High Speed Aerodynamics

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Module No. # 01

Lecture No. # 13

Waves and Supersonic Flow (Contd.)

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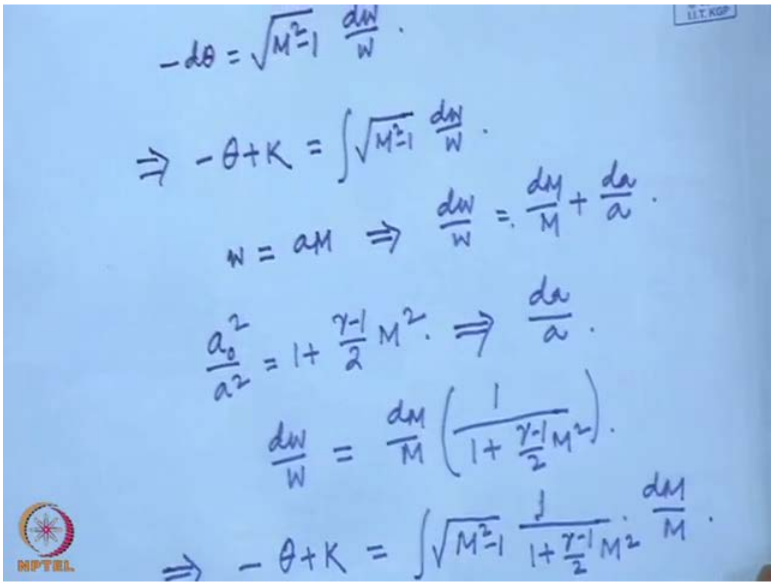

So, continuing our discussion on supersonic expansion and compression, we have seen that for isentropic compression, the relationship is given by minus d theta equal to root over M square minus 1 multiply by d W by W, where W is the flow velocity. Now, this relation holds for all isentropic turning, either compression or expansion. However, the relation changes for finite isentropic compression of… Sorry, for finite compression and that is compression by shock. The usual sign convention for the angle theta is that it is positive for compressive turn and is negative for expansive turn.

Now, integrating this relation, you will find minus theta plus K where K is the constant of integration. Now, to evaluate this integral, we write flow velocity W as speed of sound multiplied by the mach number and which gives us d W by W equal to d M by M plus d a by a. To evaluate these, in turns of the mach number M, we use a naught square by a square equal to one plus gamma minus one by two into M square. This gives us d a by a and substituting these relationship here we obtain d W by W as d M by M into 1 by 1 plus gamma minus 1 by 2 M square. Hence, we have ((no audio 03:33 to 04:19)) and this integration is a function of mach number will write as this.

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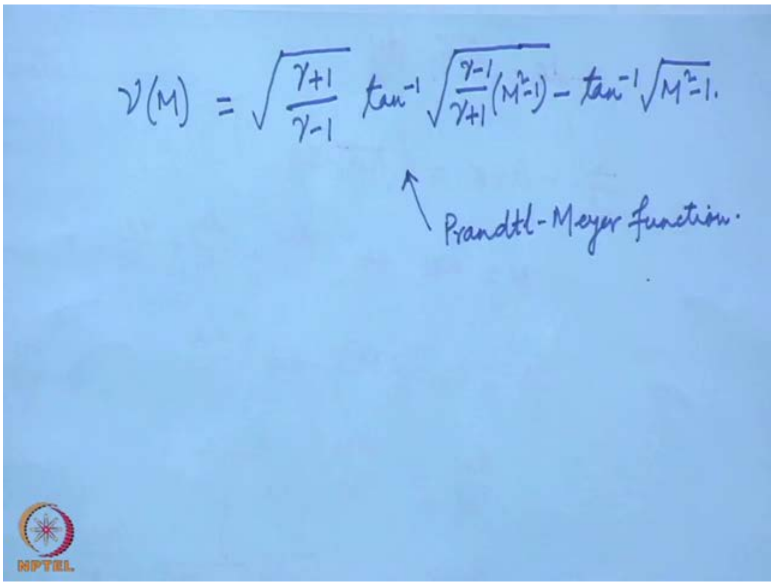

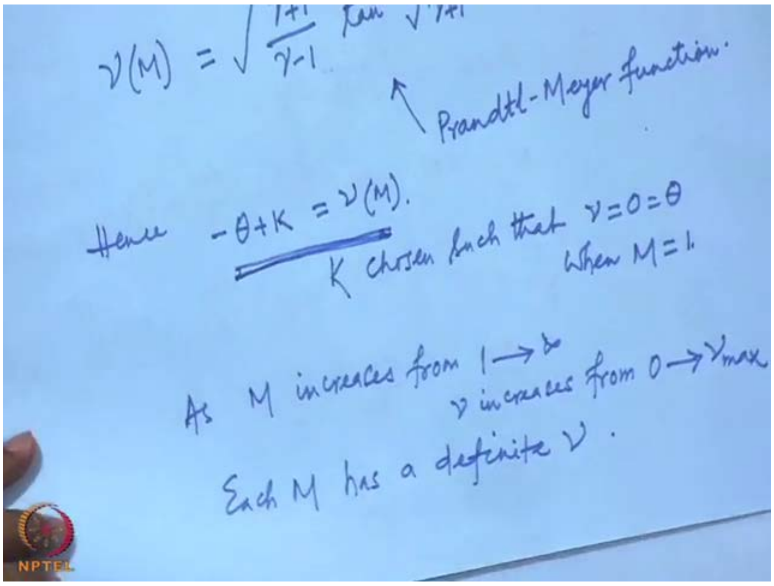

Now, that integral can be evaluated to be this root over gamma plus 1 by gamma minus 1 into inverse tangent of root over gamma minus 1 by gamma plus 1 into M square minus 1, minus inverse tangent of root over M square minus 1. This function is popularly known as Prandtl-Meyer function and as you can see how a unique value for a particular value of mach number or we can say that for each mach number there is an associated value of Prandtl Meyer function nu.

The constant of integration in this relationship or the constant of this integration that is K, is during such that the Prandtl Meyer function, nu becomes zero on M equal to one. This gives ((no audio 06:49 to 07:30)) this and this relation essentially holds for both expansion and compression turn, if the turn is isentropic; that means, it holds for smooth compressive turn and all expansive turn. Remember, that these relations do not hold for compressive turn with soft corner for which we have to use the shock relations.

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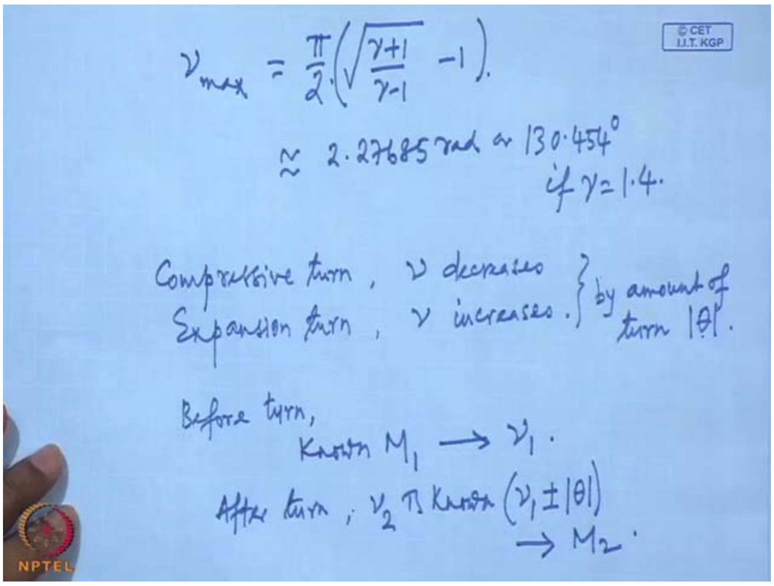

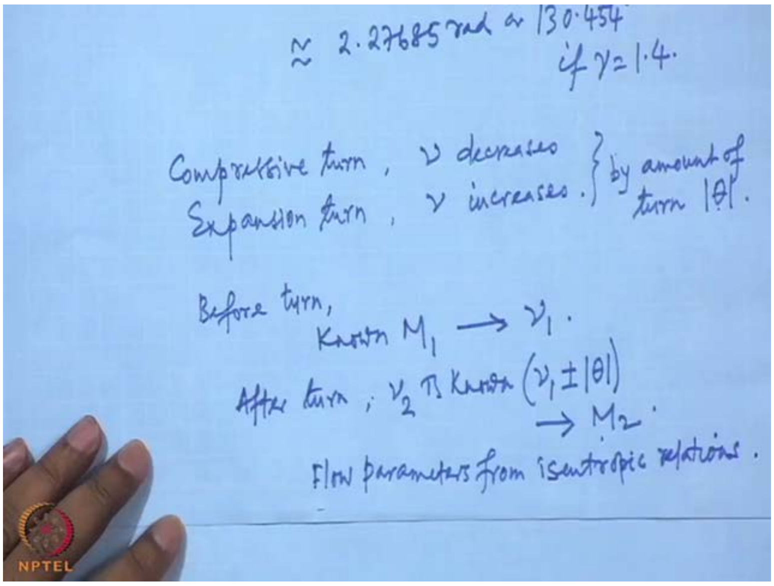

In a compression turn, the mach number decreases and so is nu; nu decreases. While in an expansion turn, the mach number increases and nu also increases. You seen that supersonic mach number is always associated with a unique value or definite value of the function nu. Now, nu is continuously increasing function as M increases from one to infinity; that is as M increases from one to infinity, nu increases from zero to say nu max and each M has a definite nu. We can also see that the value of nu max is again approaches infinity, the function becomes pi by two, that is equal to 2.27685 radian or 130.45 degree if gamma equal to 1.4. We can also see from the relationship that as nu decreases as M decreases, nu also decreases.

In a compressive turn, nu decreases and in expansion turn, the nu increases. Each case, the decrease or increase in nu is equal to the amount of flow deflection or theta. So, if you know the initial value of nu that is knowing the initial mach number M 1, we can find the initial value of nu 1, then the value of nu for a given amount of turn or given value of theta, can be obtained, and this will give us the corresponding M 2, that is before turn (no audio between: 12:12-12:38) we have known M 1 and this gives us nu 1. After turn, nu 2 is known, which will be or decrease or increase by the amount of absolute value of the flow turning (no audio between: 13:12-13:28). This will give us M 2. Knowing M 1 and M 2 using isentropic relationship, all flow parameters can be obtained. So, flow parameters can be obtained from isentropic relationship. (no audio between: 14:09-14:28)

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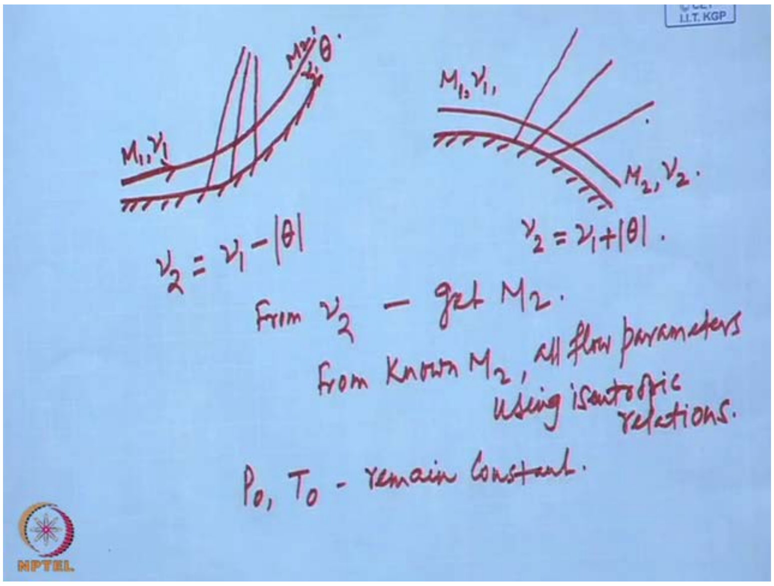

We can still here say that lets consider a compressive turn, (no audio between: 14:43-15:06) we have here M 1 and nu 1 is known, M 2 is known, (no audio between: 15:15-15:28) and nu can be found by the amount of turn. Similarly, if we consider an expansive turn ((no audio 15:40 to 16:26)) and this nu 2, in this case, can be found as nu 1 minus mod theta, (no audio between: 16:36-16:59), so from nu 2 will be getting M 2 and from known M 2, all flow parameters using isentropic relation and since in this cases you are considering only isentropic turns, p 0, T 0, where p 0 and T 0 is the total pressure and total temperature and that remain constant. (No audio between: 18:02-18:19)

Now, the special feature of these isentropic compression and isentropic expansion is that the waves or in this case, straight mach waves with constant condition on each one and by the simple relationship between the flow direction and Prandtl Meyer function. These are the special features of isentropic compression and expansion. That is they are distinguished by constant or straight mach line or mach waves with constant conditions on each one of them and the properties are obtained or computed using the simple relationship between flow deflections a Prandtl Meyer function.

Now, these mach waves as we have already discussed the mach waves belongs to one of the two families that is either plus or minus, depending on whether the wall that produces that mark wave is to the left or right of the flow.

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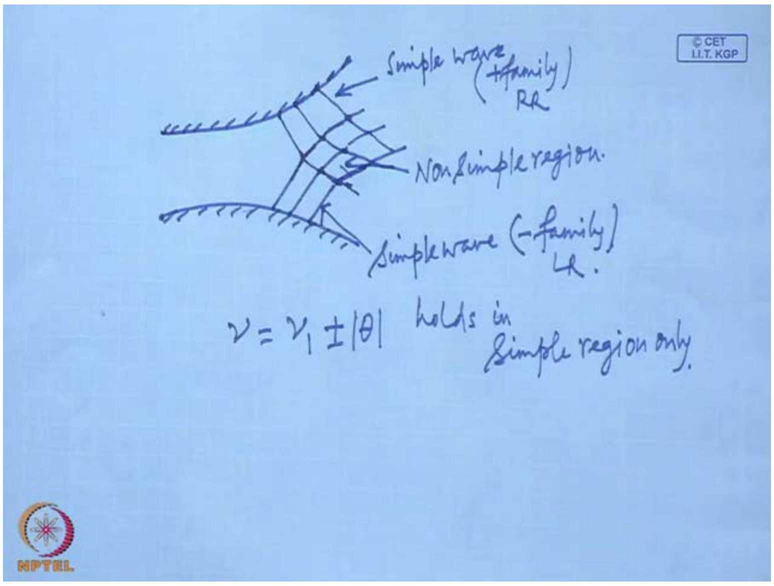

In a region, where two simple waves of opposite family interact with each other, the flow is of course is not simple in the non simple region, the relationship between nu and theta is not the simple one given by nu equal to nu one plus minus theta. So, if we can show it, we can show it by a simple, let us say consider the flow in a duct. ((no audio 20:19 to 21:34))

So, these are simple wave belongs to minus family as they originate from the wall and moves towards left in the flow. Similarly, these are simple waves of plus family or right running family, similarly, these are left running family and these are of course, two sheds of mach wave or mach lines interact. These are all non simple region and the relationship nu equal to nu 1 holds in simple region only. Now, with this we look into the possibility of reflection and intersection of oblique shock.

We have seen that the simple waves intersect. So, similarly, this to oblique shock of finite strength can also intersect and they also can reflect. We will see what happened how and when they reflect.

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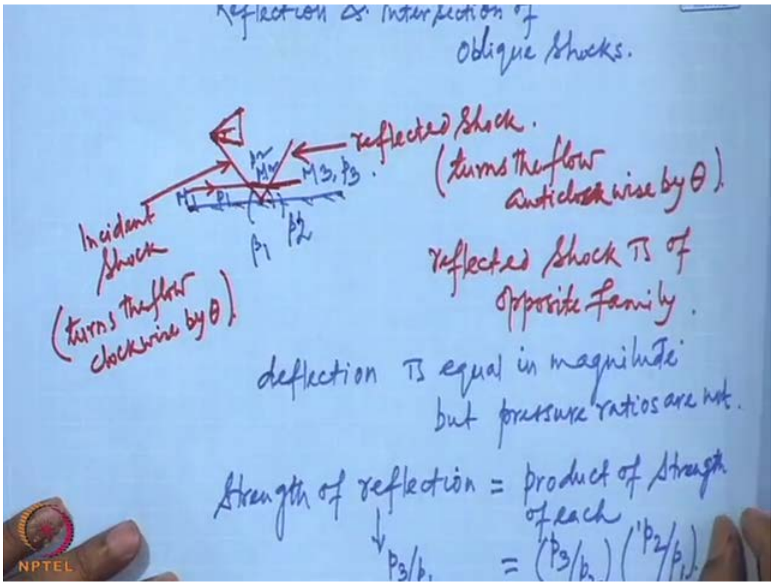

So, first of all, let us consider an reflection of oblique shock from a solid wall; reflection and intersection of (no audio between: 23:57-24:14). Of course, we will consider on some simple situation while reflection and intersection of oblique shock is quiet a vast subject and may be quiet complex. But, first of all, let us consider a simple case or let us say, we have a solid wall let us see what happens to our shock, which is being created somewhere here, may be let us say that we have a wage here, or let us forget about these and a shock is created from here and these shock as it comes it hits this wall.

Now, we see what happens to this shock. We have seen that as flow crosses this shock it turns by an angle equal to the semi vertex angle of this wage and become parallel to this wall. That is a flow, which crosses this shock turns clockwise in this case, by an angle theta. Now, the wall here sets that the flow here must be parallel to the wall. The (( )) boundary condition near a flat wall is that the flow must be parallel to that flat wall.

Now, this shock turns the flow clockwise by an amount theta; obviously, then this flow must again turn by an amount of theta in the anticlockwise direction, so that it becomes parallel to the wall and this can be achieved, if we have another shock, which turns this flow by an anticlockwise theta. So, we see that this shock, which was coming here and hits the wall, is reflected back again as an oblique shock, but which produces an anticlockwise turn of the same amount. So, this is here the incident shock and this is the reflected shock. This shock turns the flow (no audio between: 27:32-27:44) clockwise by say theta. Similarly, the reflected shock turns the flow anticlockwise by theta and it shows that the reflected shock is opposite family (no audio between: 28:24-28:44).

So, we come to the first law or first rule of shock reflection that is for an shock is incident on a solid wall, it reflects back, as an oblique shock of opposite family and pass the flow in opposite direction from the incident shock, but by the same amount that is the reflected shock in turns of flow turning has same strength. However, you can see that even though the deflection produced by this two shock are same; that is deflection is same, deflection is equal in magnitude, but the pressure ratios are not. (No audio between: 30:04-30:19)

Now, the strength of the reflection is given (no audio between: 30:25-30:40) by the product of strength of it, the strength of it which can be written as the strength of reflection and is written as p 3 by p 1 and this will be written as p 3 by p 2 into p 2 by p 1. This is let us say it as M 1, M 2 and M 3; p 1, p 2, p 3.

Now, why this p 3 by p 2 and p 2 by p 1 are not same? You can see that the shocks had same value of theta; however, this M 2 is smaller than M 1. Consequently, the two pressure ratios are also not equal.

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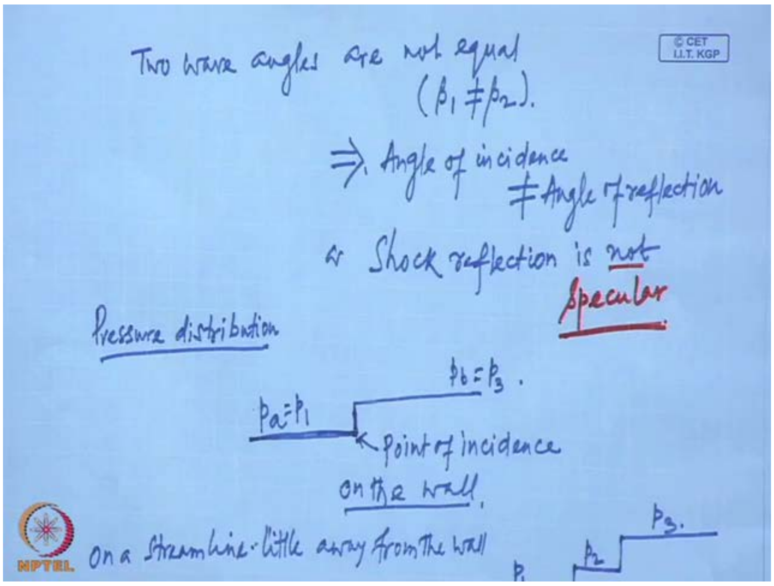

We can also see that the wave angle beta, the two wave angles are also not equal. The two wave angles are also not equal (no audio between: 32:24-32:53) and this shows that angle of incidence and angle of reflection is not equal; that is angle of incidence is not equal to angle of reflection or shock reflection is not specular (no audio between: 33:29-33:51) is not specular. A reflection is called specular when the angle of incidence is same as the angle of reflection, and since we see that in a shock reflection, these angle of incidence and angle of reflection are not same, the shock reflection is not specular. The pressure change, we can see that on the wall pressure distribution, on the wall, the pressure is uniform until the point of contact, where a pressure raises (no audio between: 34:52-35:09) p ahead and p behind.

However, on a streamline (no audio between: 35:17-35:32) on the streamline this is on the wall. On a streamline little away from the wall, we have the pressure ratio p 1, p 2 and p 3. So, what we see here that hound and oblique shock is incident upon a solid wall it reflects back; however, it reflects back as an oblique shock of different family, and this as you can very easily see, from the consideration of the boundary condition, which states that the flow must be parallel to the flat surface or that boundary condition of zero normal flow or tangential flow as usually you call it. So, according to that boundary condition the flow must remain parallel to this solid surface, and to satisfy that requirement, an incidence shock must reflect as an again an oblique shock of opposite family.

However, both will produce a turn of equal amount. One will produce, say a clock wise turn, and then the other will produce anti clock wise turn or vice versa. However, even though this flow deflection is same, the pressure ratio ratios of each of these shocks, that is incidence shock and reflected shock are not same. Since, the shock occurs at different mach number, the incidence shock occurs at mach number M 1 and the reflected shock occurs at mach number of M 2.

Also the strength of the reflection is given by the product of these individual shocks that is incident shock and reflected shock. We also see that the pressure ratios are, since the shock occurs at different mach number, with same amount of deflection, consequently the two wave angles are usually not equal, and hence the incidence angle and reflected angle are also not same, that is this shock reflection is usually not specular. Now, we will see that what happens when two shock intersect. In this earlier example of reflection, shock reflection at the wall, which can be thought of as a streamline and an in viscid flow, and so if we assume that to be the central streamline of the symmetric flow in the inter section of two shocks of equal strength, but of opposite families.

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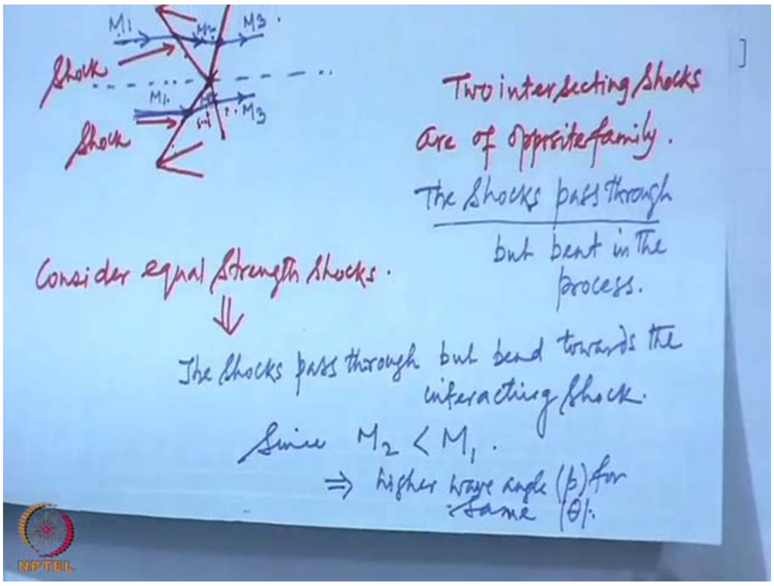

That is say that wall in the earlier case, let us consider that to be a just a central streamline and we can see that t this situation has come because of say (no audio between: 40:03-40:44) like, Here, we see that two shocks have intersected here.

However, as you can see that these two shocks are of opposite family. So, in this case you have two intersecting are of opposite family. One belongs to the plus family or the other belongs to the minus family; other one is left running, the other is right running shock. Of course, the deflection that they produce is of different sense; one produces a clockwise turn the other produces an anti clockwise turn.

Now, if these two shocks are of same strength (no audio between: 41:57-42:12), see the shocks pass through. That is these shocks will continue to move this direction and this shock will continue along this direction; however, they bend slightly. (No audio between: 42:36- 43:04) This we can see and let us say that if we consider these two of same strength. Consider them as equal strength (No audio between: 42:11- 43:26) That is this flow as turn the. This shock has turn the flow by theta in the counter clockwise direction and this shock has turn the flow by theta in the clockwise direction. Considering that if the mach number here was M 1, then after the shock, this becomes M 2 similarly, here also this is M 1 and since we are considering this shock to be of same strength and this is also M 2.

Now, we see that in this part of the flow, the mach number ahead of this shock is M 2 which is smaller than M 1. Consequently, for the same amount of flow deflection, that is same amount of flow deflection of theta at mach number of M 2, the wave angle will be slightly higher. Consequently, this instead of following this path, (Refer Slide Time: 44:48) it will now move like this. It will be bent slightly to that side and this shock, of course, turns the flow in the opposite direction by again of the same amount. So, that it will become parallel to what it was coming earlier.

Similarly, same logic applies here also and consequently the shock here also (No audio between: 45:19- 45:47), and all the flow parameters at anywhere in this configuration can be determined using the shock relations. So, knowing the value of theta here, we can find out what will be the beta here, and then we can find out what is the pressure, density, temperature, in this part of the flow. Again, knowing that mach number here and the flow deflection, by this shock, we can find out what would be the wave angle for this shock, and we can again construct this shock. Finally, again using those shock relations we can find out the properties here. (No audio between: 46:34- 46:44)The shocks pass through, but ((no audio 46:47 to 47:45)). That is this shock (Refer Slide Time: 47:47) pass through this shock; however, because of this passing through, while passing through it bents towards this interacting shock. This happens, since M 2 is smaller than M 1. This implies higher wave angle for same theta.

(No audio between: 48:36- 48:50)

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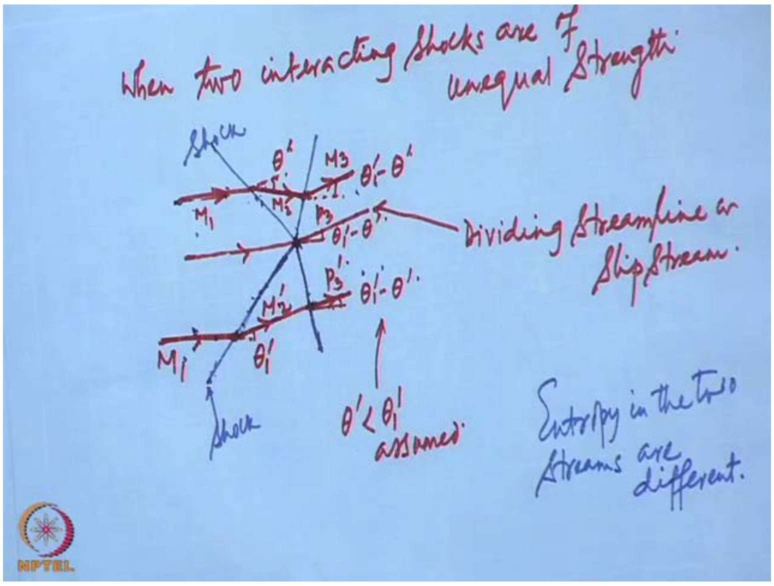

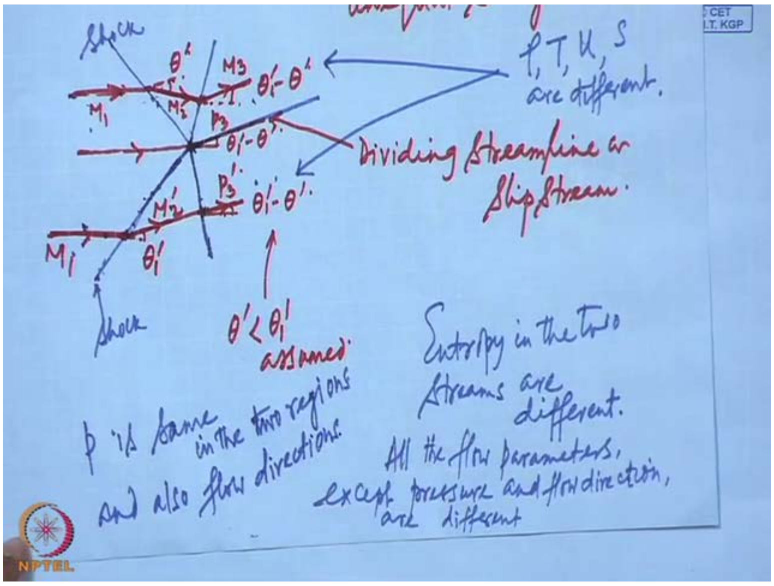

In case these two shocks are of not equal strength. What happen in this case when the two shocks are of unequal strength? ((no audio 48:58 to 49:44)) Let us say that this is the first shock and this is the other shock coming and (No audio between: 50:01- 50:16). Let us consider one particular flow streamline here as M 1.

Now (No audio between: 50:33- 50:45), let us say this turns the flow by an amount theta one prime, while it turns the flow by theta prime, and the mach number here is M 2 prime, here it is M 2. Since, the two shock are of unequal strength, M 2 and M 2 prime will also be different, and the downstream mach number will be smaller for the stronger shock case. Now, these two shocks will of course, pass through. These two shocks will pass through, but as we have seen earlier, they will bend, because now (No audio between: 52:00- 52:22) let us see through this shock, the flow will turn by theta 1 prime in the opposite direction. Now, when theta one prime and theta prime are same, then this flow will be parallel to this. But, since in this case they are of unequal strength they no longer be parallel to the upstream flow direction.

Let us say that theta 1 prime is higher than theta prime that is this shock is stronger. Consequently, this flow instead of becoming parallel will now move slightly to that direction and ((no audio 53:26 to 54:15)). So, we see that this interaction has produced on net amount of turning and in this case towards the upper side. Similarly, for a streamline, which is now crossing through this, will simply undergo a turn by this (No audio between: 54:51- 55:17) This streamline is known as a dividing streamline or slip line. (No audio between: 55:24- 55:51) This is called slip stream, because there is some essential differences between the stream enclosed in this side and enclosed in that side, which you can very clearly see. Consider this particular streamline which is crossing these two amounts of shocks.

Now, as we have assumed in this case, this shock is stronger than this shock. Consequently, the entropy rise across this shock is higher than entropy rise across this shock; however, we can see that both the streamline this as well as this streamline, both of them are crossing in the same two shock; however, the total entropy change that will be experienced by these two streamlines are not same. Here, in this lower up, the undisturbed stream is experiencing the stronger shock at a higher mach number and the lower shock, relatively weaker shock, at a lower mach number. Wwhile in this part the undisturbed stream is experiencing the relatively weaker shock at a higher mach number, but the little more little stronger shock at less mach number. Consequently, the entropy change across these two shocks is not same. Hence, the entropy in this part and entropy in this part of the flow are different. Since, entropy in two part, (No audio between: 57:55-58:15) Similarly, the density change or temperature change experienced by the flow crossing these sides of these two shocks, are different from the density, temperature or velocity changes experienced by the flow in this side of the shocks. Consequently, in these two streams, all the flow parameters except pressure and flow direction and flow direction which are different. That is if we consider this, so we have density, temperature velocity, entropy is different; however, pressure is same in the two regions and also flow direction.

Until, now we have considered interaction of shock waves of same family. Next, we will consider interaction of waves of same family. Earlier, we have considered interaction of waves of different family and next we will consider interaction of waves of same family. In out next lecture.