$$

where constant K_1, K_2, and C_{D_0} are the typically they are the function of flight Mach number and wing configuration. So when we talk about wing configuration that means this includes the flap position like that. Now C_{D_0} is the zero lift drag coefficient. So this accounts for both frictional and pressure drag in the subsonic flight.

And for supersonic flight it is the wave drag and the term $K_1$and K_2 they account for the drag due to lift. Normally K_2 is small and almost equals to 0 in most of the fighter aircrafts. Now already we have seen just to maximize the range what we need to do, we have to fly at the maximum lift to drag ratio. Second have highest possible overall efficiency, three have lowest specific fuel consumption that means TSFC or SFC. Four have highest possible ratio between the aircraft weights at the start and end of cruise. These are the things that we have already seen.

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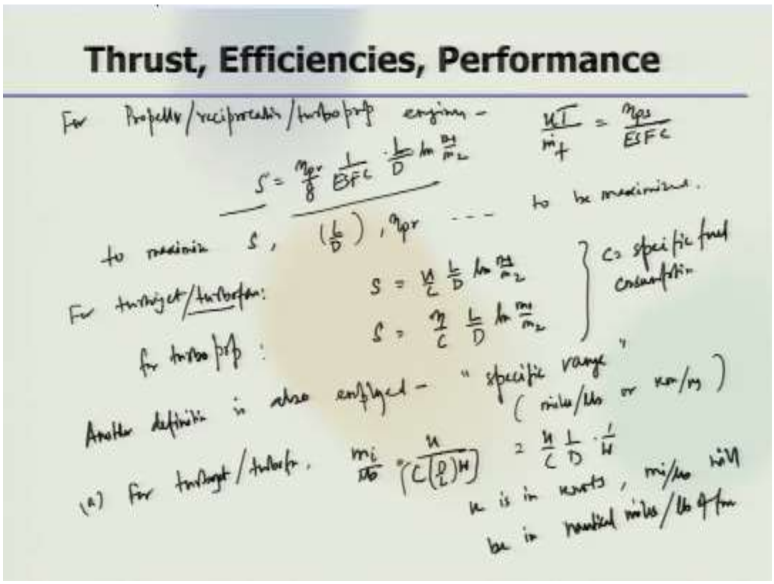

Now for a turboprop engine let say for propeller or reciprocating or turboprop engines similar analysis can be followed. Now what we can write there is that

$$\frac{uT}{\dot{m}_f} = \frac{\eta_{pr}}{ESFC}$$

So this will get us the range

$$S = \frac{\eta_{pr}}{g}\frac{1}{ESFC}\frac{L}{D}\ln\frac{m_1}{m_2}$$

Obviously, again if you look at this expression to maximize the range this lift to drag ratio to maximize S, $\frac{L}{D}$, propeller efficiency.

So these are also to be maximized. So, typically the airlines they normally follow the following Breguet relation let say for turbojet or turbofan they say

$$S = \frac{u}{C}\frac{L}{D}\ln\frac{m_1}{m_2}$$

where and for turboprop. They usually say

$$S = \frac{\eta}{C}\frac{L}{D}\ln\frac{m_1}{m_2}$$

Here C is specific fuel consumption and similarly one can define also write the specific range or another definition is also employed that is specific range so that also can be used.

So that is like which given like either miles per pound fuel or kilometer per kg of fuel consumption something like that. So for example, turbojet and turbofan, this mi / per pound of mass flow rate would be

$$\frac{m_i}{lb} = \frac{u}{C\frac{D}{L}W}$$

which is

$$\frac{m_i}{lb} = \frac{u}{C}\frac{L}{D}\frac{1}{W}$$

where u is in knots and mi per lb would be in nautical miles per pound of fuel.

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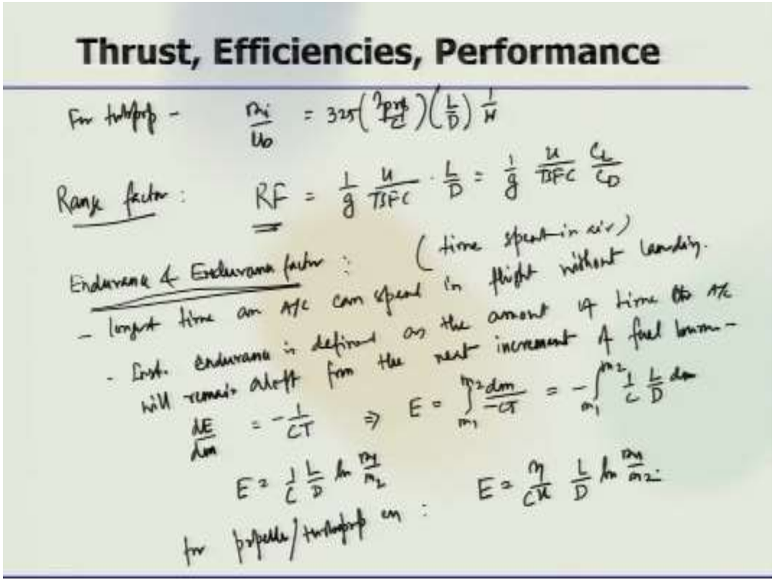

And in a similar fashion for turboprop you can write

$$\frac{m_i}{lb} = 325\,\frac{\eta_{prop}}{C}\frac{L}{D}\frac{1}{W}$$

So these are some of the commonly used by the airliner that how much nautical miles per fuel this can go. So one can define this range factor and which is say that range factor is

$$RF = \frac{1}{g}\frac{u}{TSFC}\frac{L}{D} = \frac{1}{g}\frac{u}{TSFC}\frac{C_L}{C_D}$$

So, the minimum fuel consumption for a distance it occurs at the condition where range factor is maximum, so that is what it is.

Now we can look at the other factor like endurance and endurance factor. So endurance is something the way it is defined if that time spent in the air is of interest and not the distance travel then one is concerned with the endurance. See the range when you talk, we talk about how much fuel is consumed to go certain distance. Now, here we are talking about the time which is spent in air rather the distance time spend in air.

So, endurance is the longest time an aircraft this is the longest time an aircraft can spend in flight without landing. So the maximum endurance of an aircraft or the time aloft which refers to the condition that requires the maximum fuel power. So the maximum fuel consumption for a time flight occurs when the endurance factor is also maximum we can see that.

So, the instantaneous endurance is defined as the amount of time the aircraft will remain aloft from the next increment of fuel burned. So we can express that dE let us say E is endurance is

$$dE = -\frac{1}{CT}$$

C already we have defined, that is a specific fuel consumption then what we can get

$$E = \int_{m_1}^{m_2} -\frac{dm}{CT} = -\int_{m_1}^{m_2} \frac{1}{C}\frac{L}{D}\, dm$$

So

$$E = \frac{1}{C}\frac{L}{D}\ln\frac{m_1}{m_2}$$

Now for propeller or turboprop engine this

$$E = \frac{\eta}{Cu}\frac{L}{D}\ln\frac{m_1}{m_2}$$

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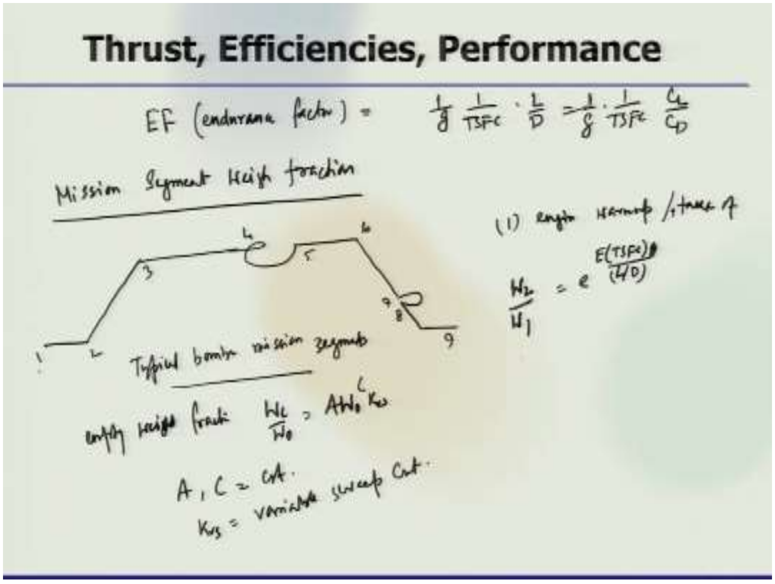

And we can define the EF which is endurance factor as

$$EF = \frac{1}{g}\frac{1}{TSFC}\frac{L}{D} = \frac{1}{g}\frac{1}{TSFC}\frac{C_L}{C_D}$$

So that is what the endurance and endurance factor is. Then we can also talk about mission segment weight fraction that is another. So for or like let us say there is a mission flight segment mission profile like 1, 3 it goes like this, go to that and go like that.

So this is 1, 2, 3, 4, 5, 6, 7 let us say here comes like this it goes like this, this is 8, 9. It is an schematic of a typical bomber mission segments. So mission segment weight fraction which is the various mission segments or legs are numbered which this 1 is the start of the mission and this means 1 is essentially the engine warm up and take off. The remaining legs are sequentially numbered since then.

And for let say if a typical passenger aircraft it would have been 1 would be again warm up and take off, 2 would be climb, 3 would be cruise, 4 would be loiter and 5 would be land. So like this and now the weight fraction or the loiter weight fractions can be found from the endurance factor so which one can express as

$$\frac{W_2}{W_1} = e^{\frac{E(TSFC)g}{L/D}}$$

where E is the endurance.

Now, it is noteworthy to mention that the empty weight fraction is given by the relation. So empty weight fraction which is given by

$$\frac{W_e}{W_0} = AW_0^C K_{VS}$$

So this A, C these are constants which are provided by the manufacturer of the aircraft and K_{VS} is the variable sweep constant.

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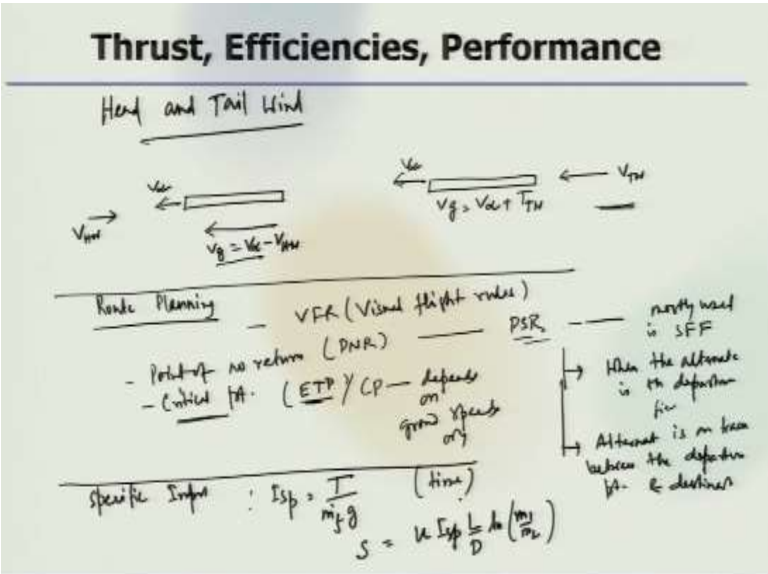

Now another thing is that what happens when you have head and tail wind. So, when aircraft is in flight it is moving relative to the body of air through which it is flying. So the maintaining the accurate ground track is not easy unless there is no wind at all which is a very rear occurrence. So, the pilot has to adjust to compensate for the wind and in order to follow the ground track.

Let us say if this is the aircraft then if this is the head wind and this is how the aircraft is moving then

$$V_g = V_\infty - V_{Hw}$$

So the pilot will calculate the head winds to fly for each leg of the trip prior to the departure and using the forecast wind conditions or direction and speed supply by the any metrological authorities for that purpose. So these are typically acquired and updated several times per day.

But obviously there is unpredicted whether that is different situation, so this is the tail wind. So the general aviation pilot will often make use of either this flight computer and a typical slide rule or purpose design to navigate to calculate the initial headings. So if there is a head wind then he has to adjust this, if there is a tail wind then the adjustment is made accordingly. So, as we can see if the flight time will depend on both the desired cruising speed of the aircraft and the wind.

So, if there is a tail wind then you can see the flight time will be shorten because this will actually provide the favorable thrust whereas if you have a head wind then it will increase the flight time so that is how it is going to affect. Now also another factor which one has to count for that is the route planning. So the pilot planning a flight under VFR which is visual flight rules which usually use an aeronautical chart of area published for use of pilots.

So the map will depict control airspace Radio Navigation Aids and the air fills prominently as well as any hazard to the flying such as mountains etcetera. So it also includes sufficient ground details like town, roads, wooded areas to aid visual navigation. So the pilot usually choose a route taking care to avoid control air space that is not permitted for the flight like restricted areas, danger areas and so on.

And the chosen route is spotted on the map and the lines drawn are called the track. During flight planning two points to be taken into consideration, one is point of no return, second is the critical point. These points we will talk in a moment. Now the point of no return so PNR is also known as PSR point of safe return. This is the farthest point along the track that the pilot can fly towards a destination.

And have sufficient fuel to divert to an alternate with safe reserves on arrival. In other words, one can say that it is pilot last chance to assess his or her prospect of a successful approach and landing at its destination and decide whether to go on or to divert. If any doubt exist he or she must divert to the alternate. So, there are number of methods which can be used to calculate PNR or PSR and one of the most favored uses what is called specific fuel flow SFF.

So this is mostly used is SFF. These are calculated by driving the plan cruise fuel flow by expected ground speeds towards the destination and towards the alternate field. Normally, two cases one can consider, one is when the alternate is the departure field and second option is the alternate is on track between the departure point and destination. So this is how point of no return or PSR is accounted for and the other one is the critical point.

So similarly equal time point refer to a ETP also critical point is the point in the flight where it would take the same time to continue flying straight or track back to the departure aerodrome. So ETP is not dependent on fuel, but wind giving chance is ground speed out form and back to the departure aerodrome. In NIL wind condition the ETP is located half way between the two aerodromes.

But in reality it actually shifted depending on the wind speed and the direction. While distance to a PNR is dependent on fuel availability and fuel flow, the critical point the CP is independent of fuel consideration and is based on mostly depends on ground speeds only and the other one which is called the specific impulse, so specific impulse is defined as that ISP which is thrust per fuel flow rate with these things.

So the quantity enters directly into the calculation of the fractional weight change of the aircraft during flight. So, we can also write back the range with

$$S = uI_{sp}\frac{L}{D}\ln\frac{m_1}{m_2}$$

So this is also equally applied for both rockets and aircraft. So specific impulse the unit will be in time. So, these are the things and now we have looked at different factors and all these things. Now we have slightly more discussion is left over for this endurance and all this that we will do in the next class.