**Introduction to Airbreathing Propulsion**
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**Lecture - 57**
**Axial Turbine (contd.,)**

So, let us continue the discussion on axial turbine. So, we are looking at different stage efficiency and their estimation and we have looked at the TS diagram and the different losses and this is where we stopped that with an argument.

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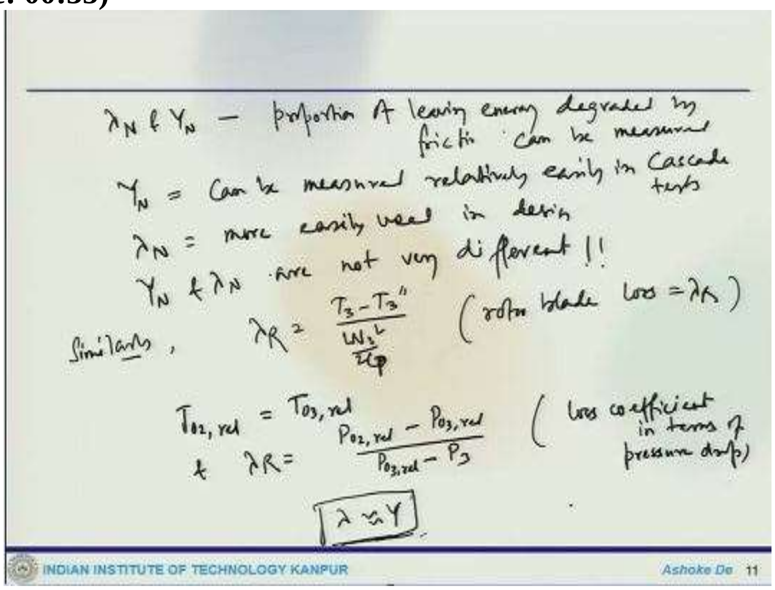


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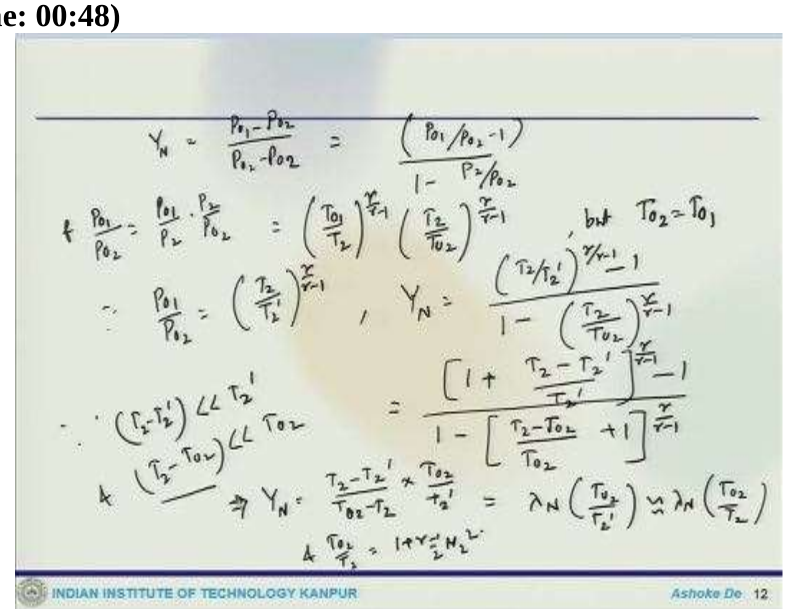


That lambda could be equals to y. Now, how we can do that, we can look at this

$$Y_N = \frac{(p_{01} - p_{02})}{(p_{02} - p_2)} = \frac{\left(\frac{p_{01}}{p_{02}} - 1\right)}{\left(1 - \frac{p_2}{p_{02}}\right)}$$

and

$$\frac{p_{01}}{p_{02}} = \frac{p_{01}}{p_2}\frac{p_2}{p_{02}} = \left(\frac{T_{01}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_{02} = T_{01}$$

since

$$\frac{p_{01}}{p_{02}} = \left(\frac{T_2}{{T_2}'}\right)^{\frac{\gamma}{\gamma-1}}$$

Then we can write

$$Y_N = \frac{\left(\left(\frac{T_2}{{T_2}'}\right)^{\frac{\gamma}{\gamma-1}} - 1\right)}{\left(1 - \left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}}\right)} = \frac{\left[1 + \frac{T_2 - {T_2}'}{{T_2}'}\right]^{\frac{\gamma}{\gamma-1}} - 1}{1 - \left[\frac{T_2 - T_{02}}{T_{02}} + 1\right]^{\frac{\gamma}{\gamma-1}}}$$

Now, one can give you an argument

$$(T_2 - {T_2}') \ll {T_2}'$$

$$(T_2 - T_{02}) \ll T_{02}$$

If that is the case, then the term which the air inside the bracket can be expanded using binomial expansion and now we can write like

$$Y_N = \frac{T_2 - {T_2}'}{T_{02} - T_2}\frac{T_{02}}{{T_2}'} = \lambda_N \frac{T_{02}}{{T_2}'} \approx \lambda_N \frac{T_{02}}{T_2}$$

So, this is although this approximation is not very accurate, but fair enough to use and we have

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2$$

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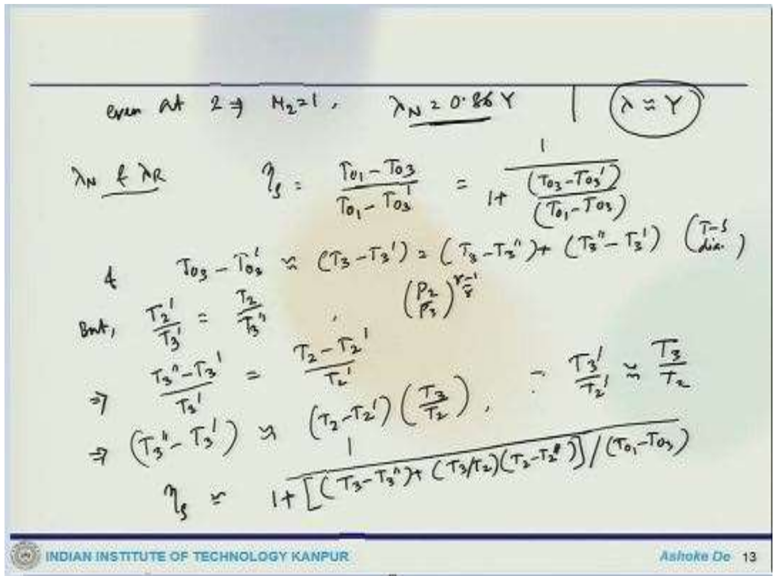


So, even at 2, where $M_2 = 1$, $\lambda_N$ is roughly 0.86Y. So, they are that is why the comment is that they are quite close enough values. Now, this $\lambda_N$ and $\lambda_R$ they can be also correlated to the isentropic efficiency of the stage $\eta_s$. now how do you know

$$\eta_s = \frac{T_{01} - T_{03}}{T_{01} - T_{03}'} = \frac{1}{1 + \dfrac{T_{03} - T_{03}'}{T_{01} - T_{03}}}$$

So, and what we can write from that TS diagram that

$$T_{03} - T_{03}' = T_3 - T_3' = (T_3 - T_3'') + (T_3'' - T_3')$$

So, this we can write from the TS diagram. Now, what we have that

$$\frac{T_2'}{T_3'} = \frac{T_2}{T_3''} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

So, we can further simplify the terms like

$$\frac{T_3'' - T_3'}{T_3'} = \frac{T_2 - T_2'}{T_2'}$$

then

$$T_3'' - T_3' = T_2 - T_2' \frac{T_3'}{T_2'}$$

$$\frac{T_3'}{T_2'} = \frac{T_3}{T_2}$$

we will write

$$\eta_s = \frac{1}{1 + \frac{\left[(T_3 - T_3'') + (T_2 - T_2')\frac{T_3}{T_2}\right]}{(T_{01} - T_{03})}}$$

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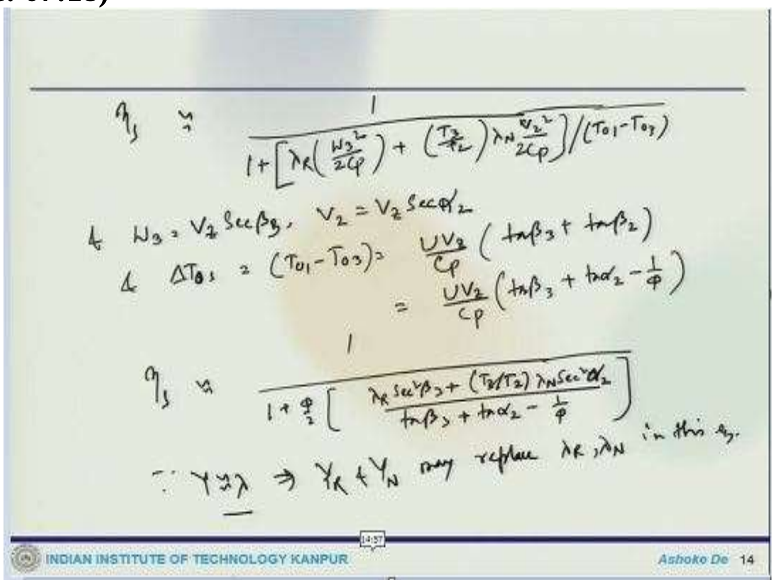


So, this further we can do

$$\eta_s = \frac{1}{1 + \frac{\left[\lambda_R\left(\frac{W_3^2}{2C_p}\right) + \lambda_N\left(\frac{W_2^2}{2C_p}\right)\frac{T_3}{T_2}\right]}{(T_{01} - T_{03})}}$$

So, if you look at all these illustrations that we are writing down here, the coming from that TS diagram. Now, what else we have

$$W_3 = V_z \sec\beta_3$$

$$V_2 = V_z \sec\alpha_2$$

and

$$\Delta T_{0s} = (T_{01} - T_{03}) = \frac{UV_z}{C_p}(\tan\beta_2 + \tan\beta_3) = \frac{UV_z}{C_p}\left(\tan\beta_3 + \tan\alpha_2 - \frac{1}{\phi}\right)$$

So, we can rewrite this expression of

$$\eta_s = \frac{1}{1 + \frac{\phi}{2}\frac{\left[\lambda_R \sec^2\beta_3 + \frac{T_3}{T_2}\lambda_N \sec^2\alpha_2\right]}{\left(\tan\beta_3 + \tan\alpha_2 - \frac{1}{\phi}\right)}}$$

Now since Y is $\lambda$, then $Y_R$ and $Y_N$ may replace $\lambda_R$ and $\lambda_N$ in this equation. So, then we can replace these things and get the stage efficiency and all this.

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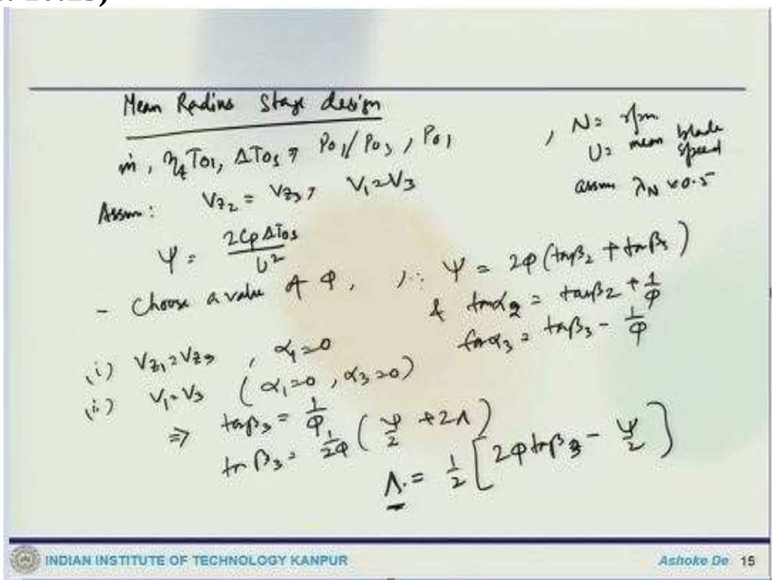


So, the next what we will look at the mean radius stage design. So, mean radius stage design. So, from cycle calculations we have already got the following parameters that $\dot{m}, \eta_t T_{01}, \Delta T_{0s}, {}^{p_{01}}/_{p_{03}}, p_{01}$. Now N is the rotational speed or rpm, mean blade speed is U. So, that is mean blade speed and this mean blade speed is restricted by the rotational criteria and they have to be satisfied. And also we assume some value of lambda N let us say 0.5 which is a reasonable guess.

And also we start with an assumption

$$V_{z2} = V_{z3}$$

and

$$V_1 = V_3$$

So, then we write this

$$\psi = \frac{2C_p \Delta T_{0s}}{U^2}$$

now we choose a value of ϕ and using that for we can calculate the degree of reaction. Now, since

$$\psi = 2\,\phi(\tan\beta_2 + \tan\beta_3)$$

and

$$\tan\alpha_2 = \tan\beta_2 + \frac{1}{\phi}$$

$$\tan\alpha_3 = \tan\beta_3 - \frac{1}{\phi}$$

Now if we consider a single state thereby and we have $V_{z1} = V_{z3}$ So, it has to be single stage and inlet is axial, so $\alpha_1 = 0$ and $V_1 = V_3$. So which means $\alpha_1 = 0$ , $\alpha_3 = 0$.

So that gives us

$$\tan\beta_3 = \frac{1}{\phi}$$

and

$$\tan\beta_3 = \frac{1}{2\phi}\left(\frac{\psi}{2} + 2\Lambda\right)$$

$$\Lambda = \frac{1}{2}\left[2\phi\tan\beta_3 - \frac{\psi}{2}\right]$$

Now, so these the degrees of reaction that increases from root to tip there for small values of this degree of reaction at mean radius must be avoided, because that will mean negative degree of reaction at the root.

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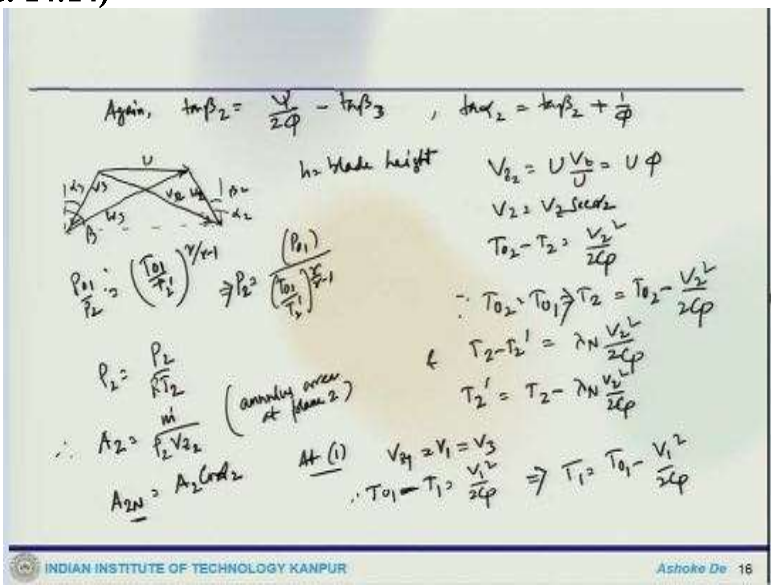


Again, we have

$$\tan\beta_2 = \frac{\psi}{2\phi} - \tan\beta_3$$

$$\tan\alpha_2 = \tan\beta_2 + \frac{1}{\phi}$$

So next we can calculate the density as station 1, 2 and 3. So let us draw the velocity triangle. So this is U $W_3$ $V_3$ so, this is $W_3$ $V_3$ $W_2$, $V_2$. So, this is $\alpha_3$, $\beta_3$, $\beta_2$, $\alpha_2$, it is blade height. Now, what we write here

$$V_{z2} = \frac{UV_z}{U} = U\phi$$

and

$$V_2 = V_z \sec\alpha_2$$

So,

$$T_{02} - T_2 = \frac{V_2^2}{2C_p}$$

Since

$$T_{02} = T_{01}$$

which is

$$T_2 = T_{02} - \frac{V_2^2}{2C_p}$$

and what we have

$$T_2 - {T_2}' = \lambda_N \frac{V_2^2}{2C_p}$$

So, we get

$${T_2}' = T_2 - \lambda_N \frac{V_2^2}{2C_p}$$

also we have

$$\frac{p_{01}}{p_2} = \left(\frac{T_{01}}{{T_2}'}\right)^{\frac{\gamma}{\gamma-1}}$$

So, which gives us

$$p_2 = \frac{p_{01}}{\left(\frac{T_{01}}{{T_2}'}\right)^{\frac{\gamma}{\gamma-1}}}$$

So

$$\rho_2 = \frac{p_2}{RT_2}$$

So that is how we get the density. Now, from the area

$$A_2 = \frac{\dot{m}}{\rho_2 V_{z2}}$$

these are 2. So, this is an annulus area at plane 2. So the

$$A_{2N} = A_2 \cos \alpha_2$$

So, this guy is not because it is not a repeating stage.

We are assuming that $V_1$ is actual and this together with the assumption the other assumption that we have made here. So, at one or $V_{z1}$ is $V_1$, which is $V_3$. So, we get

$$T_1 = T_{01} - \frac{V_1^2}{2C_p}$$

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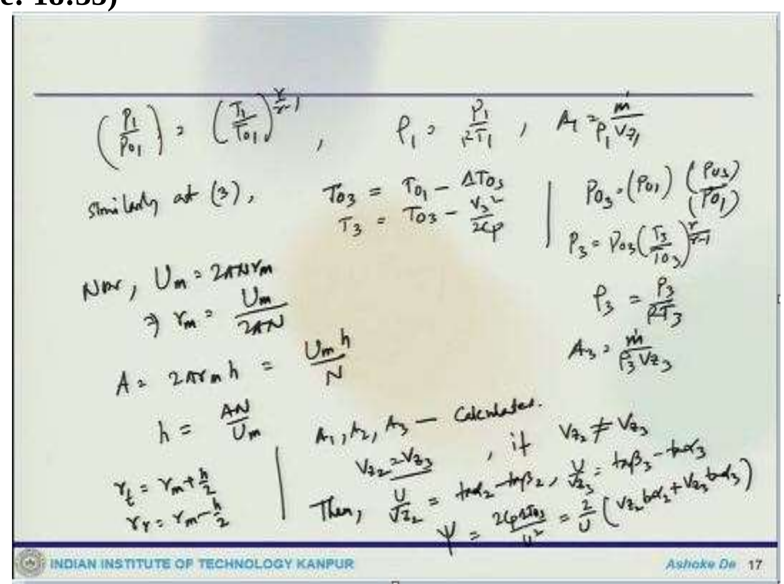


And the relationship of

$$\frac{p_1}{p_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

density we get

$$\rho_1 = \frac{p_1}{RT_1}$$

and area

$$A_1 = \frac{\dot{m}}{\rho_1 V_{z1}}$$

Similarly, at 3 we gave

$$T_{03} = T_{01} - \Delta T_{0s}$$

and

$$T_3 = T_{03} - \frac{V_3^2}{2C_p}$$

So, $P_{03}$ is now use the relationship of

$$p_{03} = p_{01}\frac{p_{03}}{p_{01}}$$

$$p_3 = p_{03}\left(\frac{T_3}{T_{03}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\rho_3 = \frac{p_3}{RT_3}$$

and area

$$A_3 = \frac{\dot{m}}{\rho_3 V_{z3}}$$

Now

$$U_m = 2\pi N r_m$$

so we get

$$r_m = \frac{U_m}{2\pi N}$$

So that is the mean radius that is what we can get.

So, area would be

$$A = 2\pi r_m h = \frac{U_m h}{N}$$

And

$$h = \frac{AN}{U_m}$$

So the tip radius is

$$r_t = r_m + \frac{h}{2}$$

and root radius is

$$r_r = r_m - \frac{h}{2}$$

now all $A_1$, $A_2$, $A_3$ all are calculated. So note that all the relationship we have derived well for

$$V_{z2} = V_{z3}$$

If

$$V_{z2} \neq V_{z3}$$

Then

$$\frac{U}{V_{z2}} = \tan \alpha_2 - \tan \beta_2$$

and

$$\frac{U}{V_{z3}} = \tan \beta_3 - \tan \alpha_3$$

and

$$\psi = \frac{2C_p \Delta T_{0s}}{U^2} = \frac{2}{U} (V_{z2} \tan \alpha_2 + V_{z3} \tan \alpha_3)$$

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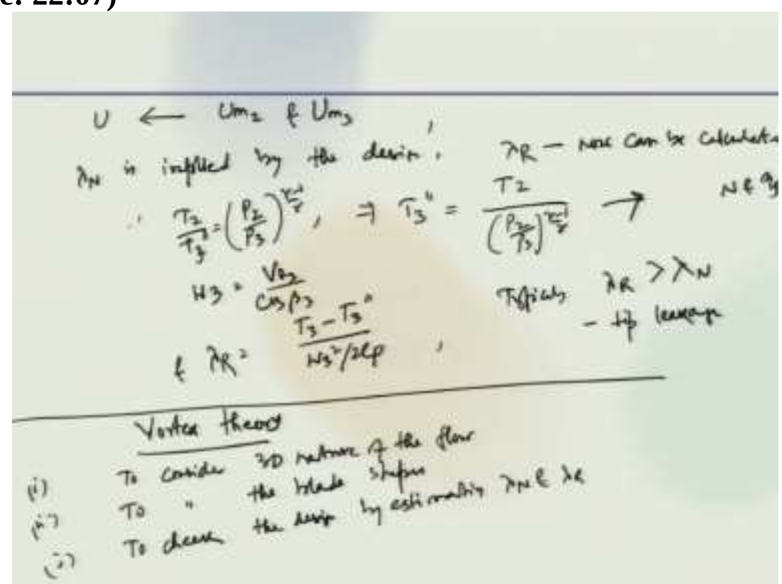


So also in the is the flare is not symmetrical. So, U must be replaced by $U_{m2}$ and $U_{m3}$. For this preliminary design we have taking losses into account via $\lambda_N$ and it is rather than $\lambda_R$ or $\lambda_N$. So, $\lambda_N$ is implied by the design and $\lambda_R$ can now we now can be calculated so we get

$$\frac{T_2}{T_3{}''} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

So, which means

$$T_3{}'' = \frac{T_2}{\left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}}}$$

So, for this preliminary design we have taken losses account buy N<sub>R</sub> $\eta_s$ other than $\lambda_R$ and V.

Now

$$W_3 = \frac{V_{z3}}{\cos\beta_3}$$

and

$$\lambda_R = \frac{T_3 - T_3''}{\frac{W_3^2}{2C_p}}$$

typically $\lambda_R$ is greater than $\lambda_N$ due to duplicate loss in the rotor blades . So, this is how we get this done. Now the next is the vortex theory. So, what is that this is the next step in the design? So, to consider the 3D nature of the flow so far as it affects the variation of the gas angles with radius. So, this is to consider 3D nature of the flow second to consider the blade shapes.

And that is also necessary to achieve the required gas angles and the effect of the centrifugal and gas bending stresses on design third to check the design by estimating $\lambda_N$ and $\lambda_R$ from the result of cascade test suitably modified to take account or take into account the 3D flows.

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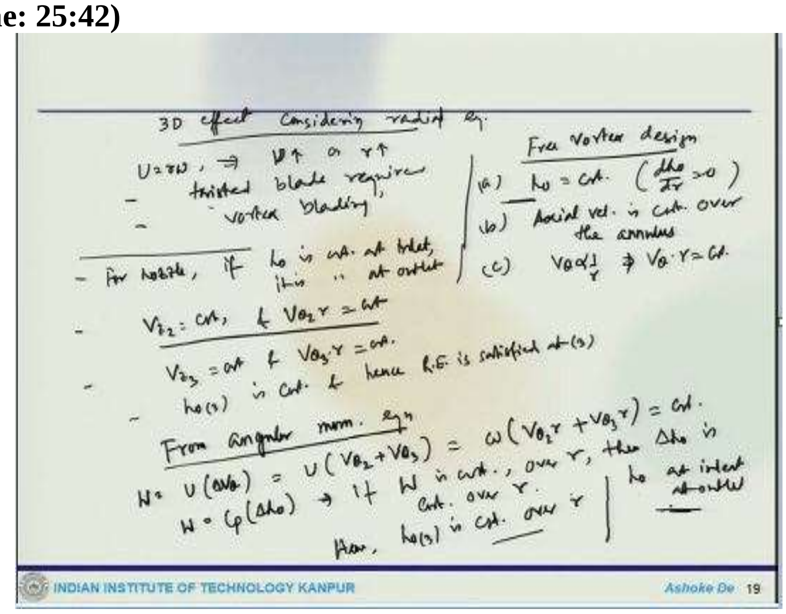


So, when we take the 3 dimensional effect considering radial equilibrium so, we have already seen in compression that $U = r\omega$. So, U goes up as rotational speed r goes up. So, to maintain a smooth flow the blade has to be twisted. So, the twisted blade required second twisted blading designed to take into account the changing gas angle which is called the vortex blading. Now, we can do free vortex design for stagnation enthalpy $h_0$ is constant over the annulus which means

$$\frac{dh_0}{dr} = 0$$

The axial velocity is constant over the annulus and $V_\theta$ is inversely proportional to radius or $V_\theta$r is constant. So, with this assumption radial equilibrium condition is satisfied and this design is called the free vortex design. Now for nozzle the $h_0$ is constant at the inlet. Then it will be constant at the so for nozzle if $h_0$ is constant at inlet, it is constant at outlet, because no work is done in the nozzle. So, again if we design the nozzle blades such that $V_z$ to his constant and $V_{\theta 2}r$ is constant.

So, this means the radial equilibrium theory satisfied at station 2. Similarly, if rotor blades are designed in such a way that these $V_{z3}$ is constant and $V_{\theta 2}r$ into r is constant, then it also satisfied the radial equilibrium theory. So, it can be shown that $h_{03}$ is also constant and hence radial equilibrium theory or radial equilibrium is satisfied at 3. So, what do we write, we write from angular momentum equation what do we write

$$W = U(\Delta V_\theta)$$

which is known.

So, you can write

$$W = U(V_{\theta 2} + V_{\theta 3}) = \omega(V_{\theta 2}r + V_{\theta 3}r) = constant$$

Now, also,

$$W = C_p(\Delta h_0)$$

So, if W is constant over r then $\Delta h_0$ is constant over r. So, hence $h_{03}$ is constant over r. So which tells me that $h_0$ must be constant at inlet and outlet to. So, that is so to maintain these the $h_0$ has to be constant at the inlet and at the outlet to so that this condition is satisfied A. So, this is what you get when you are talking about the free vortex design. So we will stop it here and continue this discussion of the design in the next lecture.