Introduction to Airbreathing Propulsion
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Lecture – 06
Review of Compressible Flows

Okay so let us continue the discussion of the compressible flow this is just we started off and what we are trying to discuss now how this sound wave is generated and how one can estimate the speed of sound. So, this is what we talked about.

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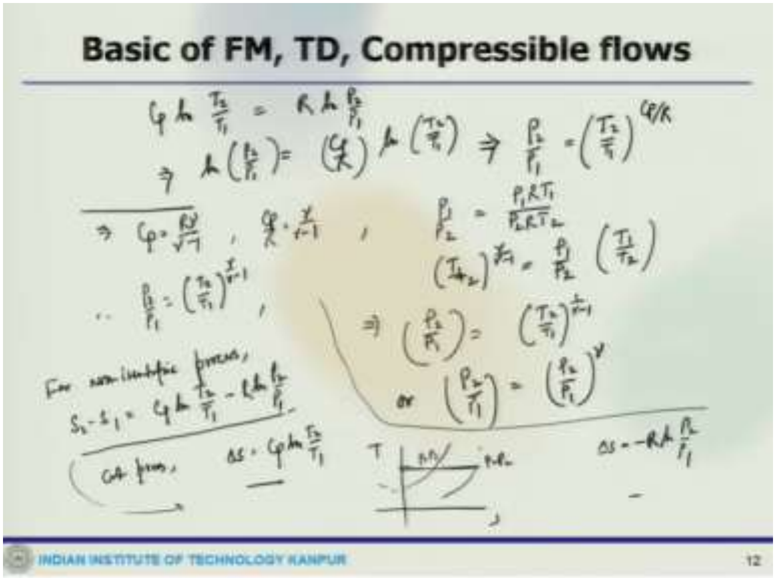

That the speed of sound and the transmission process. So the process is supposed to be an isentropic.

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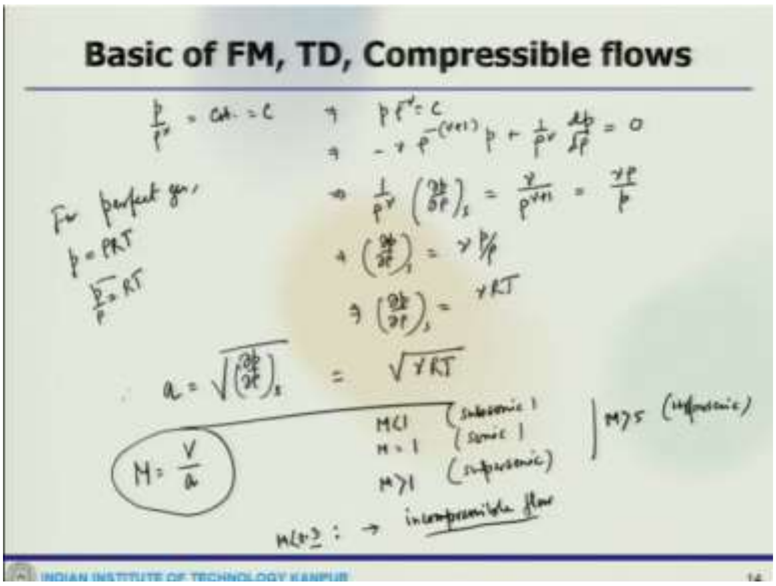

Now since this process is isentropic what one can write that

$$\frac{p}{\rho^{\gamma}} = C$$

so this is $$p\rho^{-\gamma} = C.$$

Now if we take differentiation of this so this would be

$$-\gamma\rho^{-(\gamma+1)}p + \frac{1}{\rho^{\gamma}}\frac{dp}{d\rho} = 0$$

So, taking the difference or differentiating this expression with respect to density so what I get

$$\frac{1}{\rho^{\gamma}}\left(\frac{\partial p}{\partial \rho}\right)_S = \frac{\gamma}{\rho^{\gamma+1}} = \gamma\frac{\rho}{p}$$

So, what we get from here is that that

$$\left(\frac{\partial p}{\partial \rho}\right)_S = \gamma\frac{p}{\rho}$$

again for perfect gas what we can write is

$$p = \rho RT$$

So that means

$$\frac{p}{\rho} = RT$$

So, what we will write that

$$\left(\frac{\partial p}{\partial \rho}\right)_S = \gamma RT$$

okay. So, what was the speed of sound this is

$$\left(\frac{\partial p}{\partial \rho}\right)_S = \sqrt{\gamma RT}$$

Now this is how you can estimate the local speed of sound. Now when you talk about the speed of sound or you estimate the speed of sound immediately there is another important thing which comes in place is that the Mach number.

So, the Mach number is ratio between local speed to the speed of sound and why this number is important because this can categorize the flow field in different zones like for example it could be less than 1 it could be 1 it could be greater than 1. So, when it is less than 1 typically, we call it subsonic flow 1 is sonic flow when it is greater than 1 is the supersonic flow like this.

But now when it goes beyond Mach 5 then we call it is at the hypersonic flow also. So that is what it happens in your scramjet application or the intake is now typically m < 0.3 where in this range the density variation is pretty much negligible and that is why this is where the it assumed to be a incompressible zone or incompressible flows because the variation in density is quite small or rather negligible.

Again mind it we are not talking about incompressible fluid this is in compressible flow because this is a very common mistake that one can do still the even if it is here fluid is compressible by nature but it is in that limit this is becoming in the flow happens to be the incompressible and as I said it could be another could be Mach greater than 5 which is hypersonic flow okay. Now one may think about what the physical interpretation of this Mach number.

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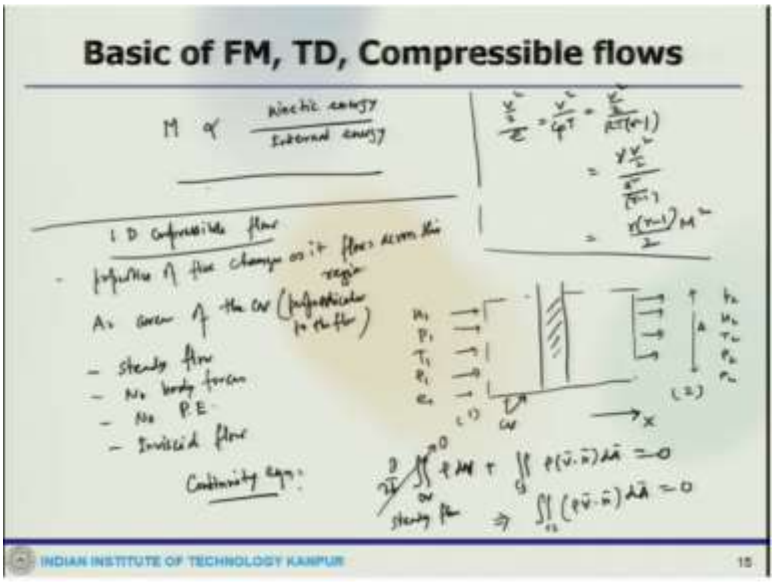

Mach number one can think this is a ratio of two energy, one is kinetic energy the other is internal energy. So how you do that I mean just a simple way to look at that I mean let us say kinetic energy by internal energy so we write

$$\frac{\frac{V^2}{2}}{e} = \frac{\frac{V^2}{2}}{RT(\gamma - 1)}$$

which would be

$$\frac{\gamma \frac{V^2}{2}}{\frac{a^2}{(\gamma - 1)}}$$

So, this would be

$$\frac{\gamma(\gamma-1)M^2}{2}$$

So that is what it happens what I said this is a ratio between kinetic energy and internal energy. So with that we will start the 1D compressible flow. Now let us consider a region here like this okay we draw a control volume okay and these sides there are flows coming in this side this goes out this is one dimensional. So, let us define the coordinate system this could be the area so at the inlet all properties are like P1, T1, rho1, e1 here p2, u2, T2, rho2, e2. So, what we are considering here the flow of a gas through this 1D region.

So, this region could be normal shock wave or region with heat addition. Now the properties of flow changes as each flow across this region. So, let us consider this control volume here which we have drawn and they are between station 1 and 2 and A is the area of the control volume which is perpendicular to the flow then we can have some more assumption like steady flow, no body forces, no potential energy, Inviscid flow.

So, these are the assumption that you can have then what you can write you write down the conservation equation of mass or the continuity equation. So, first thing that you write the continuity equation. So, if I write the complete continuity equation which we have derived now that is how it looks like which is 0. Now there are assumption of steady flow so this goes to 0 because of steady flow. So what it gives us back is the control surface v dot n which is 0.

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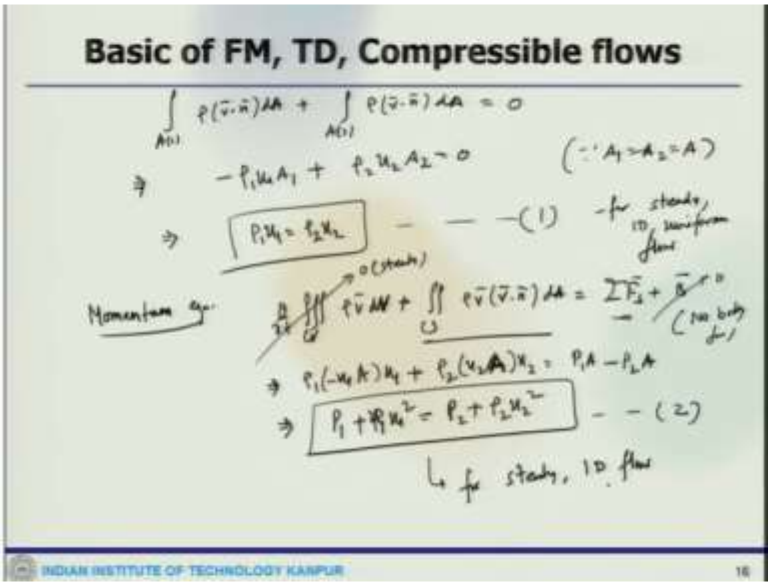

Now so we will write this one at station 1

$$\int_{1} \rho(\bar{V} \cdot n)dA + \int_{2} \rho(\bar{V} \cdot n)dA = 0$$

Now if we go about those drawn control volume so this will become

$$-\rho_1 u_1 A_1 + \rho_2 u_2 A_2 = 0$$

Now the area is same constant area so this becomes

$$\rho_1 u_1 = \rho_2 u_2$$

that let us say equation 1. So, this is an continuity equation for steady 1D uniform flow this is for steady 1D uniform flow that is the continuity equation.

Similarly, we derived the momentum equation so first we can write the complete form which is

$$\iiint_{CV} \rho\bar{V} d\forall + \iint_{CS} \rho\bar{V}(\bar{V} \cdot n)d\bar{A} = \sum F_s + B$$

Now again this goes 0 because of steady this goes to 0 because of no body force so we get this equals to sum of that now this could be the sum of the surface forces. So similarly we will write

$$\rho_1(-u_1 A_1)u_1 + \rho_2(u_2 A_2)u_2 = p_1 A - p_2 A$$

so let us replace this A1 as A and A2 as A so they become consistent so what we get

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

is an equation or the momentum equation for steady 1D flow. So, you see when we write down the complete equation system and then simplify with the assumption this makes things much simpler.

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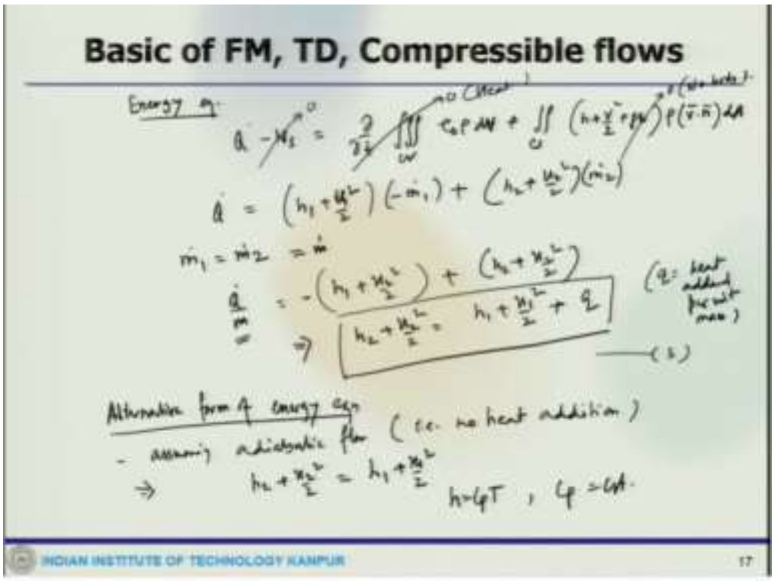

So similarly you could write the energy equation $\rho(\bar{V} \cdot n)dA$ now no factor this goes to 0 this goes to 0 for steady this goes to 0 for no body force. So, this becomes

$$\dot{Q} = \left(h_1 + \frac{u_1^2}{2}\right)(-\dot{m}_1) + \left(h_2 + \frac{u_2^2}{2}\right)(\dot{m}_2)$$

Now from continuity equation we know

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\frac{\dot{Q}}{\dot{m}} = -\left(h_1 + \frac{u_1^2}{2}\right) + \left(h_2 + \frac{u_2^2}{2}\right)$$

then you get

$$\frac{\dot{Q}}{\dot{m}} = -\left(h_1 + \frac{u_1^2}{2}\right) + \left(h_2 + \frac{u_2^2}{2}\right)$$

Now this if we write then I can write that so

$$\left(h_2 + \frac{u_2^2}{2}\right) = \left(h_1 + \frac{u_1^2}{2}\right) + q$$

where q is the heat added per unit mass. So that is equation number 3 this is for energy equation steady one-dimensional flow. Now this is one way to look at the energy equation but there could be an alternative way and which would be quite handy because when you deal with compressible flow maybe the alternative form would become handy let us look at that what is that alternative form of energy equation.

So, let us consider the same one-dimensional flow what it is there and then assuming adiabatic flow which is no heat addition. So, what we get that

$$\left(h_2 + \frac{u_2^2}{2}\right) = \left(h_1 + \frac{u_1^2}{2}\right)$$

because there is no heat addition considering the perfect gas which is

$$h = C_p T$$

where Cp is constant.

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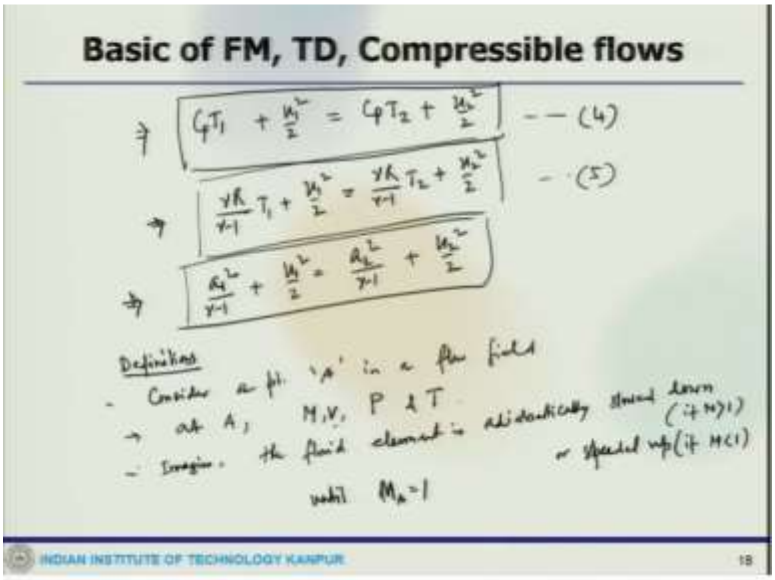

Then what I can write that

$$\left(C_p T_1 + \frac{u_1^2}{2}\right) = \left(C_p T_2 + \frac{u_2^2}{2}\right)$$

so that is let us say equation number 4. So alternatively, one can write

$$\frac{\gamma R}{(\gamma - 1)} T_1 + \frac{u_1^2}{2} = \frac{\gamma R}{(\gamma - 1)} T_2 + \frac{u_2^2}{2}$$

which is equation number 5.

$$\frac{a_1^2}{(\gamma - 1)} + \frac{u_1^2}{2} = \frac{a_2^2}{(\gamma - 1)} + \frac{u_2^2}{2}$$

So that is an alternative way one can write this energy equation.

So, let us have some other definitions in place where we consider a point let us say A in a flow field now at A field element is traveling at some velocity and speed so that M V static pressure is P and temperature is T. So, at the point A which we have considered is inside the flow field the fluid element is traveling at some Mach number and velocity pressure and temperature.

Let us imagine the fluid element is adiabatically slowed down. So, the fluid element is adiabatically slowed down or that is if Mach number at that point A if the Mach number is greater than 1 then it is slowed down or speeded up a Mach number is less than 1. So, this is done until Ma is 1. So that means to reach a sonic speed at A if the upstream fluid is at higher speed then it has to be slowed down but it is done adiabatically if the upstream fluid is at the lower speed or the subsonic then it has to be speeded up to reach that sonic speed.

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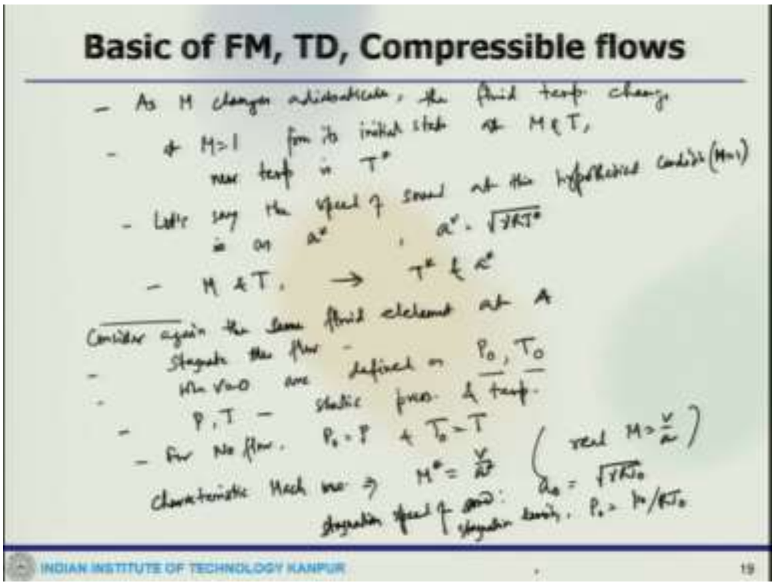

Now what happens as M changes adiabatically the fluid temperature also changes. So when the fluid element arrives at M equals to 1 from its initial state at M and T let us say the new temperature is T^* and let us say the speed of sound at this hypothetical condition which is M equals to 1 as a^* and which can be find out that

$$a^* = \sqrt{\gamma R T^*}$$

Now one can note here for any flow for any flow with Mach number M and temperature T we can associate it values of so T^* and a^*.

So, let us consider again the same fluid element at A now imagine that we isentropically slow this fluid element to 0 velocity that means stagnate the flow. So, which indirectly means isentropically the fluid element has been brought to 0 velocity. Now the pressure and temperature of the fluid element when v is 0 are defined as stagnation pressure and stagnation temperature which is not that means at the velocity 0 condition this is what the pressure and temperature would be these are called the stagnation pressure or stagnation temperature.

Now actual pressure P and T these are called now static pressure and temperature. Now for static flow or no flow for no flow P0 would be P and T0 would be T. So, the characteristics Mach number which is

$$M^* = \frac{V}{a^*}$$

so that the real M is V by a similarly stagnation speed of sound which would be a0 that is

$$a_0 = \sqrt{\gamma R T_0}$$

and stagnation density which is

$$\rho_0 = \frac{P_0}{RT_0}$$

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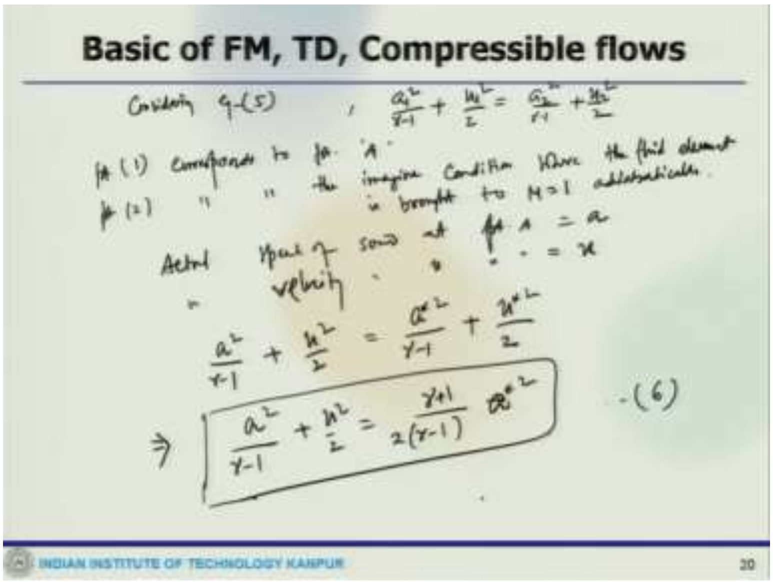

Now considering equation 5 what we can write now

$$\frac{a_1^2}{(\gamma-1)} + \frac{u_1^2}{2} = \frac{a_2^2}{(\gamma-1)} + \frac{u_2^2}{2}$$

where point 1 corresponds to point A and point 2 corresponds to the imagined condition where the fluid element it brought to M equals to 1 adiabatically okay. So actual speed of sound at point A is a actual speed of sound at actual velocity at point A is u so what we can write

$$\frac{a^2}{(\gamma-1)} + \frac{u^2}{2} = \frac{(\gamma+1)}{2(\gamma-1)} a^{*2}$$

So, this is what is equation number 6 that we get. So, for two different points we get this relationship between speed of sound and the a star. So, we will stop the discussion here and continue from where in the next session.