High Speed Aerodynamics

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Module No. #01

Lecture No. #20

Adiabatic Flow in Ducts with Friction (Contd.)

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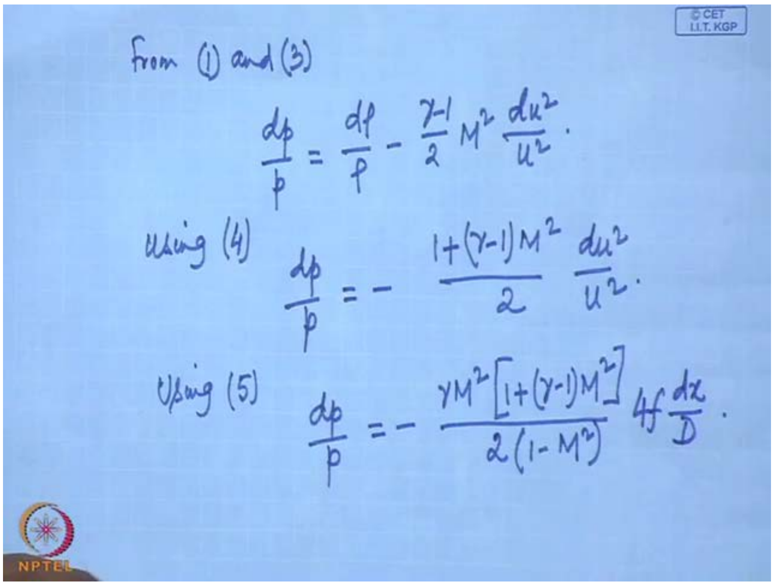

We will today, complete the mathematical solution of adiabatic flow through a duct with friction. We have derived seven equations and mention that in that seven equations, there are eight differential parameters involved and we will solve seven of them in terms of the eighth one and for that eighth one, we have chosen the geometry of the duct and the friction force, because which is the governing or driving mechanism for this flow. We have seen that, we have treated d x by D is the basic differential parameter and associated F we did. So, that f d x represents the friction force and the four u also taken with it for convenience. So, with this seven equations instead of treating them as differential equation, we have treating them as linear algebraic equations and solving for the differential parameters or differential variables.

Now, using that equation number one and three so from equation one, which was the differential form of the equation of state and equation three, which is the differential form of the energy equation. We obtain d P by P equal to d rho by rho minus gamma minus 1 by two M square d u square by u square. Now using equation four, we replaced d rho by rho using equation four we replaced d rho by rho and obtain d P by P equal to minus 1 plus gamma minus 1 by 2 d u square by u square. Now, using five to replace d u square by u square in terms of the friction parameter, 4 f d x by D we get 1 plus gamma minus 1 M square divided by 2 in to 1 minus M square 4 f d x by D.

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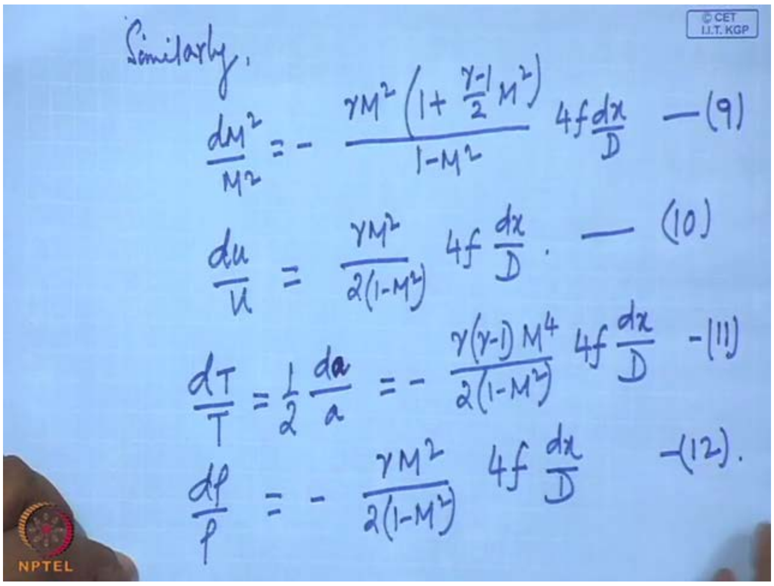

Similarly, we can use various relation to obtain d M square by M square 2 minus gamma M square into 1 plus gamma minus1 by 2 M square divided by 1 minus M square into 4 f d x by D. We number this equation as nine and the earlier equation for pressure we number it at eight. Also we can get d u by u, which is our equation number ten. The temperature differential d T by T which is also related with the speed of sound is gamma into gamma minus 1 into the power 4 by 2 in to 1 minus M square, the density differential becomes ((No audio from 06:49 to 07:19)).

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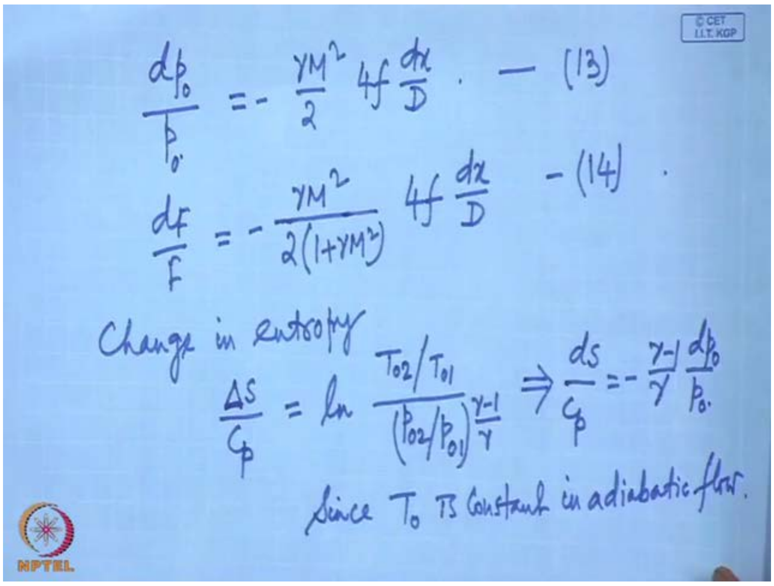

The stagnation pressure become equation thirteen and finally, the impulse function solution is ((No audio from 08:05 to 08:34)) the change in entropy can be evaluated change in entropy can be evaluated as delta S by C p into log T 0 2 by T 0 1 divided by P 0 2 by to the power gamma minus 1 by gamma. Since, you are considering adiabatic flow in this case T 0 2 by T 0 1 is one that is no change in total temperature and consequently we get d S by equal to minus gamma minus gamma by d

P 0 by P 0, since T 0 is a constant in adiabatic flow.

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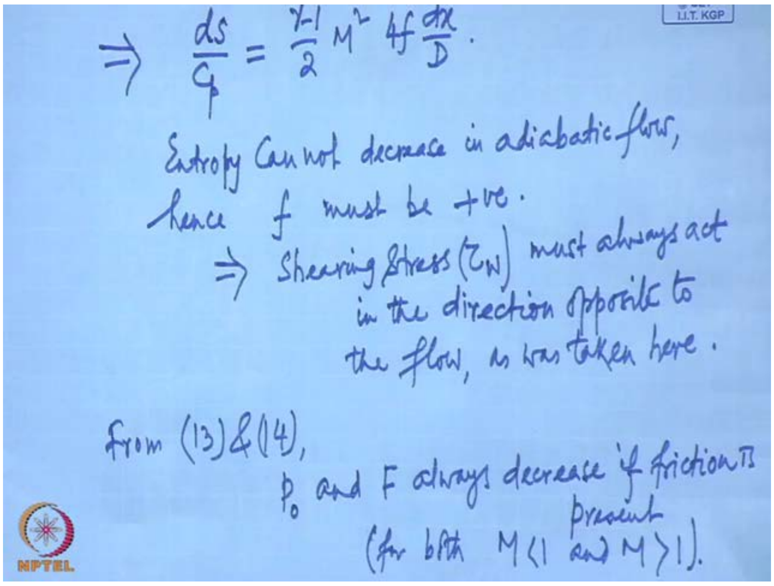

See, if we put d P 0 by P 0 from equation thirteen, if we put P 0 by d P 0 by P 0 from equation thirteen we get d S by C p, if will do gamma minus 1 by 2 M square 4 f d x by D. Now, since entropy cannot decrease in an adiabatic flow this seems that, it must be positive or the shearing stress must act on the stream in a direction opposite to the direction of flow, what you have assumed in our analysis in the beginning. So, entropy cannot decrease in adiabatic flow hence f must be positive, which implies that the shearing stress we achieve assumed tau W must always act in the direction opposite to the flow, as was taken here. And, we can see now from this relation that the isentropic stagnation pressure and the impulse function always decreases, when friction is present irrespective of subsonic or supersonic flow.

So, from thirteen and fourteen P 0 and F always decreased, if friction is present if friction is present that is for both M less than 1 and M greater than 1 and this implies, that wall friction will reduce the effectiveness of all type of flow missionary and it will also reduce the thrust to there is obtain that can be obtained from the jet propulsion devices or the jet nozzle. Of course, jet nozzle is not a uniform area duct however, this behavior remains the same that is the presence of friction will always reduce the thrust, that can be obtained and it also reduces the effectiveness of all type of flow machineries.

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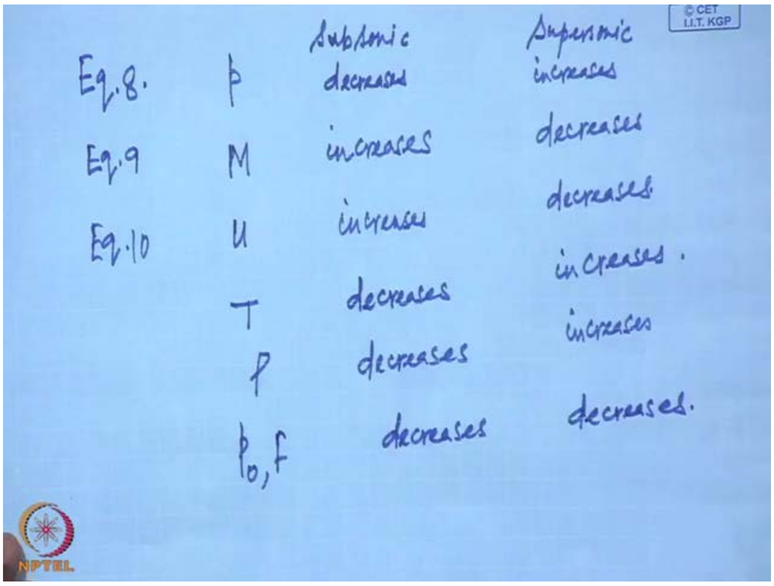

The remaining parameters that is the pressure, temperature, Mach number they; however, wave differently in subsonic and supersonic flow which can be seen from the equations, like equation eight shows P if we make a table for subsonic and supersonic, P decreases in a subsonic flow d P by P is negative and P decreases in a subsonic flow. However, it increases in a supersonic flow d P by P become positive, similarly equation nine shows Mach number d M square by M square is negative ((no audio from 17:56 to 18:26)) and Mach number increases and decreases in this case.

Similarly equation ten shows that, u increases and in supersonic fluid decreases, similarly T d T by T is negative in supersonic flow and this is decreases. However, it becomes positive in supersonic flow and it increases, rho decreases in subsonic flow, but increases in supersonic flow and the stagnation pressure and impulse function they have decreases in both cases as already mentioned. So, these also suggest that Mach number in the flow due to friction, always tends towards unity or sonic flow and consequently their transition from one region to other region is impossible.

That is for a given condition at initial section of the duct, the maximum possible duct length that can be employed without altering the given initial condition or without altering the mass flow rate or without introducing any discontinuity, the maximum length can be that for Mach number at exit is exactly unity. That is when there is friction present in a short duct, where the flow is approximately adiabatic for a given total stagnation enthalpy and that is energy and for a fixed mass flow rate, there is a maximum possible duct length higher at the exit of that duct, the Mach number will become unity.

If you try to increase the duct length, then without altering the initial conditions then the flow will adjust. So, that the mass flow rate is decreased or looking back to our Fanno curve in the h s diagram, that the flow will now shift to a different Fanno curve to the right, which represents all lower amount of mass flow rate.

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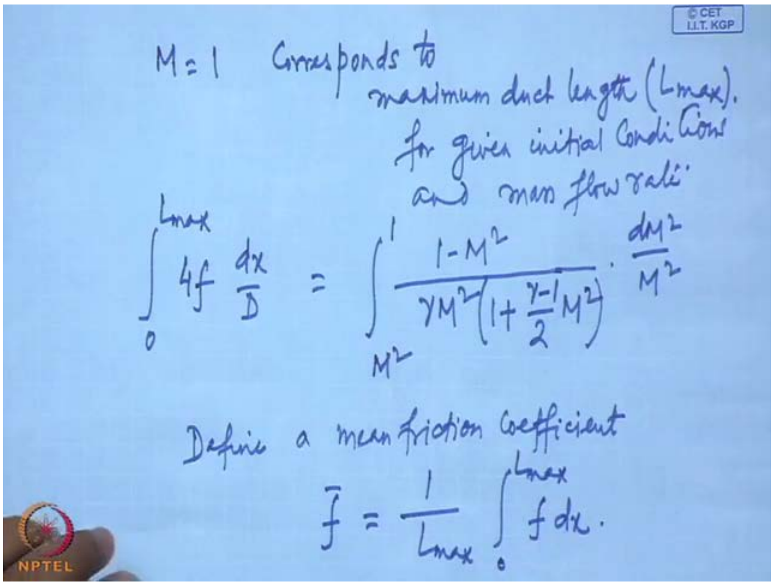

So, this equation can be integrated from a particular station to this exit condition or that is the maximum or the optimum critical Mach number, that is unity where so M equal to 1 corresponds to maximum duct length for given initial conditions and mass flow rate, maximum duct length will denote as L max. So, you can integrate that 0 to L max 4 f d x by D ((no audio from 23:43 to 24:57)). Now, the friction factor in generally will never be available as a function of duct length. However, we can define a mean section coefficient; you can define a mean friction coefficient, mean coefficient friction coefficient as f bar is 1 by L max to 0 to L max f d x.

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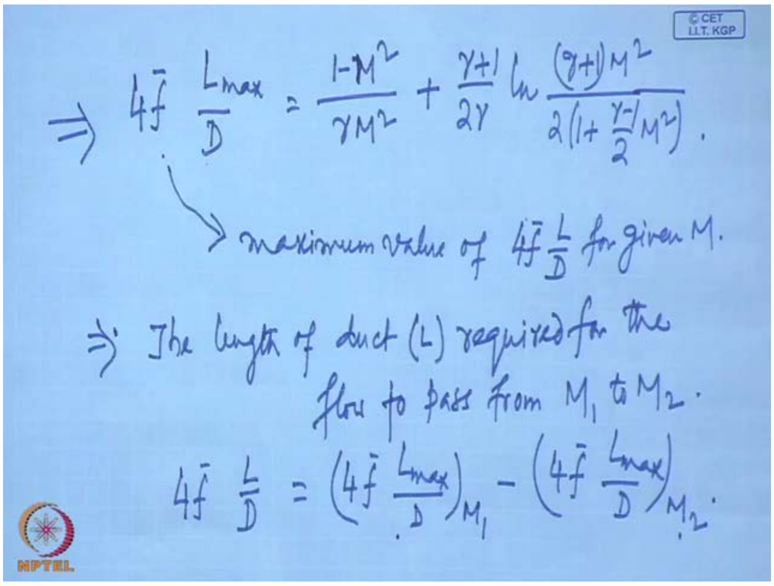

Introducing this into this integration, this relation in the integration and the integration can be competed 1 minus M square by gamma M square plus gamma plus 1 by 2 gamma log of gamma plus 1 M square by 2 in to 1 plus gamma minus 1 by 2 M square and this can also be written sometime in this form that is the maximum value of for given M. Hence, you can write the length of duct L as required for the flow to pass from M 1 to M 2; that is if we have at a certain section flow Mach number as M 1 then what will be the length, where the flow Mach number is M 2 that can be obtained using this relationship writing this for M 1 and M 2 you can write that four f bar L by D, L max by 2.

This we can find that, what would be the maximum length required from off for a flow from M reach to unity from M 1, which gives you which is given by this and the length required maximum length, that you can have for the flow to reach unity from M 2 and the difference of the two will give simply the length required for the flow to reach M 2 from M 1.

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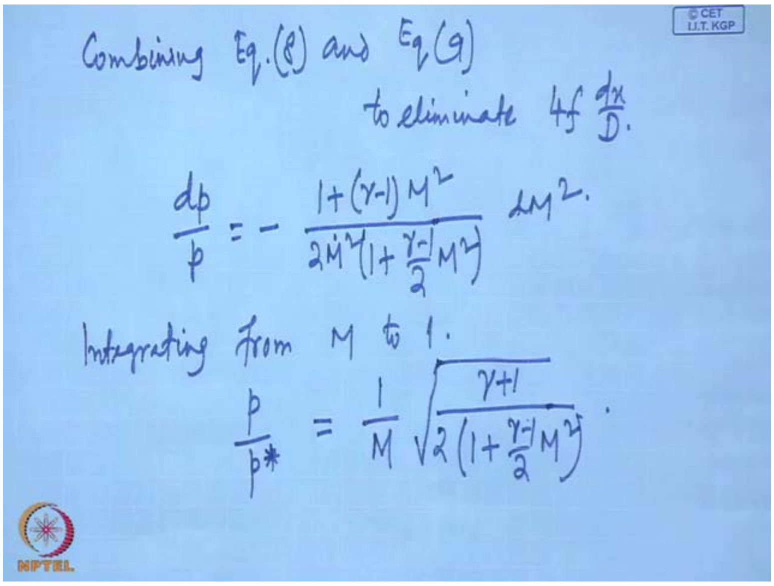

So, we can very easily find what will be the length of a duct required, if you want to accelerate the flow, accelerate in case of a subsonic flow or decelerate in case of a supersonic flow from a Mach number M 1 to different Mach number M 2. If we combine a equation eight and nine, combining equation eight that is d P by P and equation nine that is d M square by M square to eliminate 4 f d x by D, what we get (( )) is d P by P equal to minus 1 plus gamma minus 1 M square by 2 M square in to 1 plus gamma minus 1 by 2.

And again integrating this, we have P by P star where P star with the pressure higher Mach number is unity in this fluid friction. So, this P star as we mentioned earlier is different from the P star, that we obtained when the flow isentropically which is unity, is 2 in to 1 plus gamma minus 1 by 2 M square.

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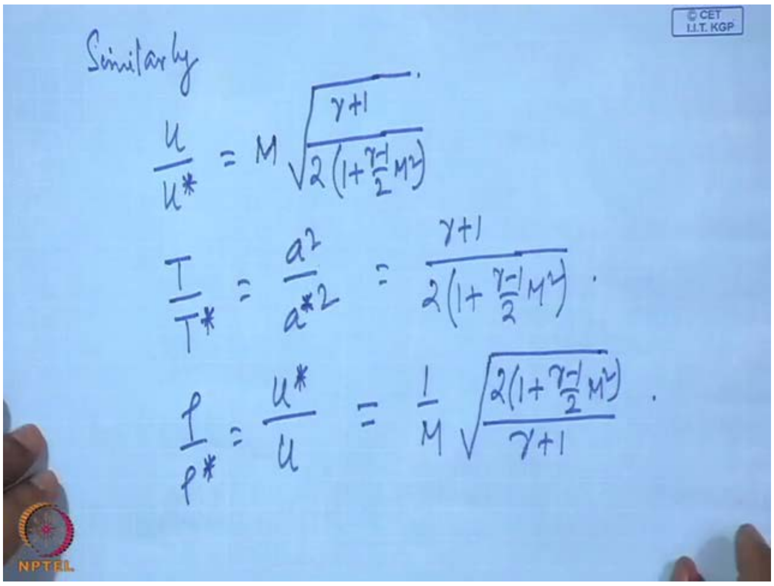

Similarly, using other equations to eliminate 4 f d x by D in favor d M square by M square and then integrating from M to 1, that is we take two or three such equations, two such equations and from them we eliminate 4 f d x by D and in favor of d M square by M square and then integrate the obtained relation from M equal to M to unity. And, we will get for different parameter u by u star aiming to gamma plus 1 divided by 2 into 1 plus gamma minus 1 by 2 M square, T by T star which correspond to a square by a star square, rho by rho star which is from continuity in uniform duct, is two into 1 plus gamma minus 1 by 2 M square by gamma plus 1.

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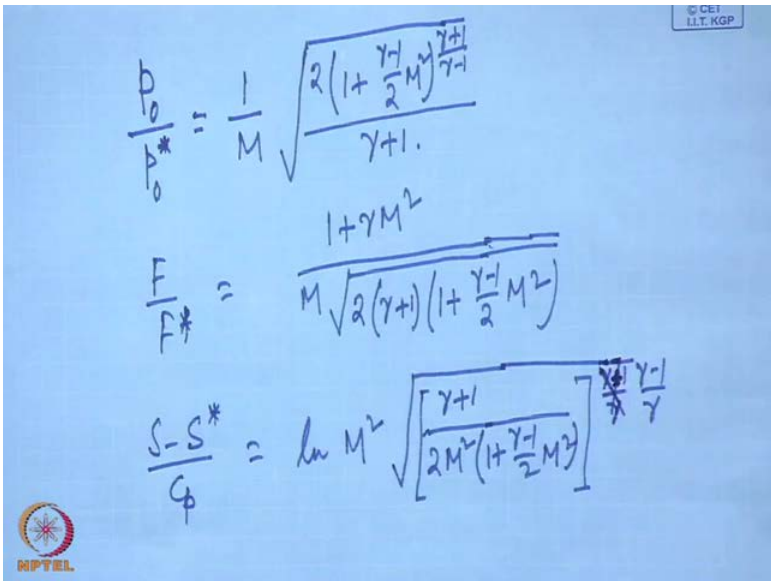

The remaining two parameter that is stagnation pressure, where the Mach number is M divided by this stagnation pressure at the end of the duct with maximum possible length, that is where Mach number is unity is 1 by M 2 in to 1 plus gamma minus 1 by 2 M square to the power gamma plus 1 by gamma minus 1 divided by gamma plus 1. And also F by F star ((no audio from 37:18 to 37:58)) and for the entropy the maximum entropy 2 M square into 1 plus gamma minus 1 by 2 M square whole to the power gamma minus 1 by gamma, clearly gamma minus one by gamma.

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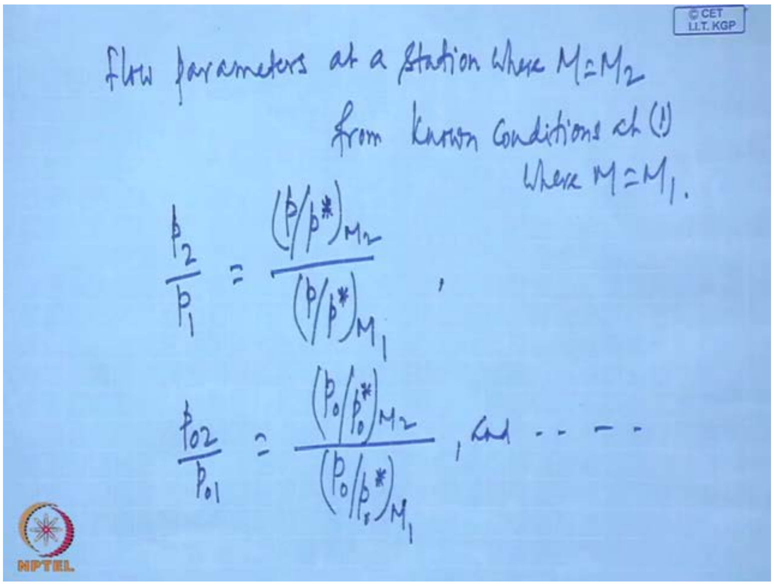

Often, it is important to find the pressure ratio between two stations however, Mach numbers are M 1 and M 2 and that can very easily be obtained from these relations, that is to find flow parameters at a station where M equal to M 2 from known conditions at 1 where M equal to M 1. These relations can very easily be used in this fashion that, say P2 by P 1 can be obtained as P by P star at M 2 by P by P star at M 1. And similarly, say P 0 2 by P 0 1 is P 0 by P 0 star at M 1 and similarly other relations and that completes our solution for flow through a uniformly duct with friction, when the flow is approximately adiabatic that, when the duct is short.

Hence, to summarize this lectures that flow through a uniform area duct with friction, what we have seen that when friction is the driving mechanism of the flow and no shaft work is done, if the duct is short the flow can be approximated as a adiabatic flow. However, if the duct is long then there is enough scope of heat transfer through the mechanism of friction and the flow cannot be treated as a adiabatic, but it can be approximated as isothermal.

With this basic assumptions, we have also assumed that the flow is one dimensional that is we are considering the average flow properties at any particular (( )) we have seen that in the h - s diagram for a given mass flow and stagnation enthalpy, there is a curve of general shape for all fluids and it has two branches. Since in adiabatic flow, the entropy cannot be reduced the flow will always move towards increasing entropy, that is on a particular Fanno curve the flow will always move towards right it cannot turn back that is irrespective of subsonic or supersonic flow, the flow approaches towards sonic condition. Otherwise, that is a subsonic flow accelerates towards sonic flow and if sufficient length is there, that is for a of course, the length is fixed for a given h naught and mass flow rate then at the exit flow can only reach sonic.

Similarly, a supersonic flow also decelerates and maximum of minimum Mach number that it can have is sonic, since the flow is adiabatic total enthalpy and total temperature of course remain constant. However, we have seen that the other parameters of course changes and the change is such that the stagnation pressure always decreases in both cases, whether it is a subsonic flow which is accelerating subsonic flow or decelerating supersonic flow, where stagnation pressure will continuously decrease.

So, will be the impulse function and from which we can conclude that the presents of friction in a duct will reduce the overall effectiveness of the fluid machinery or flow machinery. The other parameters of course, behave differently in different flow regime in a subsonic flow which accelerates towards sonic, the pressure, temperature and density they are they decreases. However, Mach number and flow velocity increases while in supersonic flow the opposite happens, that is the pressure density and temperature increases over Mach number and velocity decreases.

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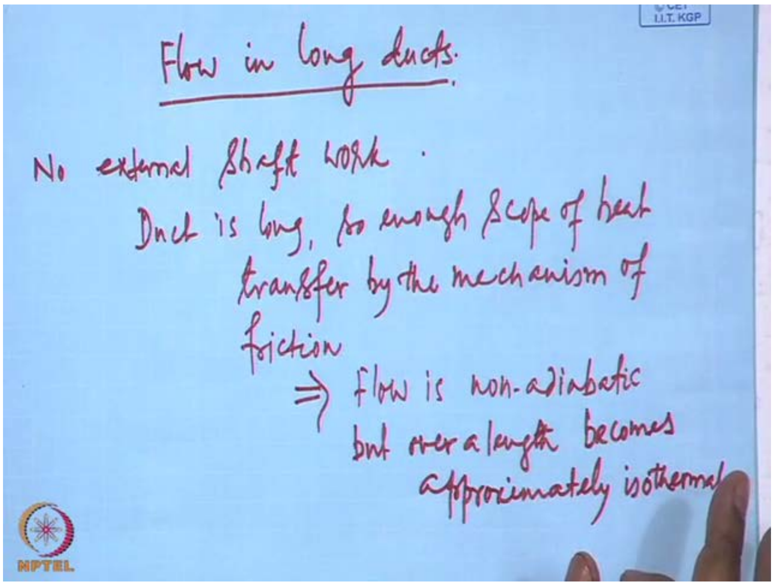

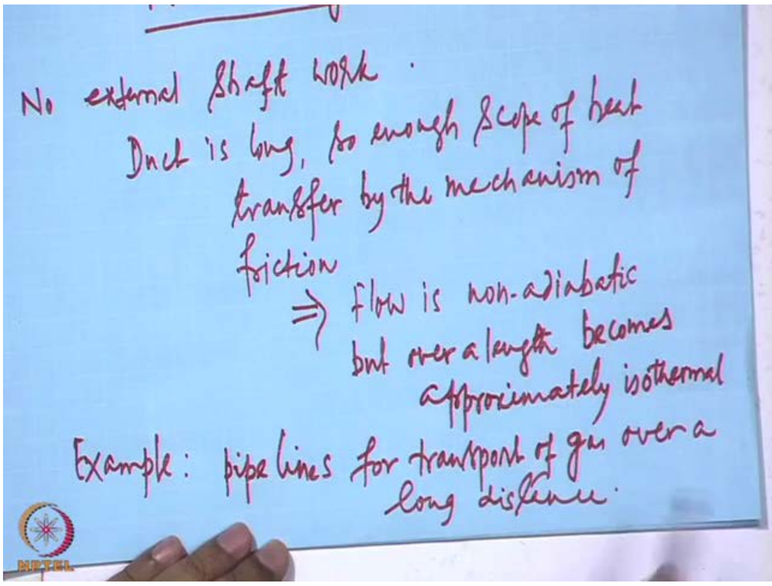

We will now consider or next we will consider, flow in long ducts next will consider flow in long ducts and as we have mentioned earlier there is again, we will consider no external shaft work. However, duct being long so enough scope of heat transfer by the mechanism of friction ((no audio from 48:25 to 48:58)). So the flow is non-adiabatic, but over a length it becomes approximately isothermal; over a length the flow becomes approximately isothermal and this type of flow is quite common in material gas transport pipelines for transport of gas over a long distance however, we will continue this discussion in our next lecture.