High Speed Aerodynamics

Prof. K. P. Sinhamahapatra

Department of Aerospace Engineering

Indian Institute of Science Kharagpur

Module No. # 01

Lecture No. # 10

One-dimensional waves (Contd.)

Continue our discussion with propagation of expansion wave through a fluid; of course, one dimensional expansion wave of finite amplitude. But, you consider that if we create a pressure disturbance suddenly particularly that is, if we lower the pressure suddenly. Then, the fluid in contact with that lower pressure region will have a ( ) order velocity and at the start or at the beginning there will be step distribution of fluid particle velocity and also a pressure the situation can quite easily be realized. If we consider a cylinder piston system and the piston is run impulsively. Let us say with the speed u p; so, at the start there will be step distribution of velocity and pressure.

However, this and a wave will be created, but since the expansion wave begins to flatten as soon as the wave starts propagating this step distribution will not be remaining and after some time the particle velocity will have a linear distribution and the pressure would have a corresponding distribution which is continuous. Now, as the wave starts traveling to the opposite direction to which the piston has moved, the front of the wave will move at the local acoustic speed other is the acoustic speed corresponding to the undisturbed pressure and density.

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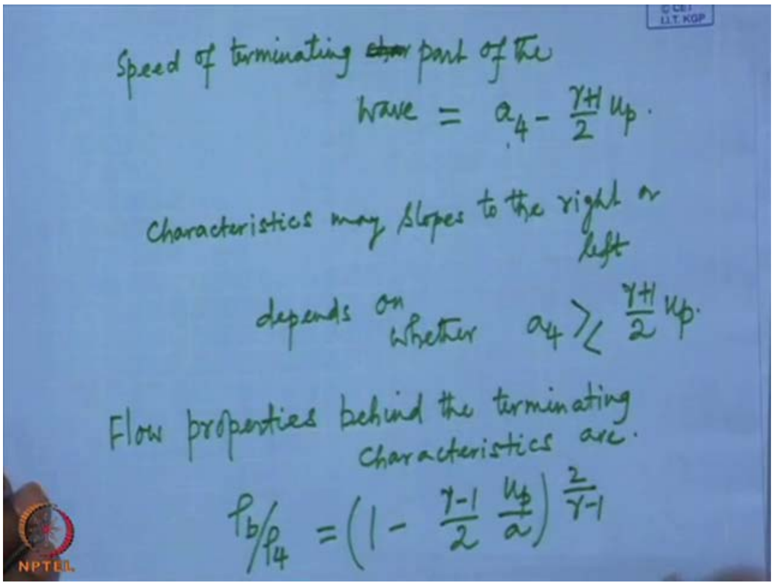

Now, as distance increases the expansion wave or the characteristics become wider because, the expansion wave flattens and these expansion waves will be limited by the terminating characteristics which has the speed as we have derived earlier will be that is the speed of the terminating wave ((no audio 03:05 to 04:04)). However, as we have mentioned earlier that a 4 is the speed of the wave front or which is the acoustic speed corresponding to an undisturbed condition. So, between the turbine written characteristics and the wave front, the speed of the wave continuously decreases. That is, the wave front have the maximum speed of a 4 and subsequent part of the wave travels at a slower speed with the terminating portion traveling by this speed.

Now, these characteristics may tilt to the left or to the right depending upon whether this a 4 is greater than or less than this. That is, characteristics tilt to the left or to the right to the right or left which depends on depends on whether a 4 is greater than or less than gamma plus 1 by 2 u p behind the terminating characteristics the flow properties are uniformed and for a perfect gas, these properties that is flow properties behind the characteristics terminating characteristics are if we denote them by say the behind and front as you have denoted by 4 is 1 minus of course, this relation comes directly from the relations that we have derived earlier for the density ratio for our finite wave amplitude for perfect gas of course, this holds for perfect gas only.

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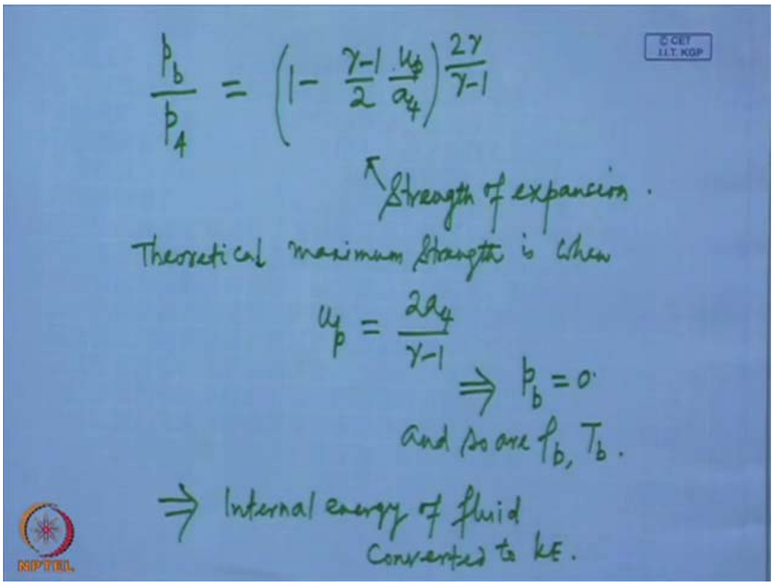

Similarly, the pressure ratio that is pressure behind this terminating characteristics and ahead of the front is similarly given by 1 minus gamma minus 1 by 2 u p by this should be a 4 these are of course, the isentropic relations because in this case the wave always remains isentropic not like the compression wave which reaches a stationary state and changes to a shock wave as we have discussed earlier that isentropic waves becomes flatter and flatter and expansion wave becomes flatter and flatter and they always remain isentropic essentially these are isentropic relations.

This pressure ratio is usually treated as or called the expansion strength. So, this is called the strength of expansion wave and from here we can see that the maximum theoretical maximum strength maximum strength is when and this implies that the pressure behind the terminating characteristics and that is when it is a become behind the terminating characteristics. So, that is the theoretical maximum possible expansion strength and this implies that the all the internal energy of the fluid is converted to kinetic energy that is this implies internal energy of the fluid converted to kinetic energy and…

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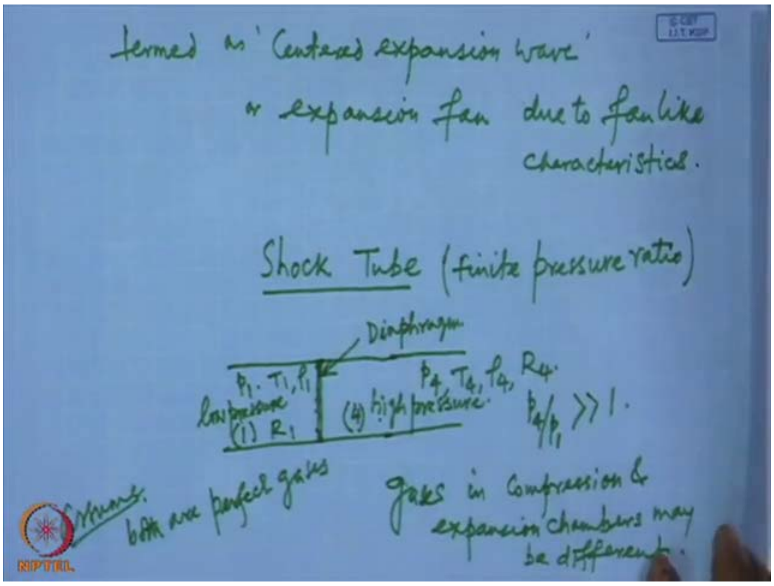

If this u p is higher than that that is in case of that withdrawing piston if the piston speed is higher than that then it will have no further effect on the flow. The wave that has been produced by this impulsive withdrawal of piston is called a centered expansion wave because, the characteristics looks like a fan the expansion process is called, this is termed as centered expansion wave or expansion fan ((no audio 13:05 to 13:41)). So, we have seen how a finite compression wave and a finite expansion wave propagates in a fluid and with this at our disposal we can now go back to a shock wave that shock tune problem, but the non-linear case.

So, we will now go to the shock tube problem with finite we have earlier discussed this shock tube problem where the pressure ratio was marginally greater than 1. That is, the pressure difference across the diaphragm, just infinitesimal the shock tube as we have earlier said is a very simple device which contains a tube separated in 2 parts by a diaphragm and the 2 ends of the tube we are connecting to 2 chambers for the gases are stored at higher pressure. In the earlier linearized case, we considered that the difference in the pressure between these 2 chambers is very small so that, when the diaphragm is removed the disturbance that is created is a very small disturbance; very small and small amplitude waves are created which could be analyzed using the linearized theory.

However what we will now consider is again that same shock tube, but connected by separated by diaphragm where this is connected to high pressure and low pressure. This low pressure region will denote as region 1 and this high pressure region will denote as region 4; so that pressure here is p 4 and here is p 1 and what we call that p 4 by p 1 is considerably larger than 1. So, this diaphragm is separating now to chamber that has high pressure and low pressures which are also called the compression chamber and expansion chamber this p 4 by p 1 is basically the characteristic parameter of a shock tube. That is what we usually control and in addition we may also consider that the temperature at the 2 chambers are different; even we may consider that the gases at the 2 chambers are different having different properties. Let us see in this case consider both the gases are perfect gas. So, gases in compression and expansion chamber may be different chambers may be different.

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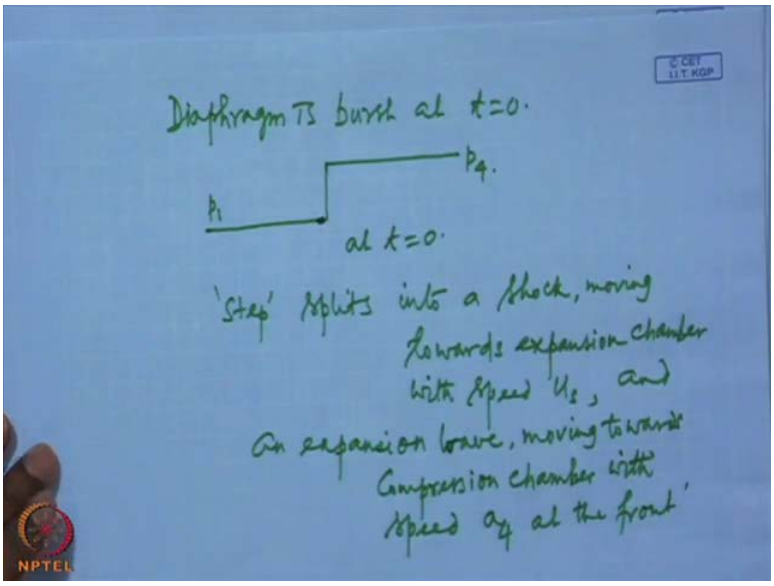

However, we consider both we consider both to be perfect gas with say gas constants r 1 and r 4 that is both are perfect gas, but the properties here, both are perfect gases. Now, let us at the time t equal to 0; we brush the diaphragm and at that instant then the diaphragm is burst at t equal to 0 and consequently the pressure distribution at time t equal to 0 can be a state distribution at time t equal to 0.

However, this step splits into a shock wave which propagates into the expansion chamber and expansion which propagates into the compression chamber, the shock propagates to the expansion chamber with speed this step splits moving towards expansion chamber with speed u s and an expansion wave moving towards compression chamber with speed a 4 at the front (Refer Slide Time: 12:39).That is, coming back to this figure when the diaphragm is burst, we have a discontinuity in pressure. Here, p 1 and p 4 a jump in pressure and consequently a shock wave moves toward of course, it may start as a finite wave compression wave which essentially will convert into a shock wave very soon and then the shock wave propagates towards this expansion chamber with speed u s. similarly, an expansion wave or centered expansion wave also be created here which will move towards this high pressure region where the front of that expansion wave will move with the local sound speed corresponding to the undisturbed condition in this high pressure chamber that is a 4.

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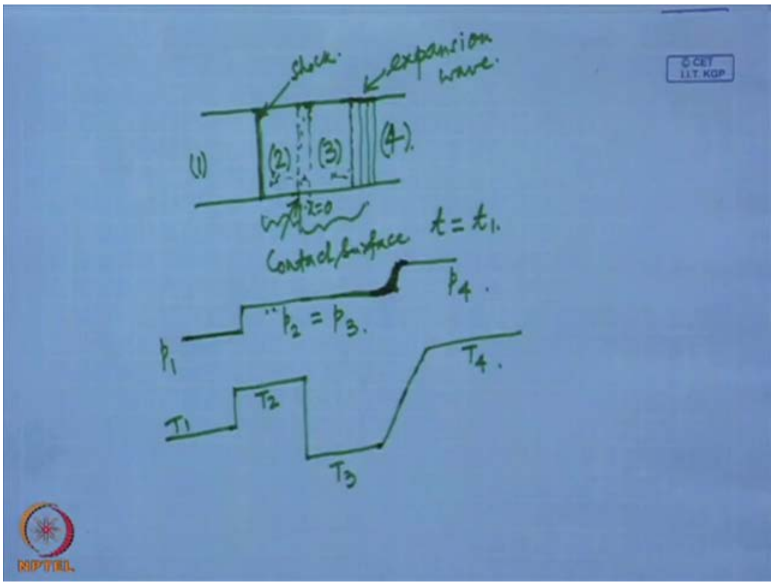

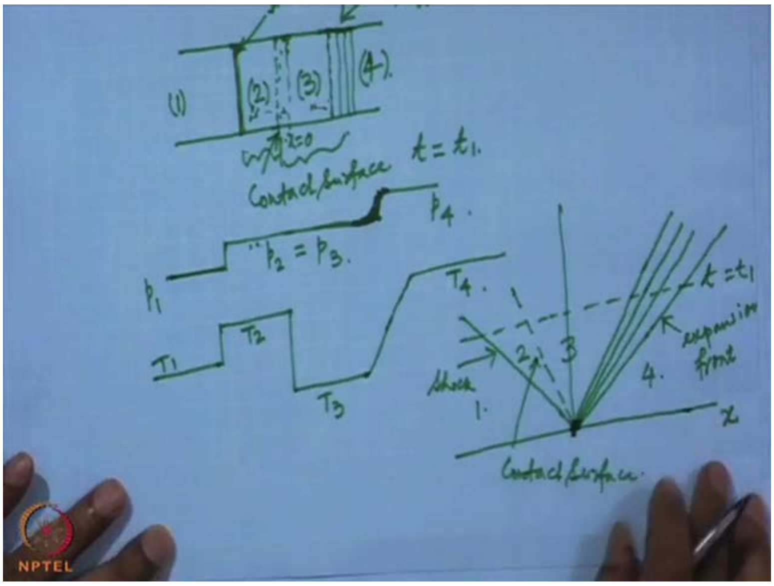

Now, after a little time t 1 the shock has moved to certain distance within the low pressure chamber or the expansion chamber and similarly the expansion has covered a certain distance in the high pressure or compression chamber and what in this sense happens? That, let us say, this shows the original diaphragm location original diaphragm location which we denote as x equal to 0 original diaphragm location through this is what is the original diaphragm location and a shock wave has moved a certain distance and similarly the expansion wave as ((no audio 26:01 to 26:49)).

That is the condition of the fluid which has traversed by the shock we are denoting by 2 and similarly, the fluid which is traversed by the expansion wave, we are denoting by 3 and the interface between these 2 regions. The interface between the these 2 region we call contact surface; that is, this part of the fluid is traversed by the expansion wave while this part of the fluid is traversed by the shock and this interface between these 2 region we called its contact surface. Essentially, it matches the boundary between the fluids which were initially on the other side of the diaphragm. That is, this fluid was initially to the left of the diaphragm. Similarly, this fluid was initially to the right of the diaphragm and since this part of the fluid traversed by a shock wave its entropy has increased while this part of the fluid is traversed by an expansion wave which are essentially isentropic. So, the entropy of this part has not changed it has remained what it was before the diaphragm was burst hence there is a difference in entropy between the region 2 and region 3. So, even though there is no physical barrier between region 2 and region 3 the fluid in region 2 and fluid in region 3 have different properties particularly their entropy are different and usually these 2 fluids do not mix they are permanently separated by this contact surface.

Let us say in this situation the this is at time say we will call it at t equal to sum time t 1 this part of the pressure still remains p 1 in this shock there is a jump ((no audio 29:31 to 30:04)) the temperature distribution can be… remember, the pressure change here is not a jump but, a continuous process ((no audio 30:37 to 31:08)), in terms of characteristics or wave propagation over time we can also depict it like this. So, this is x equal to 0 time t at time t equal to t 1 ((no audio 31:38 to 32:16)) expansion front this is the shock and let us say this might be the contact surface region 1 this is region 2 this is region 3 and this is region 4.

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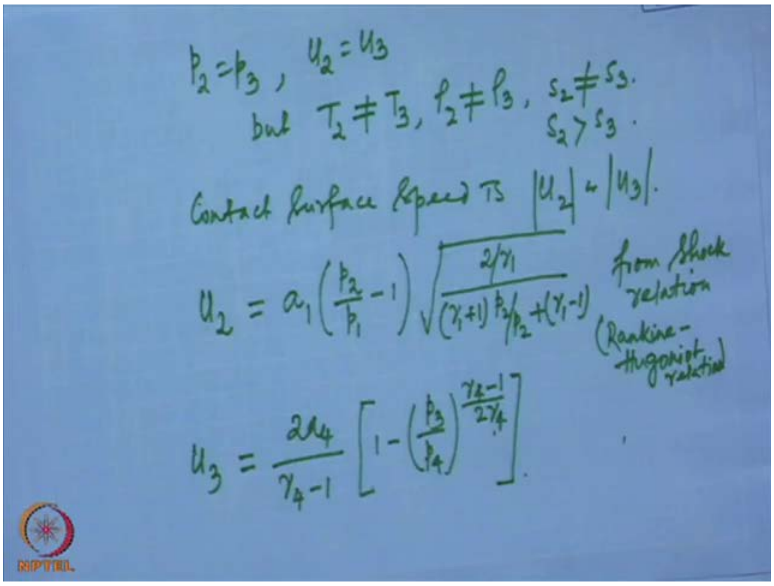

Now, on other side of this contact surface the temperature and density they can be different; however, the pressure and flow velocity they must be same that is on other side of the contact surface denoting by the appropriate symbol 2 and 3 we have p 2 equal to p 3 and u 2 equal to u 3; but, entropy as we have already seen that or other eventually entropy at region 2 is higher than region 3.

Now, the contact surface velocity must be u 2 equal to u 3. Now, if we can find this ratio of pressure between region 1 and region 2 that is p 1 by p 2 and similarly, the ratio of pressure between region 3 and 4 p 3 and p 4 using the shock wave and expansion wave relationship and which we in turn, we can express in the form of velocity. That is, the velocity 2 speeds u 2 and u 3 can be ((no audio 35:48 to 36:27)) from shock relation that is Rankine Hugoniot which we have derived earlier for the moving shock problem.

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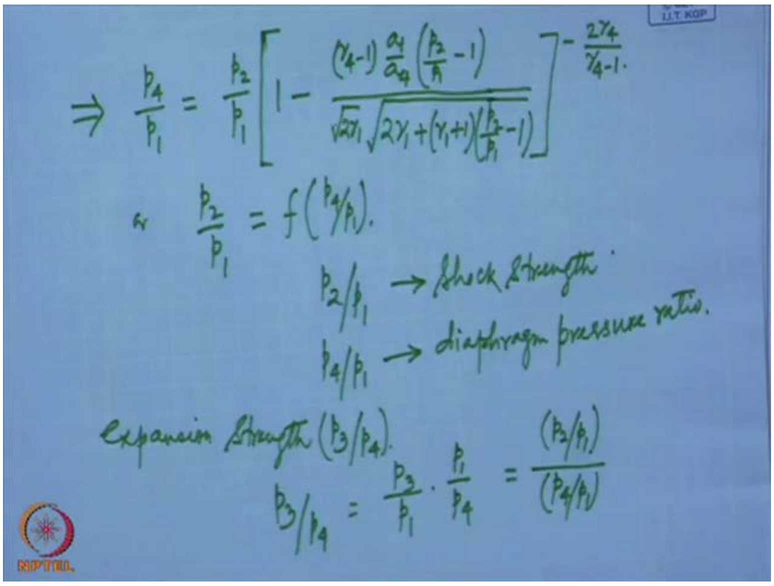

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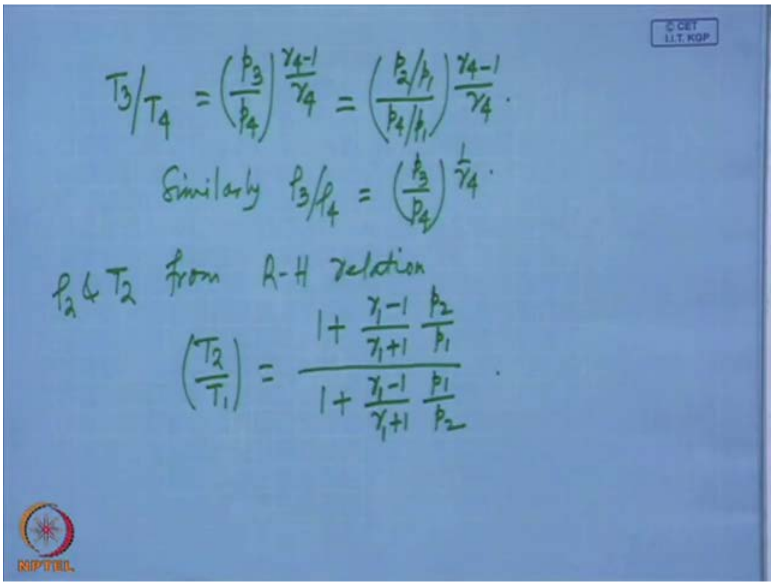

Similarly, considering the expansion part we can write this velocity 2 a 4 by it should be noted that we are considered the possibility of 2 different gasses in expansion and compression chamber which are denoted by gamma 4 and gamma 1 as we have earlier mentioned up r 1 and r 4 that is the specific ratio specific heat ratio of the gas in the high pressure chamber is gamma 4 while that at the low pressure chamber is gamma 1 now since you u 2 and u 3 are same we can equate the right hand side of these 2 relations and consequently what we get is p 4 by p 1 equal to p 2 by p 1 1 minus ((no audio 38:55 to 40:03)) that is the shock strength p 2 by p 1 we have got as an implicit function of the diaphragm pressure ratio as we have mentioned earlier this diaphragm pressure ratio p 4 by p 1 is the most basic parameter and which is in control of the experimenter which is in control of the user and hence it is the basic input for this problem and this p 2 by p 1 we can evaluate implicitly which is now an implicit function of p 4 by p 1. So, p 2 by p 1 is the shock strength is the shock strength and this p 4 by p 1 which called diaphragm pressure ratio the expansion strength the expansion strength p 3 by p 4 this can be obtained as p 3 by p 4 equal to that is the shock strength by diaphragm pressure ratio is the expansion strength shock strength by diaphragm pressure ratio gives us the expansion strength the temperature and density as we have mentioned that in region 2 and region 3 are usually different though the pressure and velocity across the shock and expansion are identical, but the pressure density and temperature they are usually different and the temperature behind the expansion we can obtain using the isentropic relation (Refer Slide Time: 43:23) which is say T 3 by T 4 can be obtained straight away by p 3 by p 4 to the power gamma 4 minus 1 by gamma 4 which have again is shock strength by diaphragm pressure ratio which we've already seen p 2 by p 1 by p 4 by p 1.

Similarly, the density ratio similarly rho 3 by ((no audio 44:10 to 44:42)) the temperature behind the shock t 2 from Rankine Hugoniot relation rho 2 and t 2 similarly can be found using the Rankine Hugoniot relationship and that is t 2 by ((no audio 45:17 to 45:46)). Now, this shock tube can be used for a short duration wind tunnel also and for various studies in shock. How about the duration of the flow? In this case is limited by the length of the expansion and compression chamber since, both these expansion waves and the shock wave they will reflect from the end of the chambers and then eventually interact with each other.

So, after a longtime this simple configuration will not exist within the tube and since in this case the flow speed can be very large or the flow speeds are very large, the duration is usually very small in terms of few micro seconds now this simple relation or 1 dimensional analysis whole squared good as long as the pressure ratio remains moderate; however, for a very large pressure ratio the effect of the starting waves the 3 dimensionality and the dissipative effect of viscosity and heat transfer they come into picture and the relations or the analysis deviates considerably from the real situation; however, as long as the pressure ratio that is the diaphragm pressure ratio remain moderate the relations are quite useful and quite accurate.

So, in what now we can see that if a body moves through a fluid, it creates a disturbance and these disturbances are propagated in an elastic media as compression and expansion wave. We have seen that as long as the disturbances are very small that is the waves are of very small amplitude they behave linearly their shape remains almost undisturbed the shape is permanent the wave speed is continuous wave speed is constant which is the local acoustic speed. However, when the wave is finite the non-linearity comes into picture and because of this non-linearity the different parts of wave travels at a different speed and from our analysis we have seen that for all real fluids the expansion part flattens and the compression part stiffened. As the compression part stiffened the gradients in velocity and temperature they become high. They can no longer be neglected and consequently the distributive action of viscosity and heat transfer or conduction comes into effect and sooner a balance between this dissipative mechanism and the non-linear stiffening effect come into balance and the compression part of the wave reaches a stationary state without any further distortion and becomes a shock wave.

However, the expansion part of the wave flattens because of non-linearity and consequently the gradients become smaller and smaller so that the dissipative actions or diffusive actions can never become considerable and the wave remains isentropic throughout and no ( ) can occur in expansion part of the wave. That is, no expansion shock can occur; shock can occur only in the compression wave. With this, we have finally, analyzed the shock tube relation. Shock tube for finite pressure ratio where we have seen that as the diaphragm is removed a shock travels to the expansion chamber and expansion travels to the compression chamber and the fluid which were behind the… which were on the two sides of the diaphragm. They do not mix; they have, there is, the contact surface is permanent. Of course, where we are considering that the wave are not come reflecting back and interacting. So, this contact surface is permanent and beside the 2 sides in the contact surface, the pressure and velocity are same.

However, temperature density and entropy - they remain different and also we have mentioned that these 1 dimensional relations obtained from a very simple analysis are quite accurate as long as the diaphragm pressure ratio remains moderate. With this we will conclude our 1 dimensional flow for the time being and we have seen that in 1 dimensional, the flow, the compression part becomes a shock wave. Since, in this case the only compatible shock wave is normal shock. ( ) 1 dimensional flow only normal shock, 1 dimensional, in this flow only normal shock can occur. Expansion we have seen that can never reach stationary or never reach a shock like condition; expansion part of the wave or expansion always remains isentropic; thank you.