High Speed Aerodynamics

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Module No. # 01

Lecture No. # 18

Flow in ducts (Contd.)

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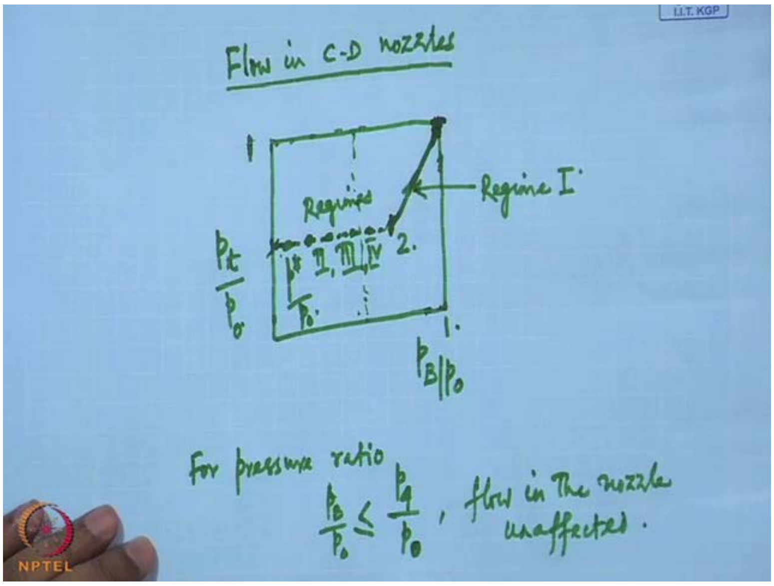

We have seen how the flow in a converging diverging nozzle behaves as we change the back pressure and we have seen, how the mass flow rate? And exit plane pressure varies as the flow develops and the pressure is continuously say decreased from pressure ratio of 1. That is pressure ratio between the pressure at the reservoir and pressure at the back will also another very important parameter.

That also needs some attention is the pressure at the throat. If we plot pressure at the throat with respect to the pressure at the back that is if we plot the pressure ratio p throat by p 0 we have seen that when there is on the back pressure is same as the total pressure that is p B by p 0 1 then there is no flow and pressure everywhere is the same and p t by p 0 is also one subsequently as the back pressure is decreased.

So, is the throat pressure and we have reached the level of p 2 pressure at the level of 2. When the throat becomes choked? or the nozzle become choked subsequent reduction in back pressure does not affect the pressure at the throat and pressure at the throat then remain constant at this level only in terms of our flow regimes, regime 1 where the flow is purely subsonic. So, this represents regime 1 and in regimes 2 3 and 4 the pressure at the throat remain constant at the critical pressure that is sonic pressure ratio p star by p 0 that is when the flow in the nozzle is choked pressure at the throat no longer changes with the back pressure it remain constant at the isentropic sonic pressure corresponding to this flow problem.

Now, we have also seen that for subsonic flow there are infinite numbers of solution, but for supersonic flow, there is only one fully isentropic solution and that is because in a supersonic flow the pressure ratio depends surely on the area ratio. In our discussion we have also seen that when the back pressure reaches a certain value which we denoted at the level 4 then the shock stands at the exit plane and any back pressure which is less than p 4 that is no more normal shock in the duct.

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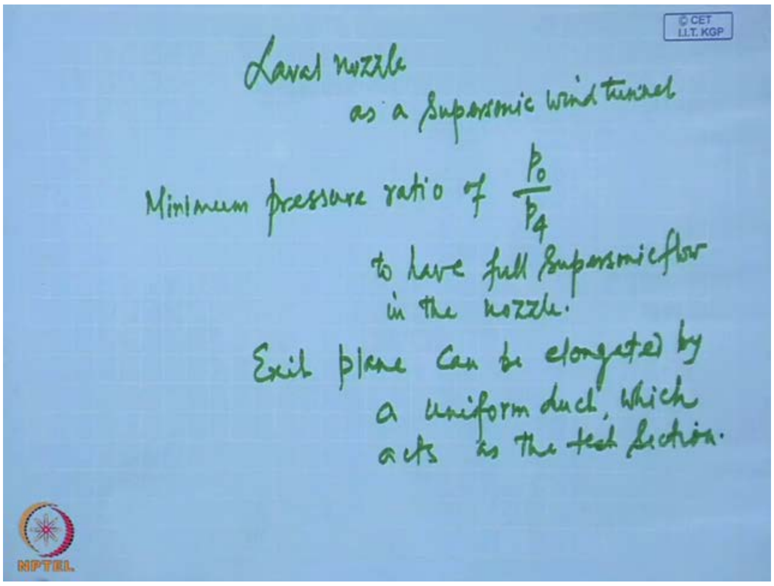

So, for pressure ratio where p 4 by p 0 flow in the nozzle is not affected by flow in the nozzle, unaffected by back pressure and this is used to design a supersonic wind tunnel. So, this Laval nozzle can act as a supersonic wind tunnel. If the back pressure is say set at p 4 or the pressure ratio as p 0 by p 4 then we seen that a shock stands at the exit.

Now, this exit plane we can elongate by a uniform duct and use that as a test section in that case the shock will be at the end of that test section. If any of course, any back pressure value of less than p 4 again an isentropic flow in the duct is available and that can be used as a wind tunnel. However, since this p 4 by p 0 which happens to be the largest of these pressure levels or in terms of p 0 by p 4 which is the minimum pressure ratio.

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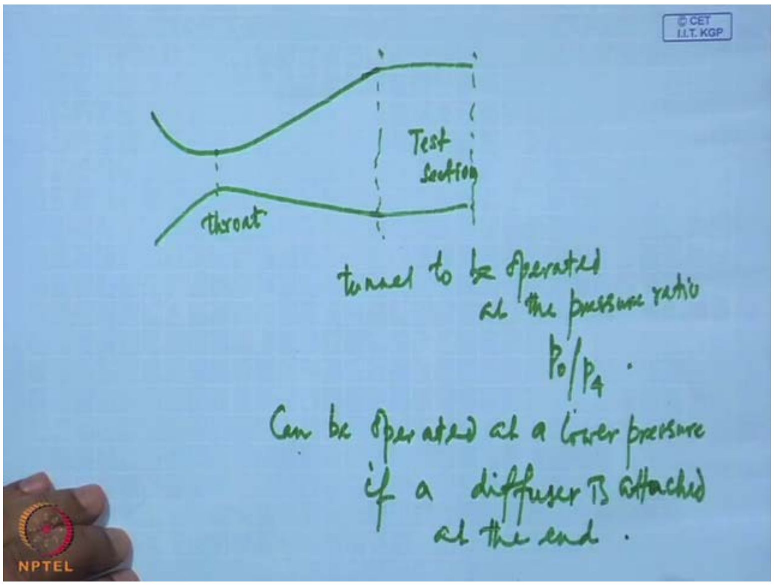

So, that is minimum pressure ratio of p 0 by p 4 to have full supersonic flow in the nozzle and then exit plane can be elongated by a uniform duct, which acts as the test section. Now, a fully isentropic expansion which is corresponds to level p 6 by p 0 since, it is considerably larger than this minimum pressure ratio of p 0 by p 4 it is really not essential to operate the wind tunnel. Because, what is required a uniform supersonic flow in that test section? Which can be attending with this way of pressure ratio? That is p 0 by p 4 the minimum pressure ratio.

So, this now becomes the test section and you see that the entire this part can be called the exit plane this is the throat. Now, if this pressure ratio p 0 by p 4 is maintained continuously. If we have sufficient power to maintain this pressure ratio continuously p 0 by p 4 we can get a continuous tunnel; however, if there is not sufficient power to maintain this pressure ratio continuously, but then we can have a short duration blow down tunnel. This is essentially the principle of designing an open circuit supersonic tunnel.

Now, as we have seen earlier that in a supersonic flow, the Mach number at any particular station is solely depends on the area ratio so, for a given wind tunnel of fixed area ratio. There is only one Mach number at which we will have full supersonic flow throughout the nozzle. Where any other Mach number there will be a shock? that is for only one particular pressure ratio will have one Mach number for which the flow is purely supersonic in the wind tunnel with that say pressure ratio we can fixed area ratio the tunnel cannot be operated at any other Mach number. To operate the tunnel at different Mach number either the area ratio need to be adjusted or the pressure ratio needs to be adjusted.

Now, see that if this supersonic wind tunnel exits directly into the atmosphere or then there is a shock at the exit and there is large amount of pressure loss and for that purpose the tunnel can be operated at pressure ratio. Since, the Mach number is fixed by the area ratio this tunnel will operate at a single Mach number which is called the design Mach number and cannot be run as a fully supersonic wind tunnel at any other Mach number.

Now, if the tunnel directly exits or discharge directly into the receiver then this is the minimum pressure ratio that has to be maintained for full supersonic flow in the test section; however, if we have a diffuser even a subsonic diffuser attached at the exit we can operate the tunnel at a lower pressure ratio that is because, at the end of the test section we will have then the normal shock and the flow downstream will become subsonic and that subsonic flow can theoretically be decelerated isentropically to a stagnation pressure with downstream of the shock .

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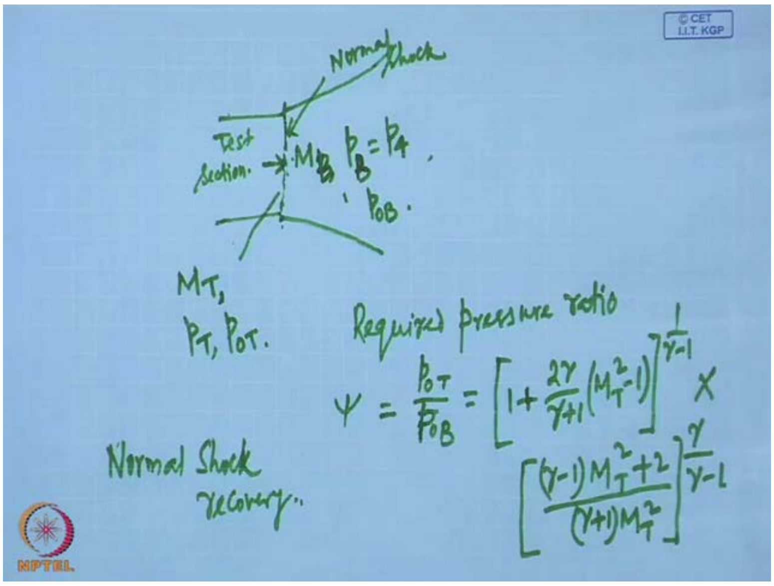

So, it can be operated at a lower pressure if a diffuser attached see what happens then let call this section and test section let us call that the Mach number in the test section is M T, pressure this and let us say the total pressure is this if you remember these are basically the same exit Mach number, exit plane pressure and exit. Total pressure that we have denoted by a substrate earlier now, there is a normal shock here a normal shock and let us say this is the downstream Mach number as before we will denote it by M 2 and this p 2 which happens to be that back pressure p 4 or to have let us say p denote it and p 0 B.

Then, the flow in this diffuser is subsonic and theoretical we can decelerate this subsonic flow isentropically to the stagnation pressure p 0 B and required pressure ratio then the stagnation pressure ratio across a normal shock required pressure ratio will denote it by phi equal to p 0 T by p 0 B which we have derived earlier to be 1 plus 2 gamma by gamma plus 1 M T square minus 1 to the power 1 by gamma minus 1 into gamma minus 1 M T square plus 2 by gamma plus 1 M T square to the power gamma by gamma minus 1.

So, using a subsonic diffuser at the end we can operate the wind tunnel at a pressure ratio which is much smaller? And this type of pressure recovery is called a normal shock recovery. Normal shock recovery; however, this is an ideal pressure recovery as it is quite clear that in our analysis we have used a very simplified model that is an 1 dimensional in viscid flow while the presence in actual case the presence of viscosity and the effect of 3 dimensionality will change this ideal flow into much more complex flow field and consequently this idealistic simplified approach gives us a value which are qualitative and approximate; however, this pressure this normal shock recovery ideal normal shock recovery is practically not achievable; however, this gives a reference that how much pressure recovery we can have? And what are the expected ranges of pressure ratio? In which we can operate for a particular Mach number.

Actually, what happens that due to the presence of viscous boundary layers there will be interaction of between shock waves and due to 3 dimensionality shocks will not be purely normal. So, we have to consider the possibilities of probably shock, shock boundary layer interaction, again shock-shock interaction which is of course, not possible to analyze within this framework of 1 dimensional flow.

In many practical cases instead of a normal shock recovery using a diverging diffuser. What is used a very long duct very long uniform duct? and then, taking the help of the boundary layer development and thickening of the boundary layer and consequently a series of shock interaction a certain amount of recompression is possible and this is a very practical use because, in there are many wind tunnel. Where this type of pressure recovery through recompression by shocks is achieved in practice and even though this in recovery through this shock interaction which is a basically dissipative process; however, in supersonic flow this gives a quite good recovery and it is quite easily achieved.

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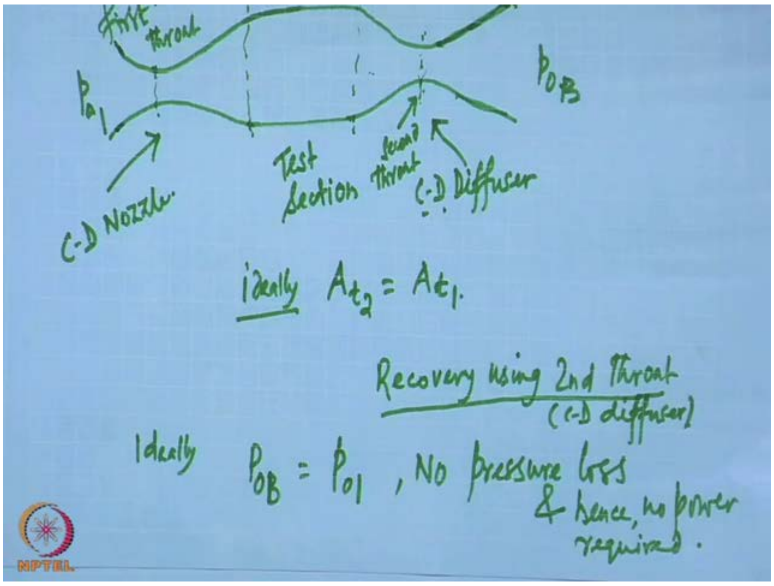

Now, theoretically even a much larger pressure where it can be achieved that is let us say now, we have a supersonic diffuser that is again a converging diverging duct at the end of the test section. That is let us say if we have so, this is a nozzle and this is the test section and this converging diverging duct. Now, acts as diffuser let us say are equal to converging diverging nozzle and this is converging diverging diffuser.

Now, ideally this supersonic flow can be decelerated in a converging duct and again at the throat. This is the second throat and this is the first throat. So, the supersonic flow in the test section can be decelerated in the converging duct and ideally this will now reach to a sonic condition at the throat and then this will further decelerated in the diverging part of the duct. Since, the flow is adiabatic or isentropic everywhere ideally the second throat should have same area as the first throat. So, if we call it A t 2 since the isentropic flow is basically reversible we are just simply reversing the same flow here and as a result theoretically the pressure here the total pressure here and the total pressure here will be same.

So, there is no pressure loss and consequently no power required to drive the same tunnel. So, ideally p 0 let us say p 0 1 and we call this at p 0 B no pressure loss and hence no power required to operate this tunnel this of course, sounds observed that how that can happen of course, in a real case there will be always some pressure loss because of the viscous action and other dissipative mechanism; however, in this ideal case also we see that these tunnel what it shows that it does not need a power to operate it, but; obviously, it must need some power to start.

Now, this starting process of a wind tunnel with second throat is quite complex essentially the same thing happens for other C-D diffuser also; however, now let us concentrate with this what happens to start this tunnel we must create some pressure difference and hence use some power, but essentially the flow at the beginning will not be at the Mach number for which the tunnel is designed initially a subsonic flow will develop and which will then reach to a supersonic case and as the flow develops the first supersonic flow that will develop in the test section will be some or in the tunnel is which belongs to regime 2 that there will be a normal shock in the diffuser or in the diverging part of the nozzle.

Now, once a normal shock develops that is when the pressure ratio is less than p 0 by p 4 or p 0 1 by p 4 then there is a normal shock at the nozzle and now, in the test section then the flow is subsonic and as you know that if the flow is subsonic that will not decelerate here, but it will let accelerate and consequently this when the shock is formed in the diverging part of the first nozzle. Then, the same mass flow that enters through the first nozzle cannot pass through this second converging diverging duct or the diffuser.

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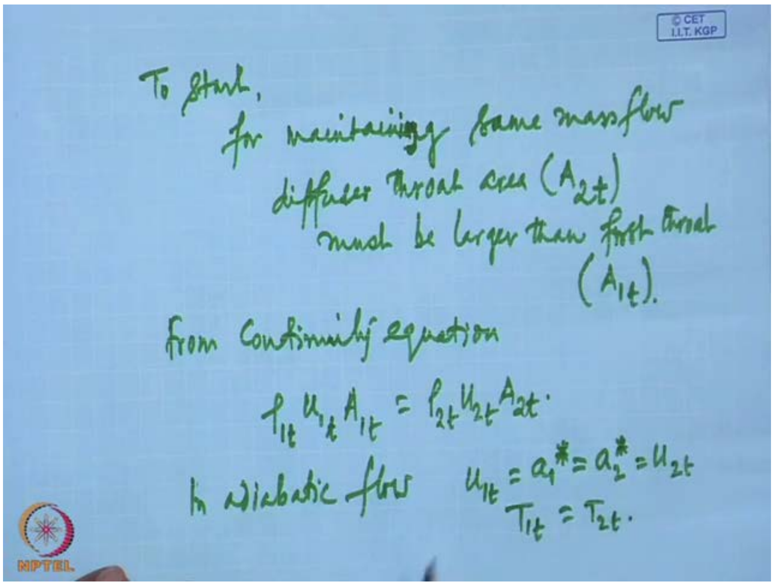

Now, to pass that same amount of mass flow through the diffuser what is required is that the area of the second throat must be larger. This we can see, to start for accumulating same amount of mass flow for maintaining same mass flow diffuser throat area, which will let us call it A 2t throat. Must be larger than first throat, that is A 1 t and this area ratio this happens as we have mentioned that because, before the final flow develops a situation comes where there is a shock in the diverging part of the first nozzle or the first converging diverging duct or we will call them nozzle and diffuser.

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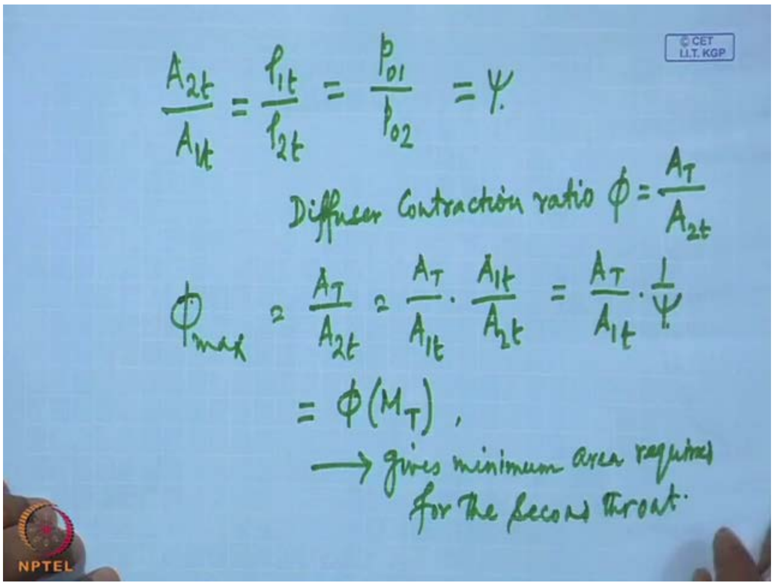

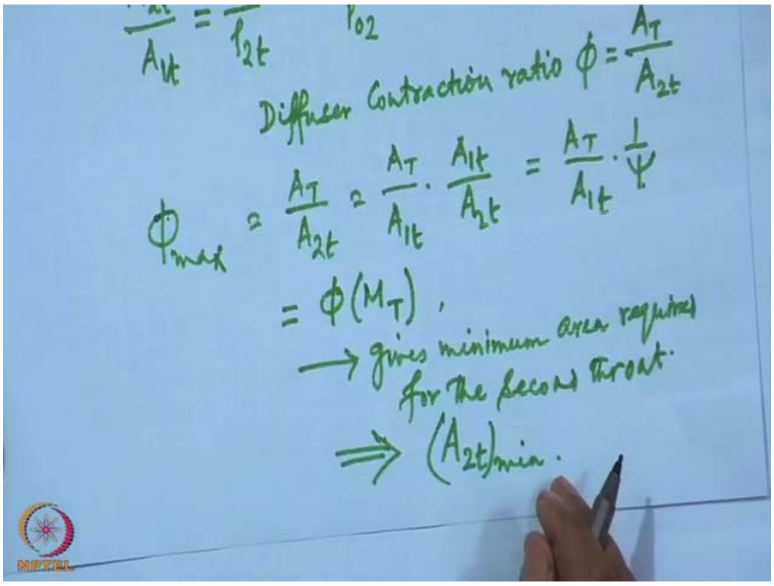

So, there is a shock in the diverging part of the nozzle and consequently a subsonic flow in the test section. Now, from continuity we can see from continuity equation. We can see that, for the same mass flow rho 1 at the throat u 1 at the throat and A 1t is rho 2t u 2t and A 2t. In adiabatic flow, since the flow is adiabatic we have u 1t equal to a 1 star and a 1 star equal to a 2 star that equal to and similarly, also T 1 throat is T 2 throat.

So, this then comes area ratio second throat by first throat and this is what we have derived earlier to be that pressure ratio. We also derived a diffuser contraction ratio phi to be test section area by second throat area. So, we

see this phi max is A T by A 2t that equal to A T by A 1t into A 1t by A 2t and we have already seen that this pressure ratio psi is just a function of Mach number and also earlier we have seen that the area is just a function of Mach number.

So, this is simply a function of Mach number in the test section Mach number. So, this gives what the maximum contraction that it can be allowed or the minimum second throat area that can be allowed. So, that the mass flow same amount of mass can flow through the entire wind tunnel. If the second throat area is less than what is required from this relation. Then, the mass flow that comes to the test section cannot pass through the diffuser and; obviously, there will be a polar motion of the flow and; obviously, a non unsteady unstable flow will be there in the test section to start with.

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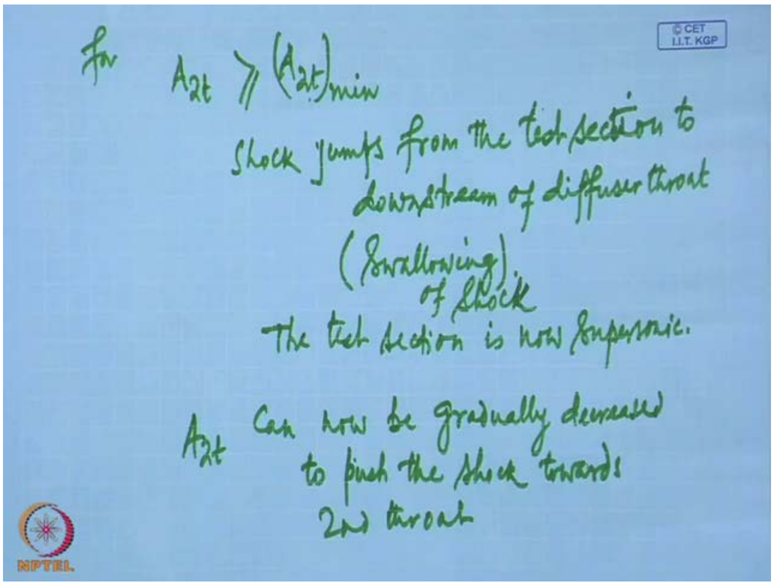

So, to start this second throat must have area larger than what is or at least equal to what is given by this contraction ratio. Now, when this larger second throat area is provided then the shock jumps from the test section to the downstream side of the diffuser throat that is shock when jumps and let us say that we call this to be A 2t minimum. So, for second throat area, shock jumps from the test section to downstream of diffuser throat. This process is known as swallowing of shock; that means, the diffuser has now swallowed this shock from the test section.

Now, the flow in the test section is now fully supersonic; however, part of the diffuser is also now supersonic that is in a sense what we have done is that we have shifted the shock from the test section to the diffuser and the laws of pressure still remains. So, what now can be done is that once the test section flow has become fully supersonic and the shock has been swallowed to the downstream of the test section. Now, the second throat area can gradually be decreased. So, that the shock is now pushed from the diverging part of the diffuser towards the throat and also simultaneously become weaker.

So, A 2t can now be gradually decrease to push the shock towards second throat and ideally when this second throat and first throat area will now become equal the shock will become extremely weak and a mark wave at the throat; however, this is a purely ideal condition and actually that reduction of A 2t from the minimum value to that equal to A 1t is not achievable. However some contraction can be achieved.

Now, this type of starting difficulties or and starting problem occurs in all supersonic diffuser. That is again because; a supersonic duct operates only at a certain Mach number and not at any other Mach number. This problem comes it is this diffuser maybe of pressure recovery diffuser in a wind tunnel or might be the intake of an gas turbine engine and this process of starting difficulty which is known as ‘unstarted’ diffuser or ‘unstarted’ intake, ‘unstarted’ wind tunnel always occurs when there is a supersonic diffusion is required. Ideally, a supersonic diffuser will work perfectly at the design Mach number, but the design Mach number is usually achieved in a continuous process.

So, assuming that if there is an intake a supersonic intake which is design to work at a certain Mach number will have this ‘unstart’ starting problem because for any Mach number less than the design Mach number. The design mass flow cannot go through that intake consequently the intake will speal some of the mass and there will be formation of shock and this shock will not remain stationary it will just move along and unsteady flow will develop; however, if as is discussed in this wind tunnel problem is the area is adjusted properly then at one time the flow will start, but if it is not then the intake may remain un started.

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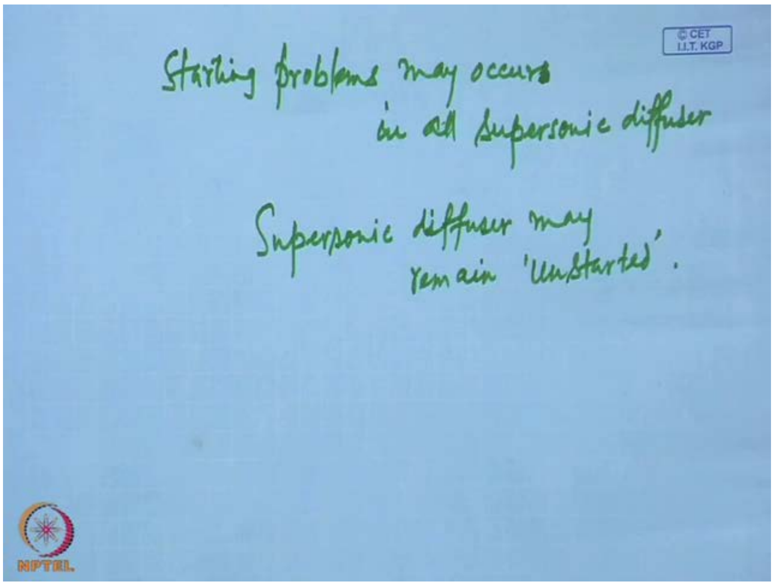

So, these starting problems may occur in all supersonic diffuser and supersonic diffuser may remain ‘unstarted’. So, we have discussed about flow through converging and converging diverging nozzle we have seen that in case of a converging nozzle. The flow can be depending upon the pressure at the exit. The flow can belong to 2 regimes in the first regime the Mach number at the exit is less than 1 and the pressure at the exit is same at the back pressure is not possible to have different exit pressure from the back pressure.

In regime 2, the flow is choked the Mach number at the exit plane is unity and there may be a difference between exit front pressure and back pressure which is the adjusted in expansion outside the duct by unsteady wave propagation mechanism and in a converging duct the maximum flow speed that can be achieved is at the exit plane Mach will to Mach one we have also considered a converging diverging nozzle and we have seen that for a very small pressure ratio that is delivery pressure to the supply pressure for small values of this delivery pressure to supply pressure ratio.

The flow through the converging diverging part of nozzle is purely subsonic where in the converging part the flow accelerates and in the downstream diverging part flow again decelerates; the exit plane pressure is same as the back pressure or the supply pressure. These flows we have designated in regime 1. So, at certain level of back pressure the Mach number at the throat reaches unity and any reduction in the back pressure or supply pressure does not affect the flow in the converging part of the converging diverging nozzle and the converging diverging nozzle is then called choked any pressure ratio below that.

Now, there are different flow regimes up to a certain pressure ratio the flow in the diverging duct portion of the duct will accelerate to some supersonic flow velocity and then if the pressure ratio is not sufficiently low there'll be a normal shock and flow will become subsonic and gradually the pressure will rise to the supply pressure level or back pressure level. As the back pressure decreases a situation comes when this normal shock stands at the exit plane and any further reduction in the back pressure the flow in the duct is not affected the flow in the duct remain as it is however, at the exit plane and the shock that changes.

So, when it reaches to the design of value or fully isentropic or fully expanded supersonic nozzle case there is no shock anywhere either within the nozzle or at the exit plane or even outside which is called fully expanded flow which in our discussion. We denoted by pressure level six and if the back pressure is anywhere between the pressure corresponding to the shock at the exit plane and a fully expanded flow the nozzle is call then over expanded and the further compression takes place in the jet outside the nozzle which is initiated by oblique shock from the lip of the nozzle.

If the back pressure is less than the fully designed level of pressure then the nozzle is called under expanded nozzle and in that case expansion takes place in the nozzle and through series of expansion fan and their interaction which originates from the lips. We have also seen the discussed that this Laval nozzle can be operated as a supersonic wind tunnel and we have discussed the operating principles of a open circuit supersonic wind tunnel for which the pressure ratio maximum pressure ratio that can be used is corresponding to the pressure at which the shock stands at the exit plane and the exit plane can be elongated to make the test section.

Then we have discussed how the tunnel can be operated at a lower pressure ratio. We have discussed about the normal shock recovery and we have also discussed about pressure recovery through a second throat and very simple idealistic discussion of the starting problem of a second throat or supersonic diffuser.